Phasor Analysis

Since coupled inductors are often used in linear circuits with sinusoidal signals in the steady-state we would like to look at the equations in the phasor domain.

Since differentiation corresponds to multiplying ~~by~~ the phasor by $j\omega$.

$$\left.\begin{aligned} \underline{V}_1 &= j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2 \\ \underline{V}_2 &= j\omega L_2 \underline{I}_2 + j\omega M \underline{I}_1 \end{aligned}\right\} \text{coupled circuit 1}$$

and similarly for coupled circuit 2, except that the mutual terms subtract rather than add.

Coupler Design.

An useful measure of the effectiveness of ~~a~~ a coupling device is the "coupling coefficient"

$$k = \frac{M}{\sqrt{L_1 L_2}}.$$

using the formulae for L_1, L_2, M,

$$k = \frac{c_m}{\sqrt{c_1 c_2}}$$ which depends only on the magnetic + geometric structure

* For good coupling we want k to be large.
  i.e. we choose materials and shapes to achieve that.

* To assess the quality of the design, we need to know how big k can be.

~~* We do so for the case of~~

* The instantaneous power absorbed ~~is~~ by the coupled circuit is

$$
\begin{aligned}
p(t) &= v_1(t)\,i_1(t) + v_2(t)\,i_2(t) \\
&= L_1 i_1(t) \frac{d}{dt} i_1(t) \pm M \frac{d}{dt}\big(i_1(t)\,i_2(t)\big) \\
&\quad + L_2 i_2(t) \frac{d}{dt} i_2(t)
\end{aligned}
$$

+ for coupled circuit 1
− for coupled circuit 2

* The energy stored in the system is

$$
w(t) = \int_{-\infty}^{t} p(\tau)\,d\tau = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2
$$

* Since the system is passive (i.e does not generate power) $w(t)$ must be ≥ 0

Taking the worst case (coupled circuit 2), we must have

$$\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 - M i_1 i_2 \geq 0$$

This can be algebraically manipulated to.

$$\underbrace{\left(\sqrt{\frac{L_1}{2}}\, i_1 - \sqrt{\frac{L_2}{2}}\, i_2\right)^2} + i_1 i_2 \left(\sqrt{L_1 L_2} - M\right) \geq 0 \quad *$$

This term is ≥ 0
but can be zero
in many applications

$\Rightarrow$ to ensure that $*$ holds, we require

$$M \leq \sqrt{L_1 L_2}$$

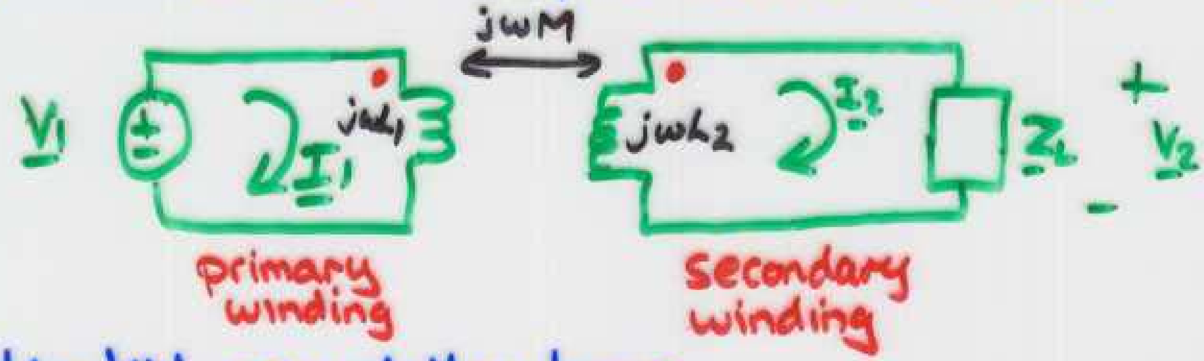
Hence $\quad 0 \leq k \leq 1$

Power transformers have $k \to 1$

Coupled ~~radio~~ radio circuits have small k.

Applications of magnetic coupling.

Consider the following "phasor domain" representation of a coupled inductor circuit



Writing KVL around the loops

$\underline{I}_1$ enters dotted terminal
$\underline{I}_2$ leaves dotted terminal

Hence $$\underline{I}_2 = \left[\frac{j\omega M}{(j\omega)^2(L_1L_2 - M^2) + j\omega L_1 Z_L}\right]\underline{V}_1$$

If the coupling coefficient $\to 1$, then $L_1L_2 \to M^2$

In that case,

$$\underline{I}_2 = \left[\frac{j\omega M}{j\omega L_1 Z_L}\right]\underline{V}_1 = \left(j\frac{\omega\sqrt{L_1L_2}}{j\omega L_1 Z_L}\right)\underline{V}_1$$

$$= \frac{\sqrt{L_2}}{Z_L\sqrt{L_1}}\,\underline{V}_1$$

Hence $\underline{V}_2 = \underline{Z}_L \underline{I}_2 = \sqrt{\frac{L_2}{L_1}}\, \underline{V}_1$

Now $\frac{L_2}{L_1} = \frac{c_2 N_2^2}{c_1 N_1^2}$

If the geometry + the magnetic properties of the core are the same ~~as~~ for both coils, $c_1 = c_2$

In that case.

$$\underline{V}_2 = n\, \underline{V}_1 \qquad (1)$$

where $n = \frac{N_2}{N_1}$ is the "turns ratio"

Similarly, $\underline{I}_1 = -n \underline{I}_2 \qquad (2)$

Equations (1) and (2) describe the operation of an ideal transformer, which we now study in more detail!