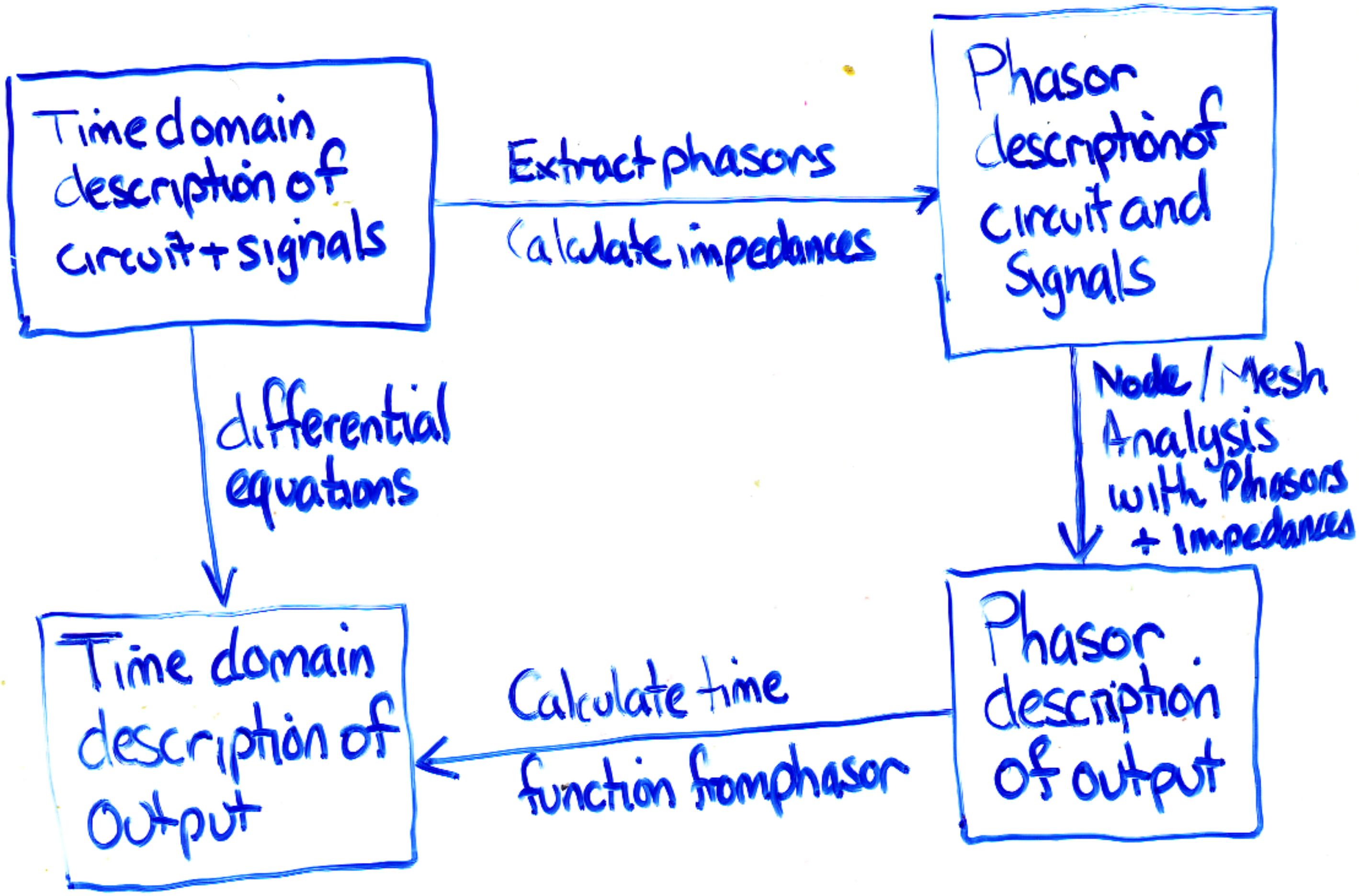
Simplified Analysis of Linear Circuits with Sinusoidal inputs in steady-state using phasors



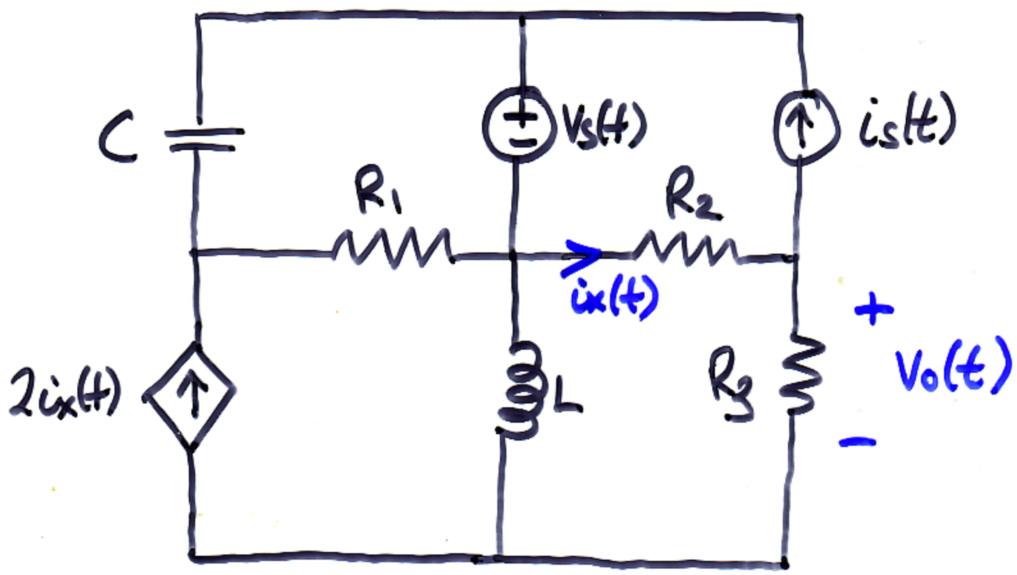
Why does this work?

How do phasors allow us to avoid differential equations?

Phasor analysis examples

The following circuit is in the steady state.

Find $v_o(t)$



$v_s(t) = 12\cos(100t)$

$i_s(t) = 4\cos(100t)$

$R_1 = R_2 = R_3 = 1\Omega$

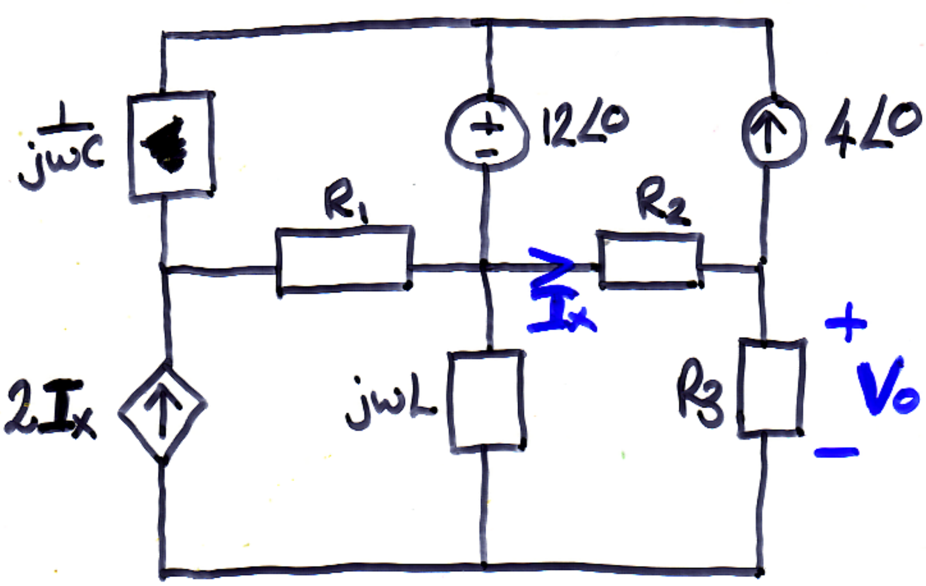
$C = 10mF$

$L = 10mH$

STEP 1:

- Identify that circuit is in steady state
- Identify that all sources are sinusoidal with the same frequency, 100 rads^{-1}
- Therefore we can solve via phasor analysis

STEP 2: Draw phasor equivalent circuit at 100 rads^{-1}

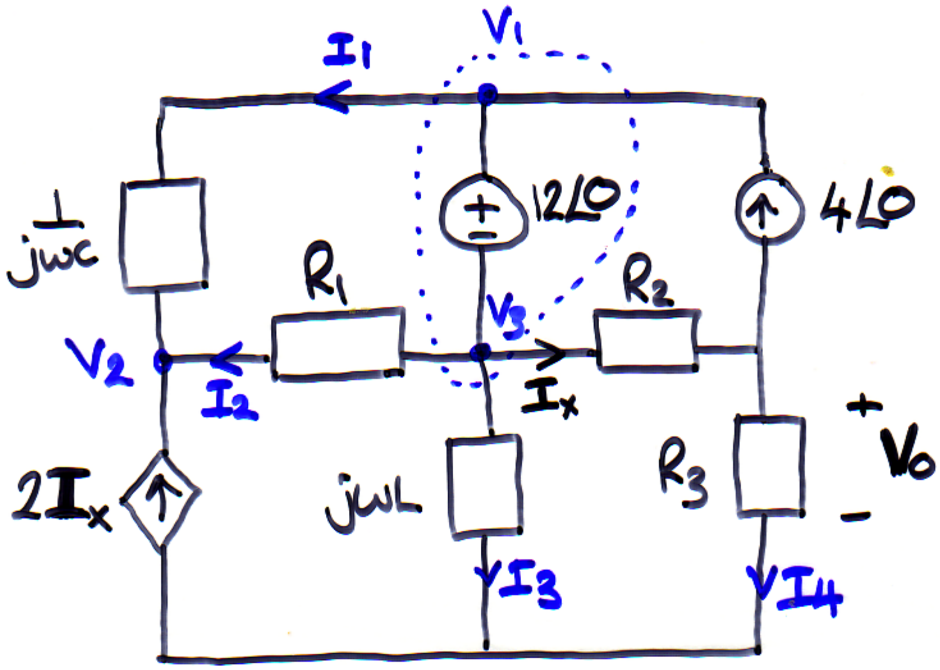


STEP 3: Solve for V_o

- Node, Mesh, Thevenin, Norton

STEP 4: Convert V_o to $v_o(t)$

Step 3, Node analysis



KCL at supernode:

$$I_1 + I_2 + I_x - 4\angle 0 = 0$$

KCL at node 2: $I_1 + I_2 + 2I_x = 0$

KCL at output node: $I_x - 4\angle 0 - I_4 = 0$

Inside supernode: $V_1 - V_3 = 12\angle 0$

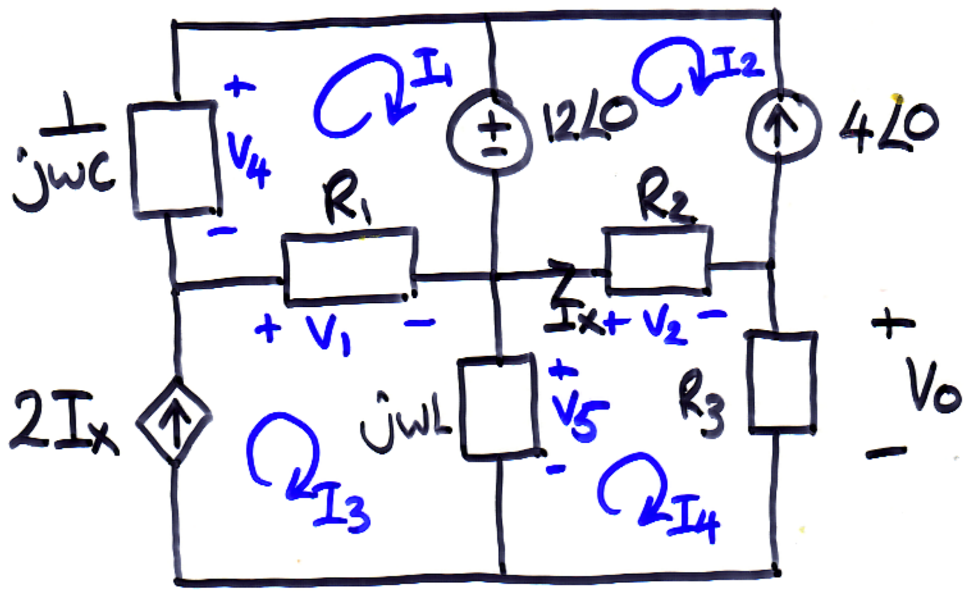
Elements:

$$I_1 = \frac{V_1 - V_2}{1/j\omega c} = j\omega c\,(V_1 - V_2)$$

$$I_2 = (V_3 - V_2)/R_1 \;; \quad I_4 = V_o/R_3$$

$$I_3 = \frac{V_3}{j\omega L} \;; \quad I_x = (V_3 - V_o)/R_2$$

Step 3, Mesh Analysis



$I_2 = -4\angle 0$

$I_3 = 2I_x \quad ; \quad I_x = I_4 - I_2$

KVL loop1 $\quad -V_4 + 12\angle 0 - V_1 = 0$

KVL loop4 $\quad -V_5 + V_2 + V_o = 0$

Elements

$V_1 = R_1(I_3 - I_1)$

$V_2 = R_2(I_4 - I_2)$

$V_o = R_3 I_4$

$V_4 = -\frac{1}{j\omega C} I_1$

$V_5 = j\omega L(I_3 - I_4)$

STEP 3, solving equations

yields

$$V_o = 4 \angle 143.13^\circ$$

STEP 4, convert back to time domain

$$v_o(t) = 4\cos(100t + \text{[redacted]}\ 143.13^\circ)$$

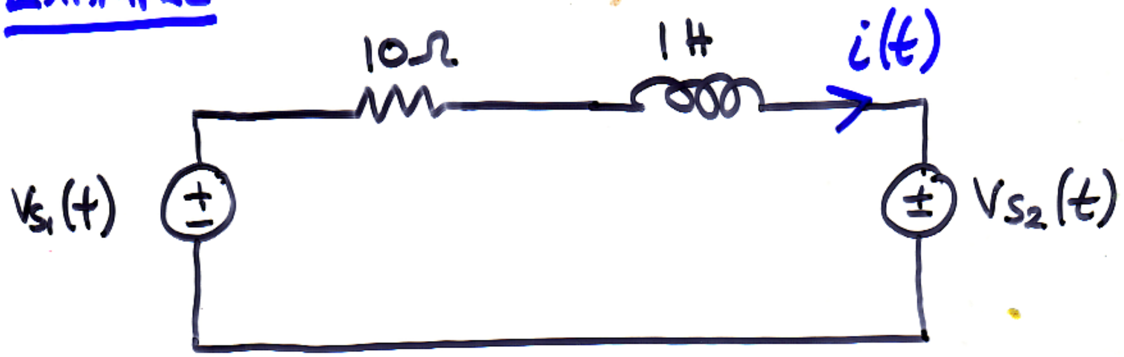
SUPER POSITION

* when the circuit contains sources with more than one frequency, we must use super-position

* However, super position applies only to the time domain signals, we <u>cannot</u> superpose the phasors if they apply to signals with different frequencies

* For circuits with sources of more than one frequency, we must:
    1. Collect sources into groups with same frequency
    2. Use phasor analysis for each group alone
    3. Calculate time responses for each group alone
    4. Add <u>time</u> responses.

EXAMPLE:



The above circuit is in the steady state.

Find $i(t)$, when $V_{s_1}(t) = 100\cos(10t)$

$V_{s_2}(t) = 50\cos(20t - 10°)$

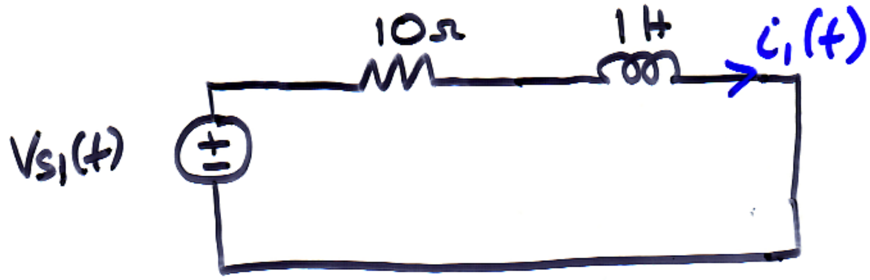
STEP 1 - Identify that the circuit is in the steady-state

- Identify that there are two sinusoidal sources of different frequencies

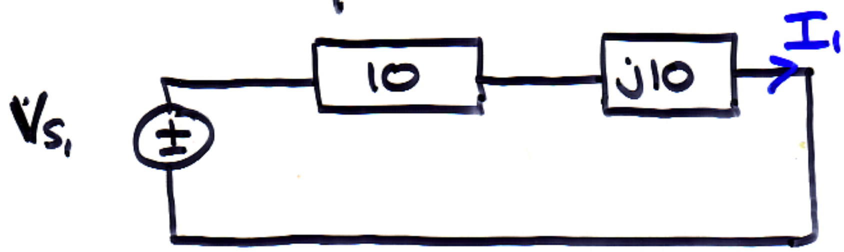
$\Rightarrow$ Solve via superposition in time domain

- For each frequency use phasor analysis.

For the 10 rad s^{-1} source:



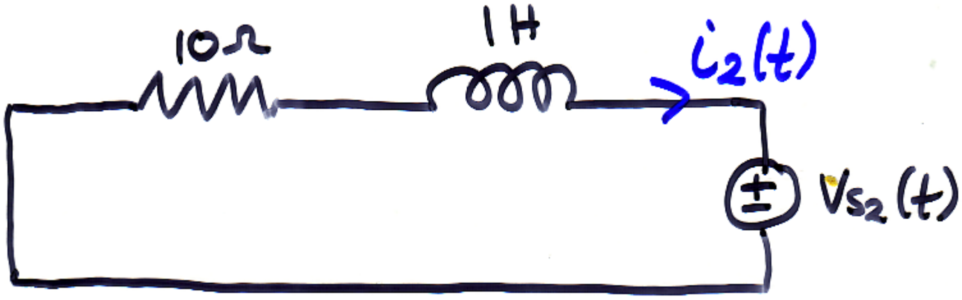
Phasor domain equivalent



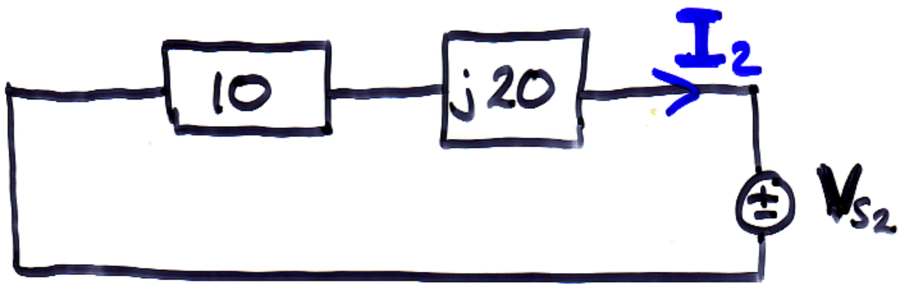
$$I_1 = \frac{V_{s_1}}{10+j10} = \frac{100\angle 0}{10+j10} = 7.07\angle -45^\circ$$

$$\Rightarrow \quad i_1(t) = 7.07\cos(10t - 45^\circ)$$

For the 20 rad s^{-1} source



Phasor domain equivalent



$$I_2 = \frac{-V_{s2}}{10+j20} = \frac{-50\angle -10}{10+j20} = 2.24\angle 106.6^\circ$$

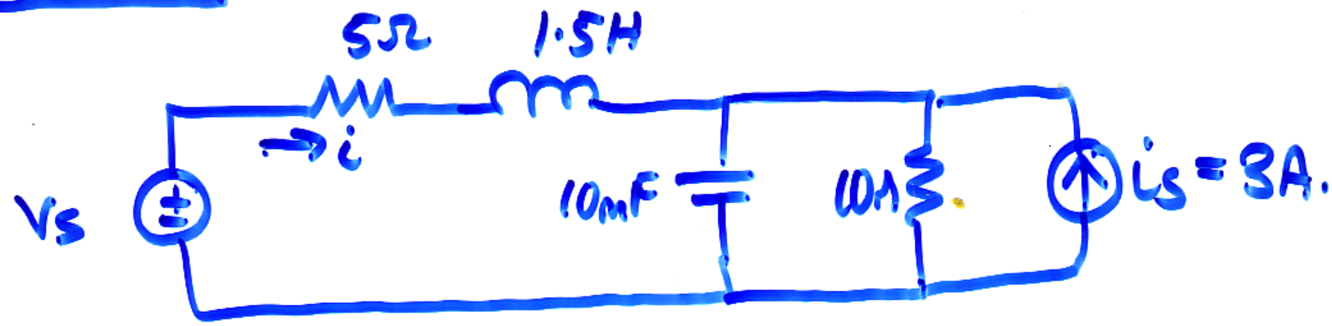
$$\Rightarrow i_2(t) = 2.24\cos(20t + 106.6^\circ)$$

Final answer

$$i(t) = i_1(t) + i_2(t)$$

$$= 7.07\cos(10t - 45^\circ) + 2.24\cos(20t + 106.6^\circ)$$

EXAMPLE



$V_s = 10\cos 10t$.

Find $i(t)$.

Step 1 - look at Vs alone.

$\underline{V_s} = 10$

using Phasor analysis, we find that

$$\underline{I}_1 = \frac{10}{\sqrt{200}} e^{-j45°}$$

where $\underline{I}_1$ is the component of $\underline{I}$, due to $\underline{V_s}$

Hence $i_1(t) \approx 0.71 \cos(10t - 45°)$

Step 2 - i_s alone

$\underline{I}_s = 3$, actually this is DC!

Can find that $\underline{I}_2 = -2$. $\Rightarrow i_2(t) = -2$

Step 3 - Add time domain

$i(t) = i_1(t) + i_2(t) = 0.71\cos(10t - 45°) - 2$ A.