ENERGY STORAGE

- Apply a constant voltage to two parallel conducting plates and wait a while

- What do we have?
  - no current flowing
  - positive charge on top plate
  - negative charge on bottom plate

  ⇒ electric field across the gap.

- Now remove/disconnect the source
  - nowhere for charge to flow
  - electric field remains
  - energy stored in potential of charges.

- How to use this?

Interesting idea! Let's try to model it so that we can use it.

Idealized setup

- infinite plates
- perfect conductors
- perfect insulator
- perfectly uniform dielectric

In that case,

charge stored $\propto$ voltage applied

Actually,

$$q = CV$$

$$C = \frac{\varepsilon A}{d}$$

ε = permitivity
A = area.
d = separation

CIRCUIT MODEL

$$q(t) = C(t)\,v(t)$$

what about current?

$$i(t) = \frac{dq(t)}{dt} = \frac{d}{dt}\left[C(t)\,v(t)\right]$$

If capacitance is constant,

$$i(t) = C\,\frac{dv(t)}{dt}$$

So what is $v(t)$? Assume $v(t)\big|_{t=t_0}$ is known.

$$v(t) = V(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)\,dx$$

Sometimes choose $t_0 = -\infty$
and assume $v(-\infty) = 0$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(x)\,dx$$

The idea was to store energy

How much did we store?

$$E(t) = \int_{-\infty}^{t} p(x)\,dx.$$

where $p(t)$ is power.

$$p(t) = v(t)\,i(t) = C\,v(t)\,\frac{dv(t)}{dt}$$

$$\Rightarrow \quad E(t) = \int_{-\infty}^{t} C\,v(x)\,\frac{dv(x)}{dx}\,dx$$

$$= C \int_{v(-\infty)}^{v(t)} v(x)\,dv(x)$$

$$= \frac{1}{2} C\,v(x)^2 \Big]_{v(-\infty)}^{v(t)}$$

If we assume $v(-\infty) = 0$,

$$E(t) = \frac{1}{2} C\,V(t)^2$$

* Sometimes desirable + engineered
* Sometimes undesirable + to be engineered away

PROPERTIES

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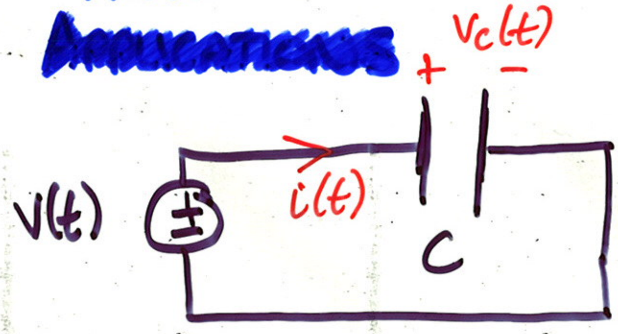

$$i(t) = C \frac{d\,v_c(t)}{dt}$$

* what if $v(t)$ is constant?

- current = 0, no matter how big $v(t)$ is

⇒ open circuit

- have you seen this idea in use?

* what if $v(t)$ changes rapidly?

- $|i(t)|$ is large, even if $v(t)$ is small

→ short circuit

* $$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(x)\,dx$$

what does this tell us about $v(t)$?

Continuous