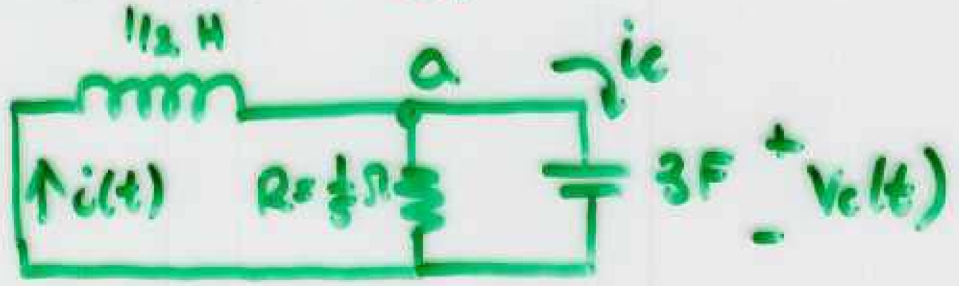
Using Laplace Transforms for Circuit Analysis

We will look at two methods.

① Write differential equations for the circuit, then solve using transforms

② Develop notions of impedance in "s-domain" to aviod having to write differential equations.

An example of method ①.



Find $v_c(t)$, $t \geq 0$, if $v_c(0) = 12V$
and $i(0) = 60A$

KVL for left mesh

(1)

KCL, node a.

$$\Rightarrow C\frac{dv_c}{dt} + \frac{v_c}{R} - i = 0 \qquad (2)$$

Take Laplace transforms of (1) and (2).

$$L[sI(s) - i(0)] + V_c(s) = 0 \qquad (3)$$

$$C\left[sV_c(s) - v_c(0)\right] + \frac{V_c(s)}{R} - I(s) = 0 \qquad (4)$$

(3) + (4) are two linear equations in two unknowns, $V_c(s)$ and $I(s)$

Solve for $V_c(s)$

$$V_c(s) = \frac{36s + 60}{3\left(s + \frac{2}{3}\right)(s+1)}$$

$$= \frac{36}{s + 2/3} - \frac{24}{s+1}$$

$$\Rightarrow v_c(t) = 36e^{-2t/3} - 24e^{-t}, \quad t \geq 0.$$

Method 2 - use impedance and initial conditions

* We have seen how useful the concept of impedance was for steady-state analysis of sinusoidal signals

* Now we want to do the same for general signals

* Resistor $\quad v(t) = R\, i(t)$

$\Rightarrow \quad V(s) = R\, I(s)$ } take Laplace transform of both sides

* Suggests a notion of impedance in the "s-domain"

* Definition

If the initial current through an element and voltage across it are zero, the impedance is defined to be

$$Z(s) = \frac{V(s)}{I(s)}$$

In the case of the resistor, there is no initial condition to set.

Capacitor

$$V_c(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau + V_c(0)\,u(t)$$

Take Laplace transforms of both sides

$$V_c(s) = \frac{1}{Cs} I(s) + \frac{V_c(0)}{s}$$

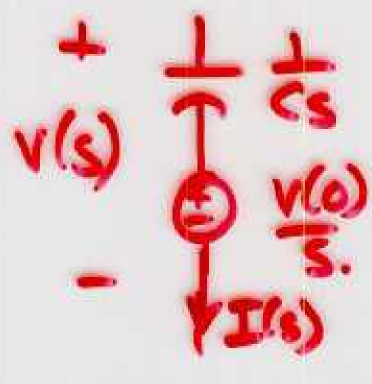
$\Rightarrow$ Impedance is $\frac{1}{Cs}$

$\Rightarrow$ Equivalent models.

Time Domain

$$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau + V_c(0)$$

"s-domain"

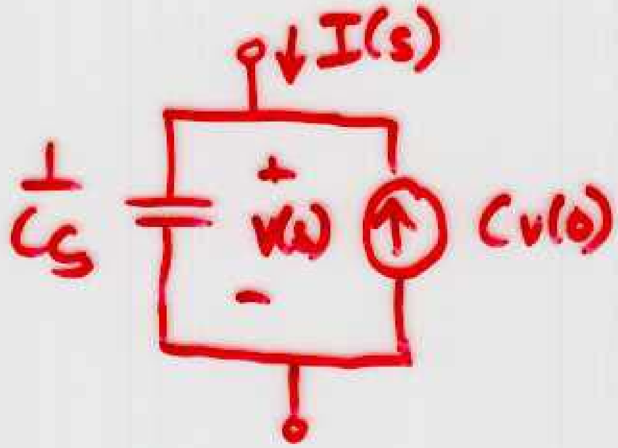


$$V(s) = \frac{1}{Cs} I(s) + \frac{V(0)}{s}$$

Since $V(s) = \frac{1}{Cs} I(s) + \frac{V(0)}{s}$

$$\Rightarrow \quad I(s) = CsV(s) - Cv(0)$$

$\Rightarrow$ Third equiv. model.



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Now do a similar thing for inductors

$$v(t) = L \frac{di(t)}{dt}$$

Take Laplace transforms.

$$V(s) = LsI(s) - Li(0)$$

$\Rightarrow$ Impedance of inductor is $\frac{V(s)}{I(s)}$ with zero initial condition $= Ls$

Equivalently,

$$I(s) = \frac{1}{Ls} V(s) + \frac{i(0)}{s}$$

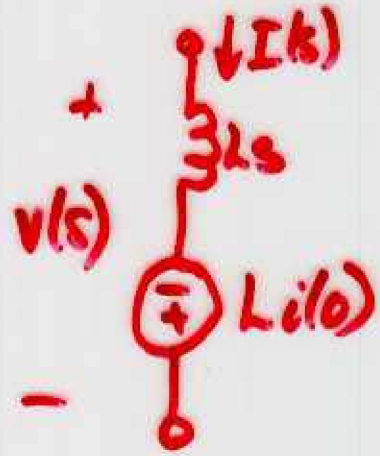
Hence three equivalent models.

Time domain

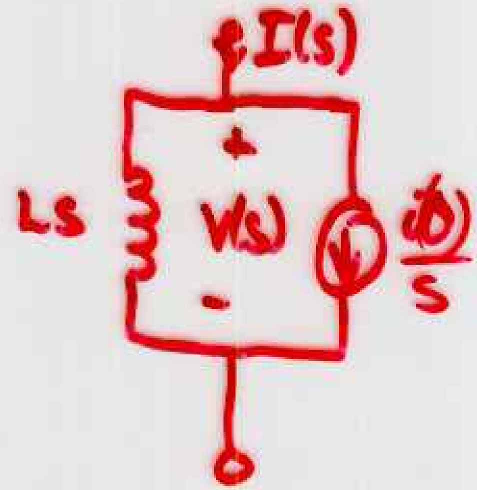
$$v(t) = L \frac{di(t)}{dt}$$

"s-domain"

Series



Parallel.



$$V(s) = LsI(s) - Li(0)$$

See also Table 14.8-1