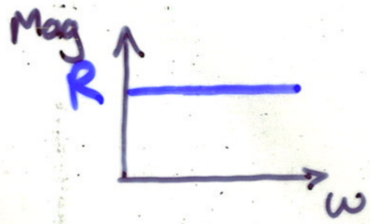
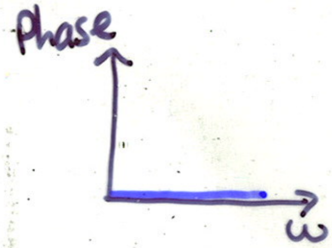
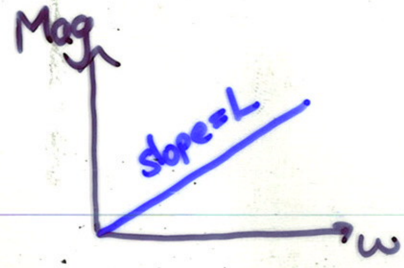
FREQUENCY RESPONSE

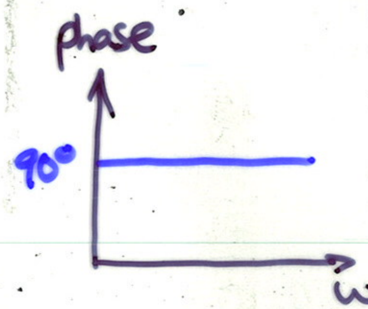
$Z = R$



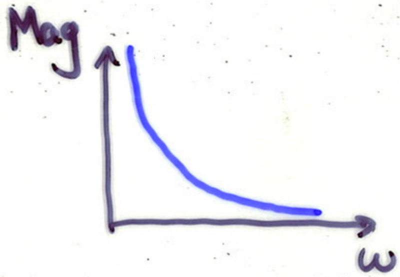


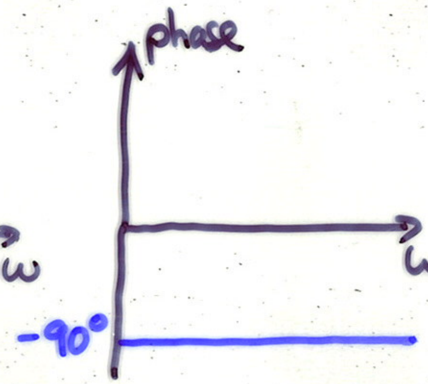
$Z = j\omega L$



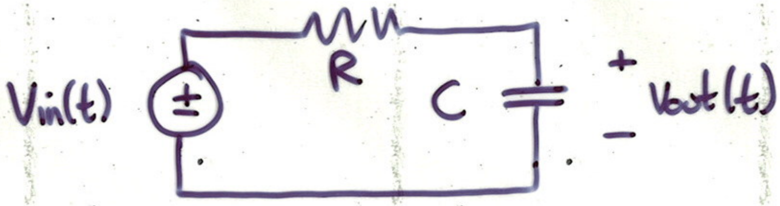


$Z = \frac{1}{j\omega C}$





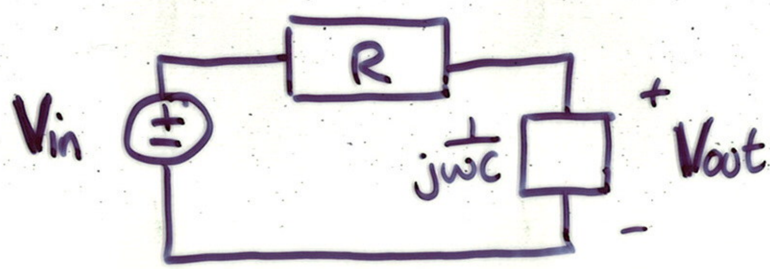
Consider the following circuit



Find $V_{out}(t)$ if circuit is in the steadystate and

$$V_{in}(t) = A\cos(\omega t + \theta)$$

Phasor equivalent



By voltage division:

$$V_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_{in}$$

$$= \frac{1}{1 + j\omega CR} V_{in}$$

$$= \frac{1}{\sqrt{1+\omega^2C^2R^2}} e^{-j\,\mathrm{atan}(\omega CR)} V_{in}$$

$$= \frac{A}{\sqrt{1+\omega^2C^2R^2}} e^{j(\theta - \mathrm{atan}(\omega CR))}$$

$$\Rightarrow V_{out}(t) = \frac{A}{\sqrt{1+\omega^2C^2R^2}} \cos(\omega t + \theta - \mathrm{atan}(\omega CR))$$

Look at magnitude : $\dfrac{A}{\sqrt{1+\omega^2 R^2 C^2}}$

At low frequencies $\approx A$

At high frequencies $\approx \dfrac{A}{\omega RC}$

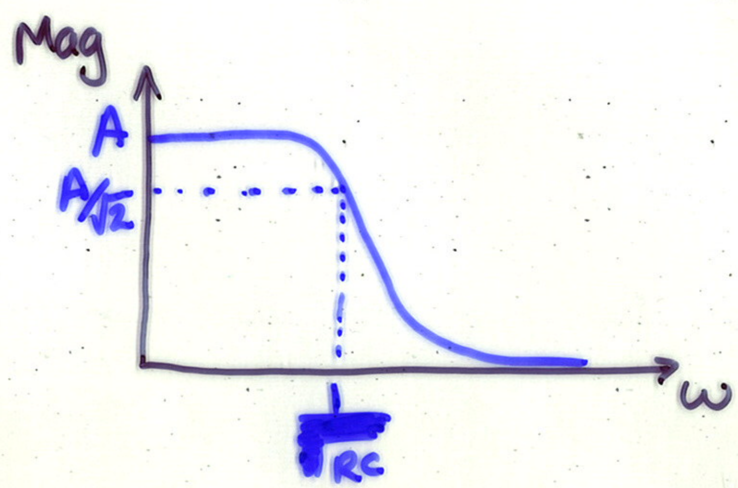
At $\left(\omega = \dfrac{1}{RC}\right) = \dfrac{A}{\sqrt{2}}$

Look at phase $\theta - \operatorname{atan}(\omega CR)$

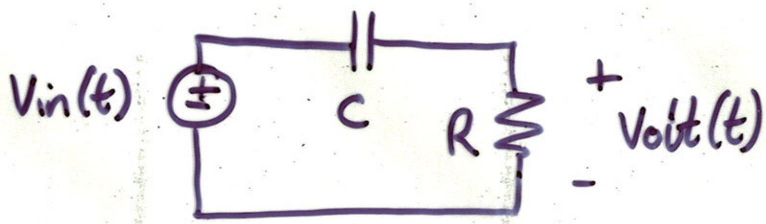
At low frequencies $\approx \theta$

At high frequencies $\approx \theta - 90^\circ$

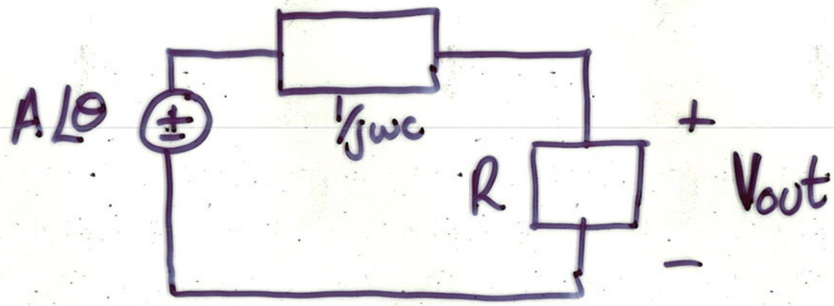
At $\left(\omega = \dfrac{1}{RC}\right) = \theta - 45^\circ$



Now what about:



Phasor domain



$$V_{out} = \frac{R}{R + 1/j\omega C} \cdot A\angle\theta$$

$$= \frac{j\omega CR}{1 + j\omega CR} \, A\angle\theta$$

$$= \frac{\omega CR}{\sqrt{1+\omega^2 C^2 R^2}} \, e^{j(90^\circ - \mathrm{atan}(\omega CR))} \cdot Ae^{j\theta}$$

$$\Rightarrow V_{out}(t) = \frac{A\omega CR}{\sqrt{1+\omega^2 C^2 R^2}} \cos(\omega t + \theta + 90^\circ - \mathrm{atan}(\omega CR))$$

MAGNITUDE

$$\frac{A\omega CR}{\sqrt{1+\omega^2C^2R^2}}$$

At low frequencies $\approx A\omega CR$

At high frequencies $\approx \dfrac{A\omega CR}{\omega CR} = A$

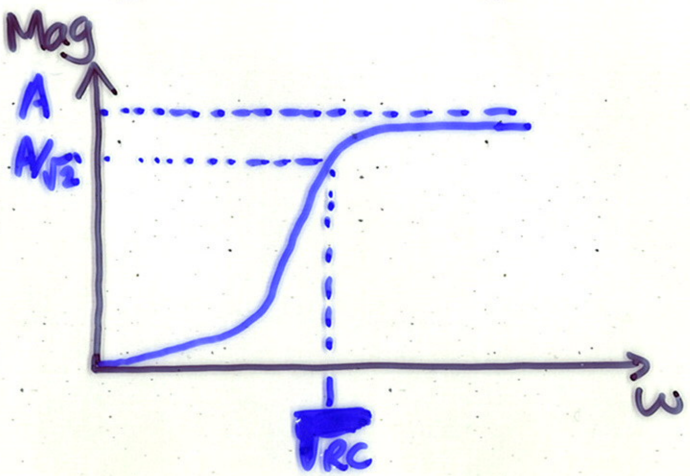
At ($\omega =$ [redacted] RC) $= \dfrac{A}{\sqrt{2}}$

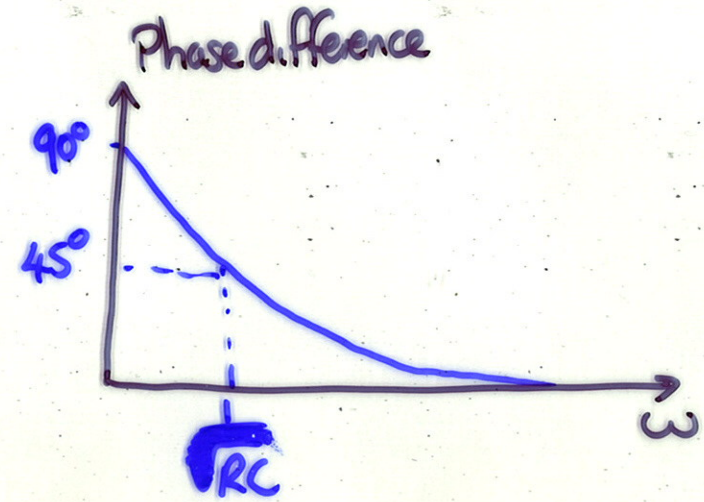
Phase difference: $90° - \text{atan}(\omega CR)$

At low frequencies $\approx 90°$

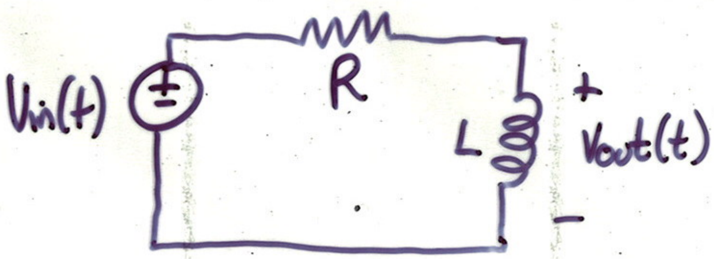
At high frequencies $\approx 0°$

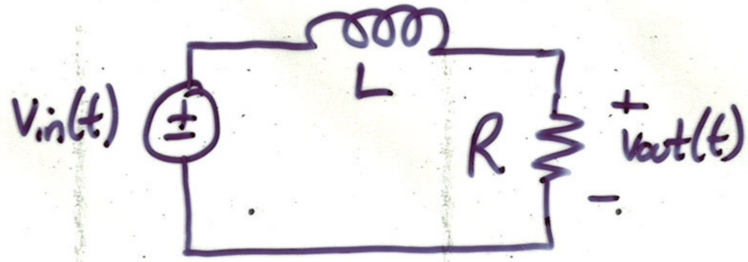
At ($\omega =$ [redacted] RC) $= 45°$





RELATED CIRCUITS



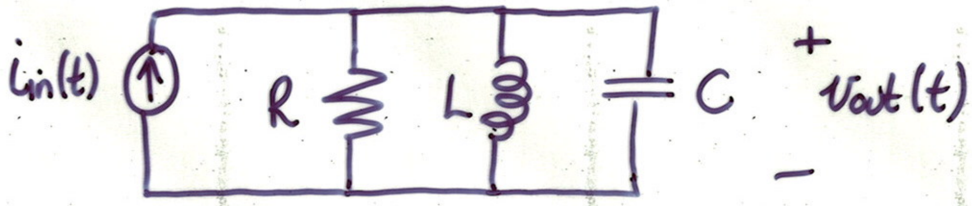


SHOW THAT:

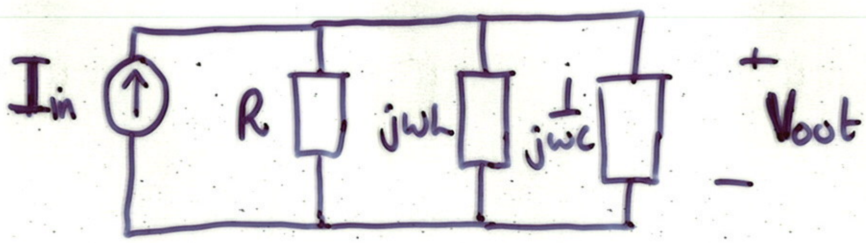
$$\frac{V_{out}}{V_{in}} = \frac{j\omega L/R}{1+j\omega L/R}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1+j\omega L/R}$$

Another Example



Find: $\dfrac{V_{out}}{I_{in}}$



$$V_{out} = Z_{eq}\, I_{in}$$

$$\frac{1}{Z_{eq}} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$= \frac{RL/C}{R + j(\omega L - 1/\omega C)}$$

$$= \frac{RL/C}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\, e^{-j\,\mathrm{atan}\left((\omega L - \frac{1}{\omega C})/R\right)}$$

MAGNITUDE

$$\frac{RL/C}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

At low frequencies $\approx \omega L R$

At high frequencies $\approx \frac{R}{\omega C}$

Where is the max magnitude?

where $\omega L - 1/\omega C = 0 \implies \omega = \frac{1}{\sqrt{LC}}$

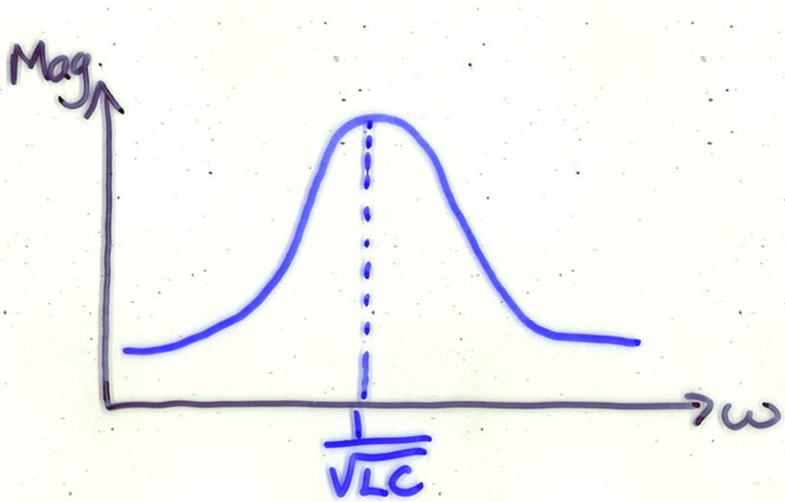
Phase difference

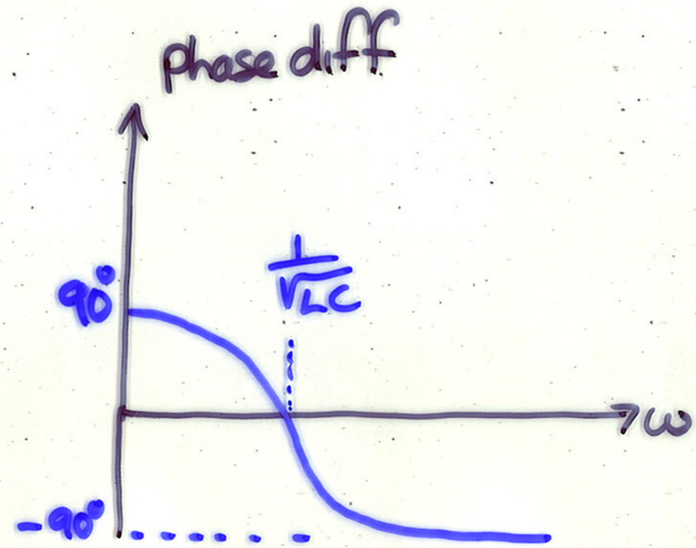
$-\text{atan}\left((\omega L - 1/\omega C)/R \right)$

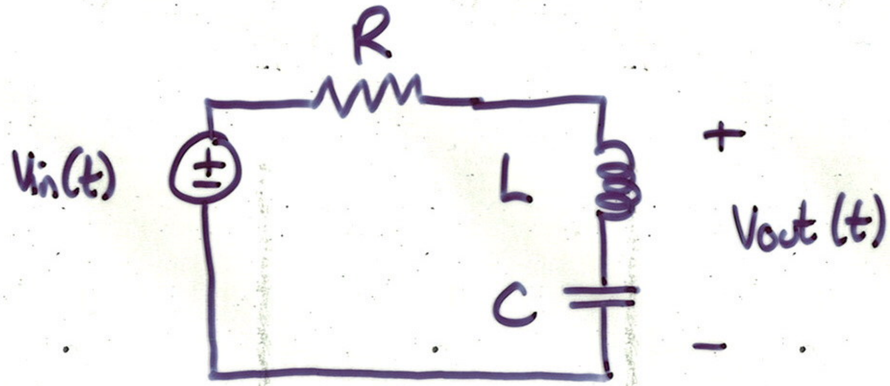
At low frequencies $\approx 90°$

At high frequencies $\approx -90°$

At $\left(\omega = \frac{1}{\sqrt{LC}}\right) = 0°$







Show that this circuit has a band stop characteristic

Hint: First show that

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{LC} - \omega^2}{\frac{1}{LC} - \omega^2 + j\omega R/L}$$