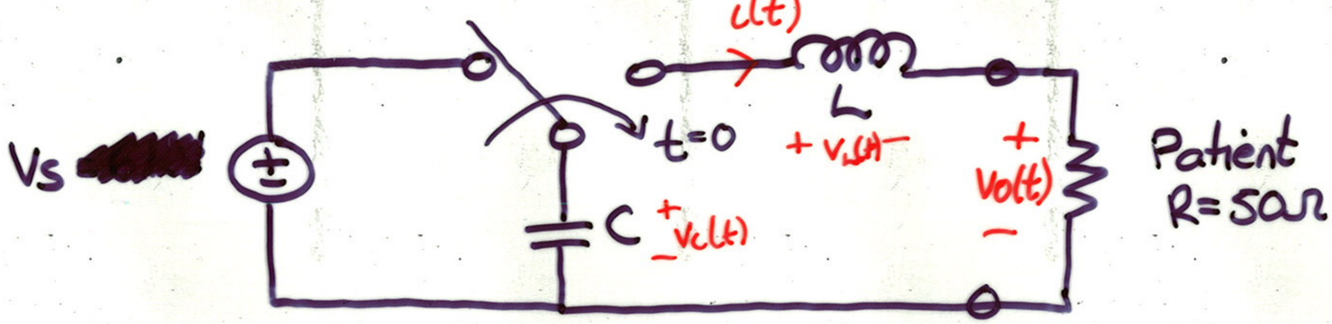
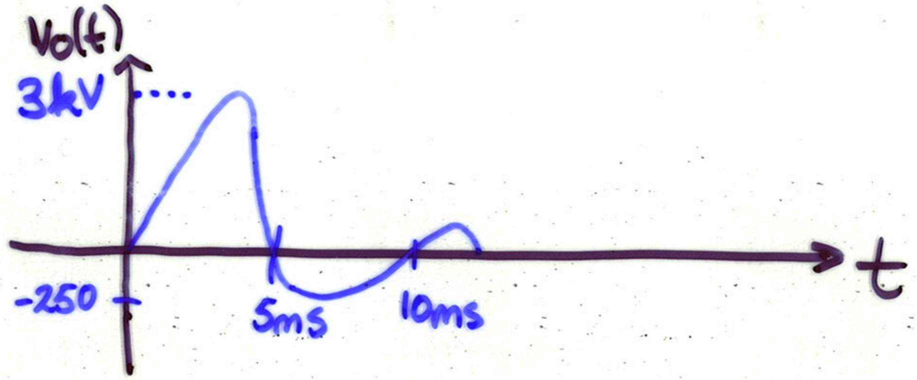
Building a Defibrillator

[redacted]
Circuit topology (Lown defibrillator)



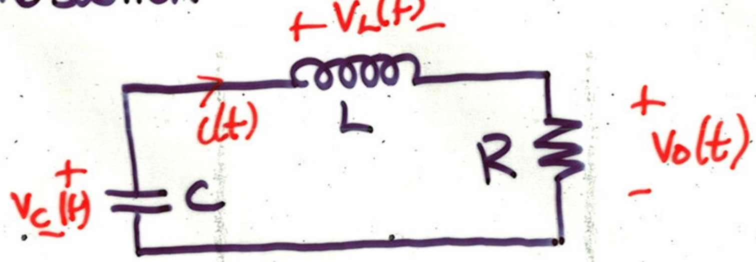
Desired response



Design L and C so that this is well approximated

This will also require design of Vs

After the switch



KVL $\quad -V_c(t) + V_L(t) + V_o(t) = 0$

$$-V_c(0) + \frac{1}{C}\int_{0}^{t} i(x)\,dx + L\frac{di(t)}{dt} + Ri(t) = 0$$

Differentiate and divide by L.

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = 0$$

We need an underdamped circuit; i.e., $\zeta < 1$

$$\Rightarrow \quad \frac{R/L}{2\sqrt{\frac{1}{LC}}} < 1$$

$$\Rightarrow \quad C < 4L/R^2$$

For an underdamped circuit, with no source.

$$i(t) = A_1 e^{-\sigma t}\cos(\omega_d t) + A_2 e^{-\sigma t}\sin(\omega_d t)$$

where

$$\omega_d = \omega_0\sqrt{1-\zeta^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\sigma = \zeta\omega_0 = \frac{R}{2L}$$

Since in this circuit, the inductor current at $t=0^-$ is zero, this means that $i(0^+)=0$

Hence $A_1=0$

Therefore, this circuit has a response

$$i(t) = A_2 e^{-\sigma t}\sin(\omega_d t)$$

and hence

$$v_0(t) = RA_2 e^{-\sigma t}\sin(\omega_d t)$$

- what should ω_d be?

Period desired is 10ms

$$\Rightarrow \omega_d = \frac{2\pi}{10 \times 10^{-3}}$$

$$\Rightarrow \omega_0 \sqrt{1-\zeta^2} = 200\pi \qquad (A)$$

- What should σ be?

Evaluate where $\sin(\omega_d t) = \pm 1$

That is, $t = T/4$ and $t = 3T/4$

From expression: $$\frac{V_0(T/4)}{-V_0(3T/4)} = \frac{RA_2 e^{-\sigma T/4}}{RA_2 e^{-\sigma 3T/4}} = e^{\sigma T/2}$$

From desired: $V_0(T/4) = 3000$; $V_0(3T/4) = -250$

$$\Rightarrow e^{\sigma T/2} = \frac{3000}{250} = 12$$

$$\Rightarrow \zeta\omega_0 = 497.0 \qquad (B)$$

From (B) we have

$$\zeta\omega_0 = \frac{R}{2L} = 497.0$$

Since $R = 50\Omega$, we have

$$L = 50.3\,mH$$

From (A) we have

$$\omega_0^2 - (\omega_0\zeta)^2 = (200\pi)^2$$

$$\Rightarrow \frac{1}{LC} = (200\pi)^2 + (497.0)^2$$

Hence

$$C = 31.0\,\mu F$$

Sanity check: Is circuit underdamped?

i.e., is $C < 4L/R^2$?

$$C = 31.0 \times 10^{-6}$$

$$4L/R^2 = 80.5 \times 10^{-6}$$

We still have to design V_s

To do so, we need to find the required value of A_2

Requirement is

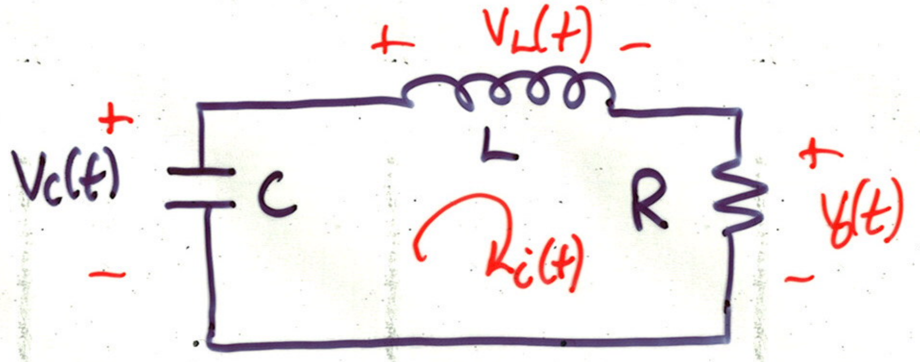
$$V_o(T/4) = 3000$$

$$V_o(t) = RA_2 e^{-\sigma t} \sin(\omega_d t)$$

$\Rightarrow$ we require $RA_2 e^{-\sigma T/4} = 3000$

$$\Rightarrow A_2 = 207.86$$

Now, how to choose V_s such that $A_2 = 207.86$?



$$i(t) = A_2\, e^{-\sigma t} \sin(\omega_d t)$$

$$V_L(t) = L \frac{di(t)}{dt}$$

$$= L A_2 \left[(-\sigma e^{-\sigma t}) \sin(\omega_d t) + e^{-\sigma t} \omega_d \cos(\omega_d t) \right]$$

What happens at $t=0$?

$$i(0) = 0 \Rightarrow V_o(t)\Big|_{t=0} = 0$$

$$\Rightarrow \text{at } t=0 \quad V_L(t)\Big|_{t=0} = V_c(t)\Big|_{t=0}.$$

but $V_c(t)\Big|_{t=0} = V_s$

Therefore, we can find the required value for V_s by solving

$$V_s = V_L(0)$$

$$= L A_2 \omega_d$$

$$= 50.3 \times 10^{-3} \times 207.86 \times 200\pi$$

$$= 6.569 \text{ kV}$$