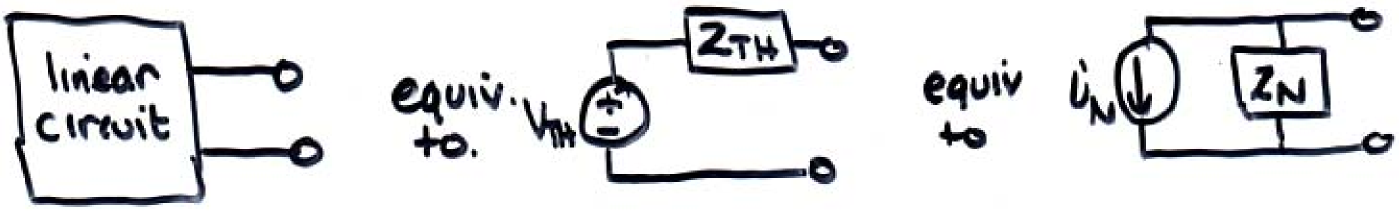
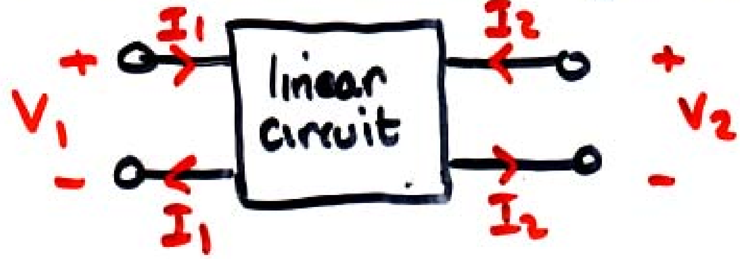
Two Port Networks

- When we developed the theory of Thevenin and Norton equivalents, we considered circuits with a single port:



- These equivalents provided a structured way to analyse single port networks.

- We now develop a structured way to look at two-port networks



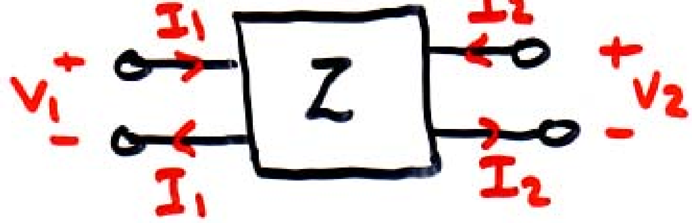
- [redacted] Our models will describe the relationships between. V_1, I_1, V_2, I_2

- This is important because two port networks are often interconnected, and our models will simplify the analysis of these interconnections

- We will focus on 4 models. Each one is easier to use in certain situations

- The models are related because they describe the same underlying circuit

The Impedance model.



DEFINE

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \ \Omega \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \ \Omega$$

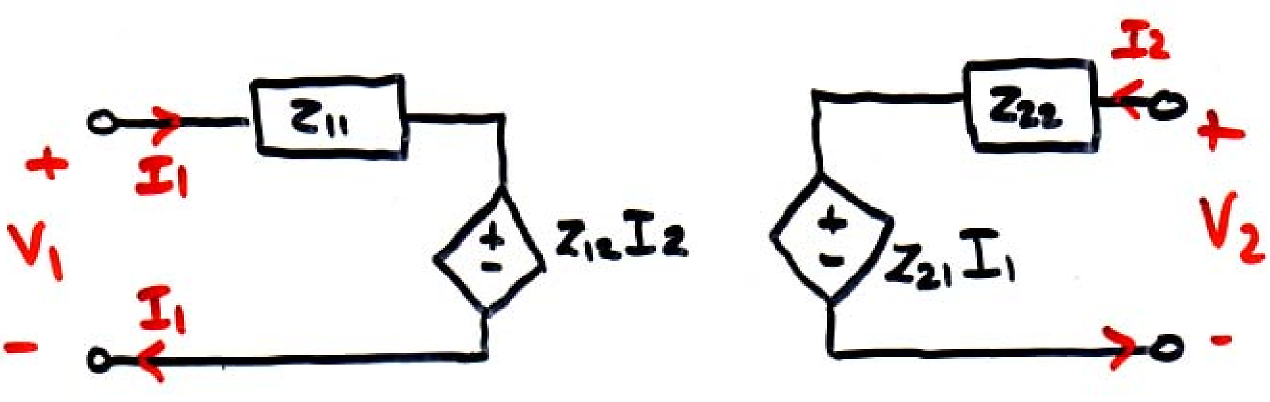
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \ \Omega \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \ \Omega$$

What does this mean?

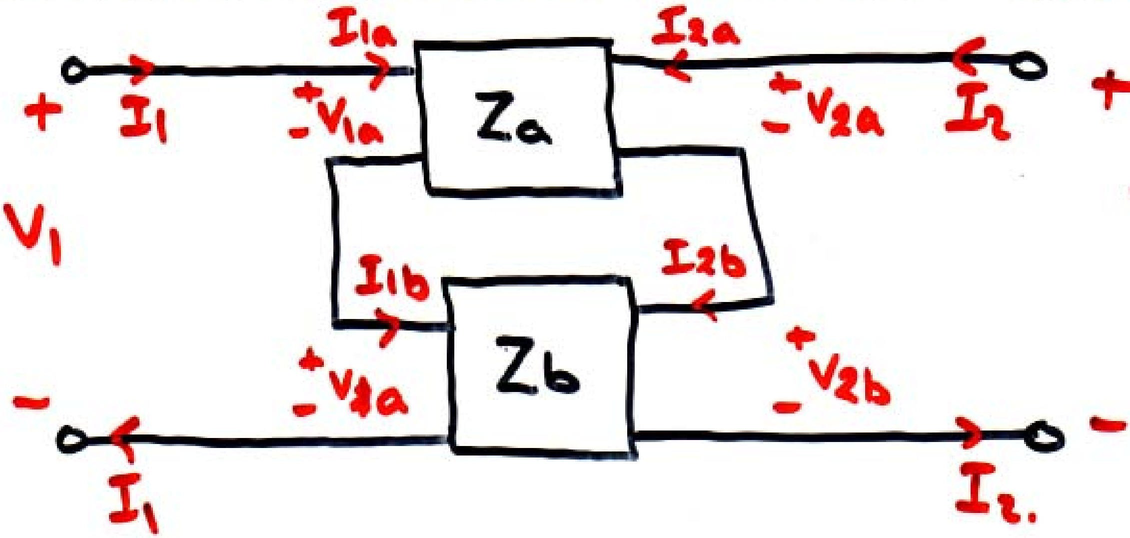
$$\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned} \quad \longleftrightarrow \quad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Can we make a circuit model for this?

$$V_1 = \underbrace{Z_{11} I_1}_{\text{a voltage that depends on } I_1} + \underbrace{Z_{12} I_2}_{\text{a voltage independent of } I_1}$$



SERIES CONNECTION OF TWO PORT NETWORKS.



what is V_1 ?

$$V_1 = V_{1a} + V_{1b}.$$

$$I_{1a} = I_1$$

$$I_{1b} = I_{1a}$$

$$\Rightarrow \quad V_1 = Z_{11a} I_{1a} + Z_{12a} I_{2a} + Z_{11b} I_{1b} + Z_{12b} I_{2b}$$

$$= (Z_{11a} + Z_{11b}) I_1 + (Z_{12a} + Z_{12b}) I_2$$

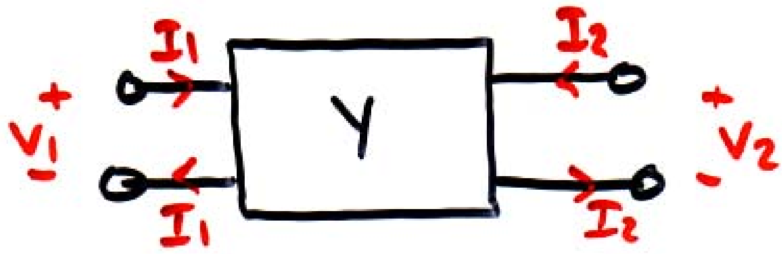
Similarly

$$V_2 = (Z_{21a} + Z_{21b}) I_1 + (Z_{22a} + Z_{22b}) I_2$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = (Z_a + Z_b) \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For series connections, Z parameters add

The admittance model.



Define:

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \ S \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \ S$$

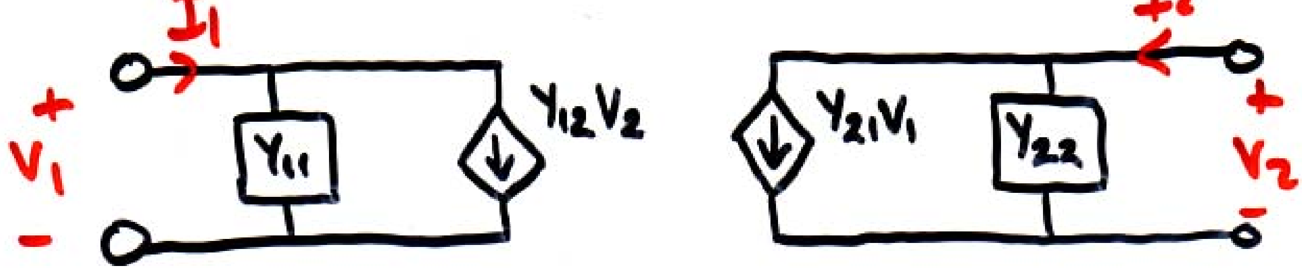
$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \ S \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \ S$$

Therefore

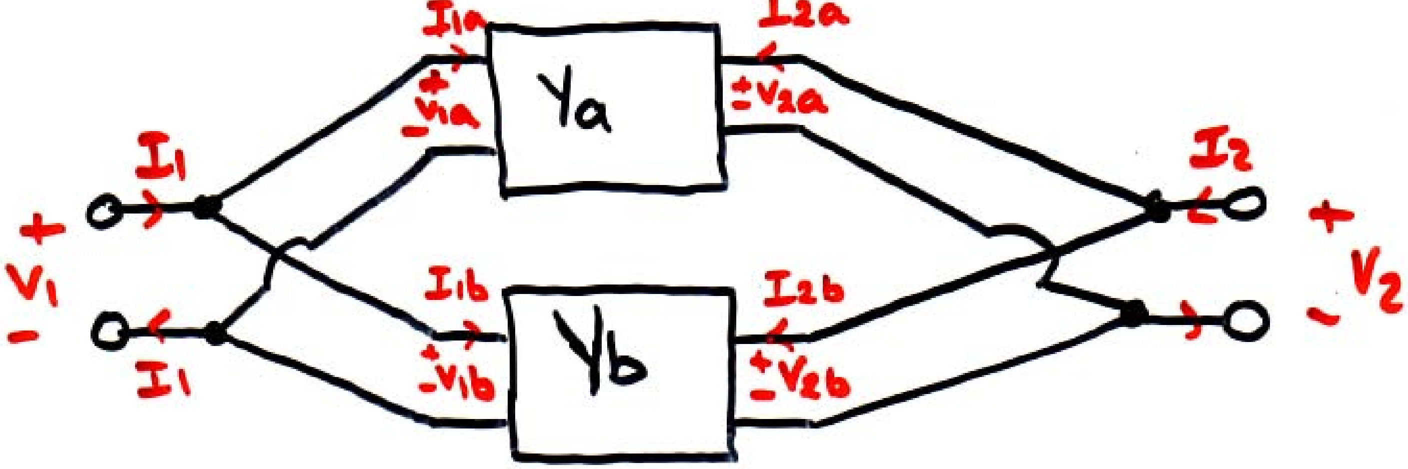
$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$
$$\Longleftrightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

A circuit model

(Note that the quantities in the boxes are admittances)



Parallel Connections

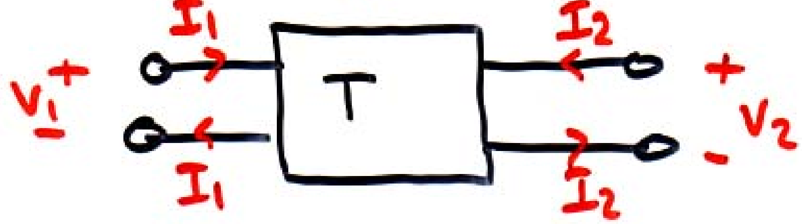


$$I_1 = I_{1a} + I_{1b} \qquad V_{1a} = V_{1b} = V_1$$
$$I_2 = I_{2a} + I_{2b}. \qquad V_{2a} = V_{2b} = V_2$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11a} + Y_{11b} & Y_{12a} + Y_{12b} \\ Y_{21a} + Y_{21b} & Y_{22a} + Y_{22b} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

For parallel connections, [redacted].
Y parameters add.

Transmission parameters.



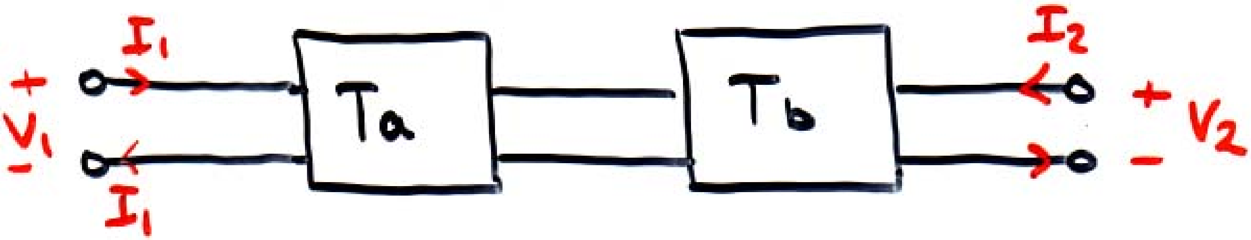
Define

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad ; \quad B = -\left.\frac{V_1}{I_2}\right|_{V_2=0} \ \Omega$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \ S \quad ; \quad D = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$

$$\Rightarrow \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{T} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Cascade connection



$$V_1 = V_{1a} \qquad I_1 = I_{1a}$$
$$V_{2a} = V_{1b} \qquad I_{2a} = -I_{1b}$$
$$V_2 = V_{2b} \qquad I_2 = I_{2b}.$$

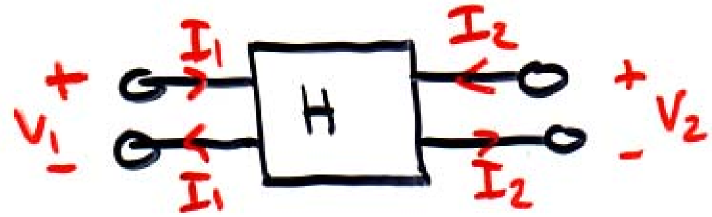
Therefore

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

For cascade connections, T matrices multiply

Hybrid parameters.

useful in transistor models



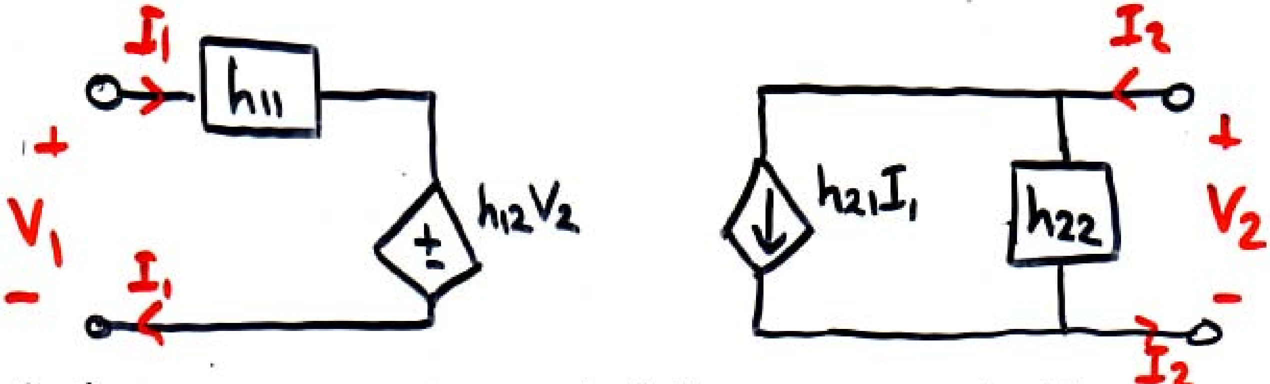
Define:

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \ \Omega \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \ S$$

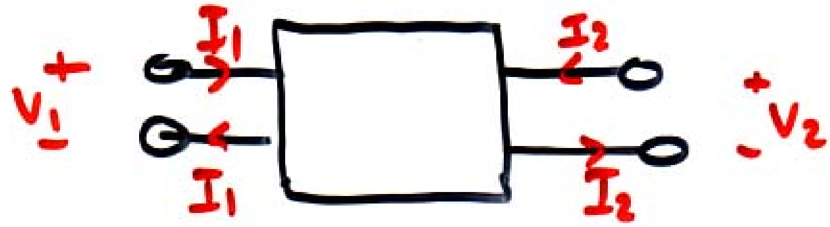
$$\Rightarrow \quad \begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned} \quad \Longleftrightarrow \quad \begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Model



Note that h_{11} is an impedance, but h_{22} is an admittance

Relationships between models



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad ; \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = Y \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = T \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad ; \quad \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = H \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

These models all describe the same circuit, so they must be related. (see Table 17.8-1)

Example

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = ZY \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow \quad Y = Z^{-1}$$

For a 2x2 matrix, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Let $\Delta_Z = Z_{11} Z_{22} - Z_{12} Z_{21}$

$$\Rightarrow \quad Y_{11} = \frac{Z_{22}}{\Delta Z} \qquad Y_{12} = \frac{-Z_{12}}{\Delta Z}$$

$$Y_{21} = \frac{-Z_{21}}{\Delta Z} \qquad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

Reciprocal Two Port Networks

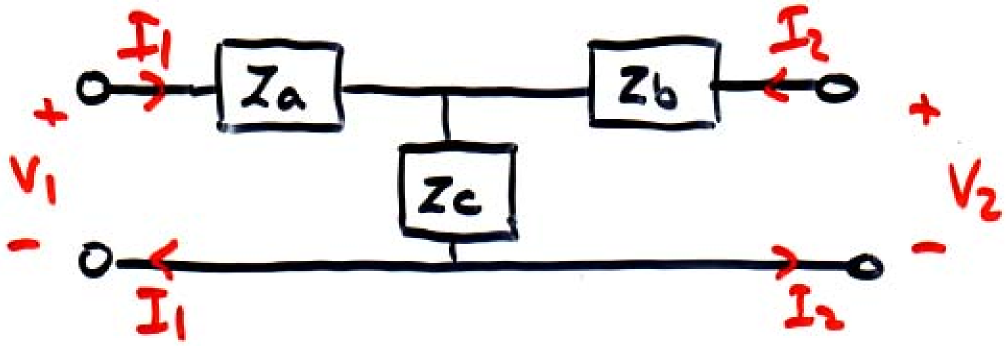
- A twoport network is said to be reciprocal if $Z_{12} = Z_{21}$

- Networks with passive components are generally reciprocal

- Those will dependent sources are typically non-reciprocal

- We now consider two standard structures for reciprocal networks

The T Network



Find the Z parameters

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} =$$

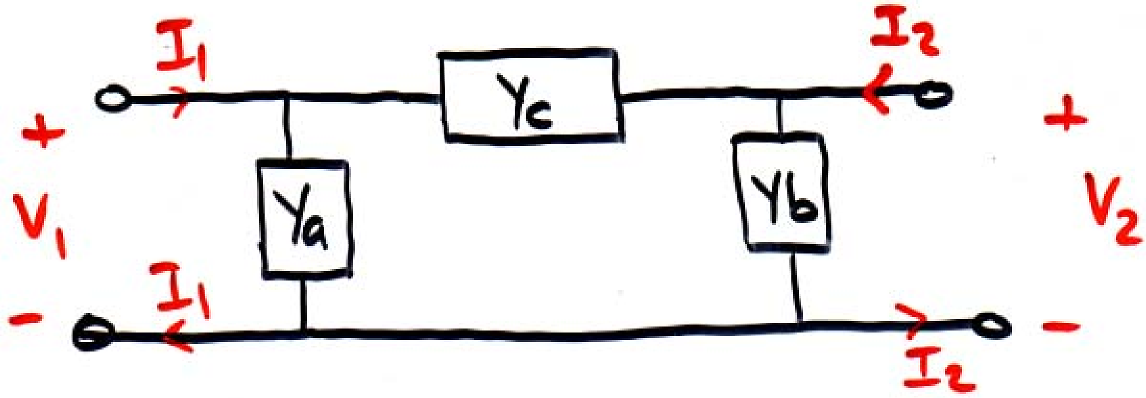
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} =$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} =$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} =$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_a + Z_c & Z_c \\ Z_c & Z_b + Z_c \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

The π Network



Find the Y parameters

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} =$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} =$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} =$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} =$$

$$\Rightarrow \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_a + Y_c & -Y_c \\ -Y_c & Y_b + Y_c \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$