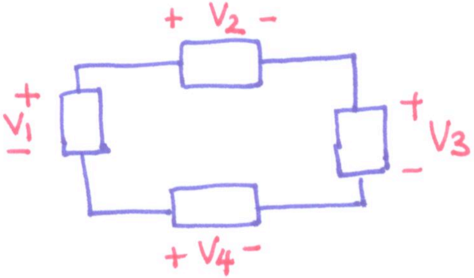
MESH CURRENT ANALYSIS

Kirchoff's Voltage Law

"The algebraic sum of drops in potential around a closed path is zero"



$$-V_1 + V_2 + V_3 - V_4 = 0$$

Why does it hold?

conservation of work done in a closed system

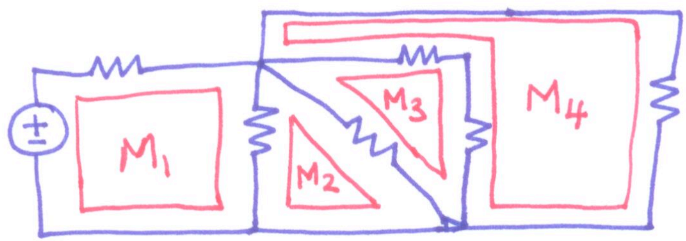
MESH CURRENT ANALYSIS

- A complementary technique to NODE VOLTAGE ANALYSIS
- Systematic application of Kirchoff's voltage law
- applies to planar (two-dimensional) circuits

TERMINOLOGY

CLOSED PATH – a path along circuit branches beginning and ending at a given node and passing through any other node at most once

MESH – a closed path that does not contain any other closed path within it; e.g.,



MESH CURRENT

- the current that flows through every element of the mesh
- often the clockwise direction is chosen to be positive, but this is not required

MESH CURRENT ANALYSIS

- find all the mesh currents in the circuit

RECIPE

* As in the case of node voltage analysis, we want a systematic procedure

* Start with the simplest case

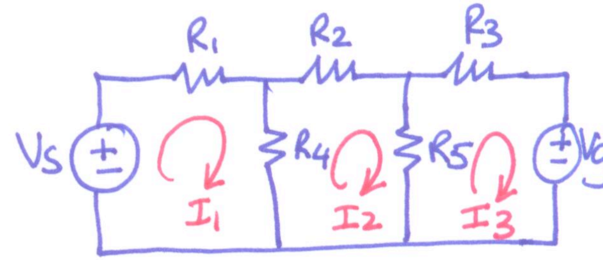
MESH CURRENT ANALYSIS WITH INDEPENDENT VOLTAGE SOURCES

Consider a circuit with no current sources and no dependent voltage sources; e.g.,

STEPS

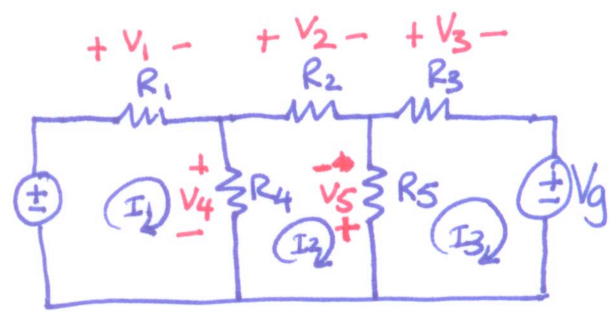
① Identify meshes

② Label the current in each mesh, e.g., i_1, i_2, i_3.
Direction is arbitrary, but often chosen to be clockwise



③ • Label the voltage drop across each component.
• For components on the "edge" the direction is often chosen according to passive sign convention and mesh current

• For elements in two meshes, direction is arbitrary



(4) Write KVL for each mesh in terms of voltages.

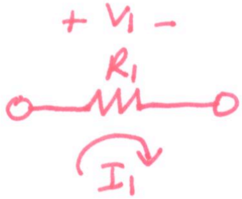
Loop 1: $-V_s + V_1 + V_4 = 0$

Loop 2: $-V_4 + V_2 - V_5 = 0$

Loop 3: $V_5 + V_3 + V_g = 0$

(5) For branches that contain resistors, apply Ohm's Law.

At this stage the passive sign convention is critical.

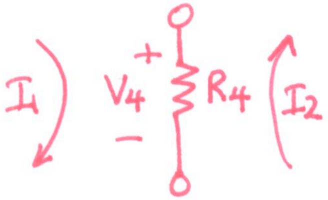


$V_1 = R_1 I_1$

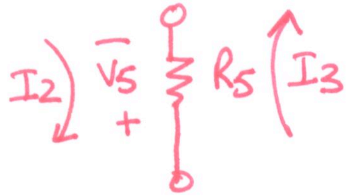
Similarly

$V_2 = R_2 I_2$

$V_3 = R_3 I_3$



$V_4 = R_4 (I_1 - I_2)$



$V_5 = R_5 (I_3 - I_2)$

(6) Make sure that the number of equations equals the number of unknowns

UNKNOWNS: i_1, i_2, i_3

V_1, V_2, V_3, V_4, V_5

KVLs:

$-V_s + V_1 + V_4 = 0$

$-V_4 + V_2 - V_5 = 0$

$V_5 + V_3 + V_g = 0$

Ohm's Law:

$V_1 = R_1 I_1$

$V_2 = R_2 I_2$

$V_3 = R_3 I_3$

$V_4 = R_4 (I_1 - I_2)$

$V_5 = R_5 (I_3 - I_2)$

(7) Solve the linear system

Completing the example

Substituting Ohm's Laws into KVLs

$$-V_s + R_1 I_1 + R_4(I_1 - I_2) = 0$$

$$-R_4(I_1 - I_2) + R_2 I_2 - R_5(I_3 - I_2) = 0$$

$$R_5(I_3 - I_2) + R_3 I_3 + V_g = 0$$

Now only 3 equations in 3 unknowns.

In matrix form:

$$\begin{array}{l} \text{Loop 1} \\ \text{Loop 2} \\ \text{Loop 3} \end{array} \begin{bmatrix} R_1 + R_4 & -R_4 & 0 \\ -R_4 & R_2 + R_4 + R_5 & -R_5 \\ 0 & -R_5 & R_3 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ -V_g \end{bmatrix}$$

$$\mathbf{R}\,\boldsymbol{\iota} = \mathbf{v}$$

Note:
- R is symmetric
- Diagonal elements are sum of resistances in the mesh
- off diagonals ~~of th~~ are negatives of resistors shared by appropriate meshes.
- This structure applies in general.

Above also suggests a streamlined procedure in which we apply Ohm's Law within the KVLs, directly.

*Use this only if you are confident that the signs will be correct