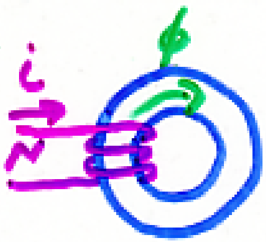
MAGNETIC CIRCUITS

- For applications with symmetry, calculating H is reasonably straightforward.
- Not so for more general case
- ⇒ we develop magnetic circuit theory so we can solve magnetic circuits like electrical ones

Components.

Electrical	Magnetic
Voltage	Magneto motive force (MMF) For a coil of N turns, with current i $\mathcal{F} = Ni$
Resistance	Reluctance Measures ratio between $\mathcal{F}$ and ϕ For a straight component, $\mathcal{R} = \frac{l}{\mu A}$, where l is the length
Current	Flux, ϕ
Ohm's Law	$\mathcal{F} = \mathcal{R}\phi$

EXAMPLE



Since the flux is always along a circle, if we take the centreline, the equiv. length of the torroid is

$$l = 2\pi R$$

It area is $A = \pi r^2$

$\Rightarrow$ Reluctance is $\mathcal{R} = \dfrac{l}{\mu A} = \dfrac{2R}{\mu r^2}$

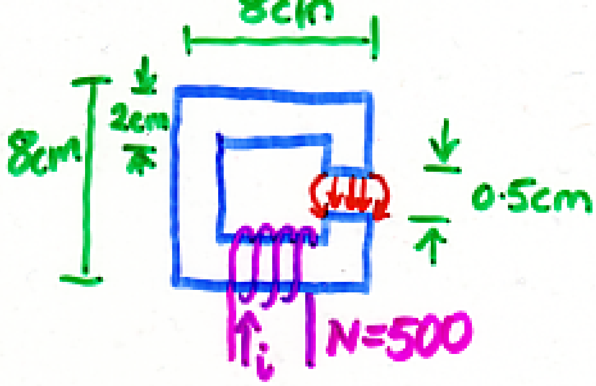
MMF is $\mathcal{F} = Ni$

$\Rightarrow$ Flux is $\phi = \dfrac{\mathcal{F}}{\mathcal{R}} = \dfrac{\mu N r^2 i}{2R}$.

Equiv. circuit

The big advantage is that we can use these concepts for non-symmetric arrangements

Example. – Iron loop with air gap and 2cm x 3cm cross-section

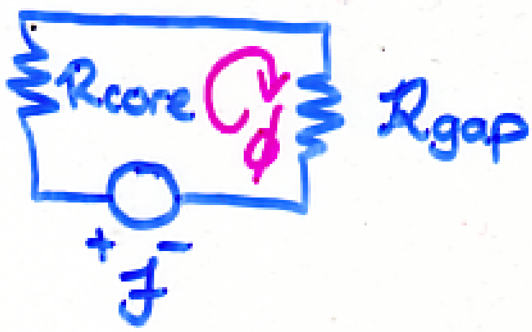


Find the current required to produce a flux density of 0.25 T in the air gap.

$\mu_{core} = 6000\,\mu_0$

We would like to do this using magnetic circuit concepts.

Equiv. circuit



Thus circuit techniques will give relationship between i and ϕ.

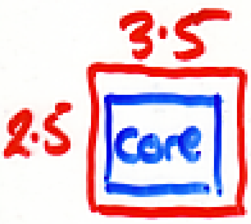
What ϕ do we need?

$$\phi_{gap} = B_{gap} A_{gap}$$

$B_{gap} = 0.25\,T$, what is A_{gap}

Cross sectional area of iron loop is 2cm x 3cm.

However flux lines "bow" out in air gap.
- called fringing
- $\Rightarrow$ effective area is larger than core cross-section
- although we could try to compute these quantities via integration, a standard approximation is to form effective area by adding length of gap to each dimension



ie, effective $A_{gap} = 2.5 \times 3.5 \text{ cm}^2$
$= 8.75 \times 10^{-4} \text{ m}^2$.

Thus the required flux is

$$\phi = BA = 2.188 \times 10^{-4} \text{ Wb}$$

Now we can answer question by finding the required MMF.

$$\mathcal{F} = \phi \mathcal{R}$$

and $\mathcal{F} = Ni$

$$\Rightarrow i = \frac{\phi \mathcal{R}}{N} = \frac{2.188 \times 10^{-4} \mathcal{R}}{500}$$

All we need now is $\mathcal{R}$

As you might guess, reluctances in series add.

$$\Rightarrow \quad \mathcal{R} = \mathcal{R}_{core} + \mathcal{R}_{gap}$$

$$\mathcal{R}_{gap} = \frac{l_{gap}}{\mu_{gap} A_{gap}}$$

$\mu_{gap} \approx \mu_{\text{vacuum}} = 4\pi \times 10^{-7}$

$l_{gap} = 0.5cm$, $\quad A_{gap} =$ effective area.

$$\Rightarrow \quad \mathcal{R}_{gap} = 4.547 \times 10^{6}$$

If we model the core as a sequence of straight reluctances,

$$\text{mean length} = 4 \times 6 - 0.5 \text{ cm} = 23.5 \text{ cm}$$

$$A_{core} = 2 \times 3 \text{cm}^2 = 6 \times 10^{-4} \text{m}^2$$

$$\mu_{core} = 6000 \mu_0 = 7.54 \times 10^{-3}$$

$$\Rightarrow \quad \mathcal{R}_{core} = 5.195 \times 10^{4}.$$

Note that $\mathcal{R}_{core} \ll \mathcal{R}_{gap}$

$$\mathcal{R} = \mathcal{R}_{core} + \mathcal{R}_{gap} = 4.6 \times 10^{6}$$

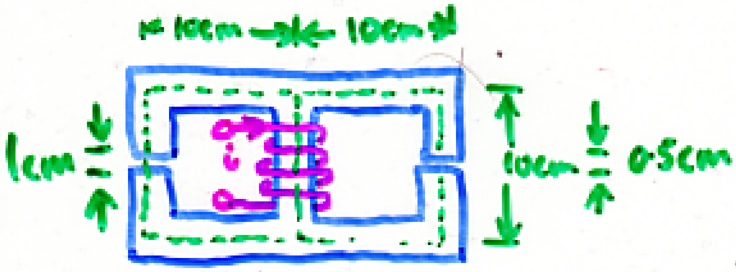
$\Rightarrow$ required MMF is

$$\mathcal{F} = \phi \mathcal{R} = 1006 \text{ A turns.}$$

$$\Rightarrow \quad i = \frac{\mathcal{F}}{N} = 2.012 \text{ A.}$$

Note that most of the MMF is 'dropped' over the air gap.

EXAMPLE



An iron core has a 2cm x 2cm cross section, mean dimensions as shown, permeability $1000\mu_0$

Coil has 500 turns, 2A
Find B in each gap

Solution:

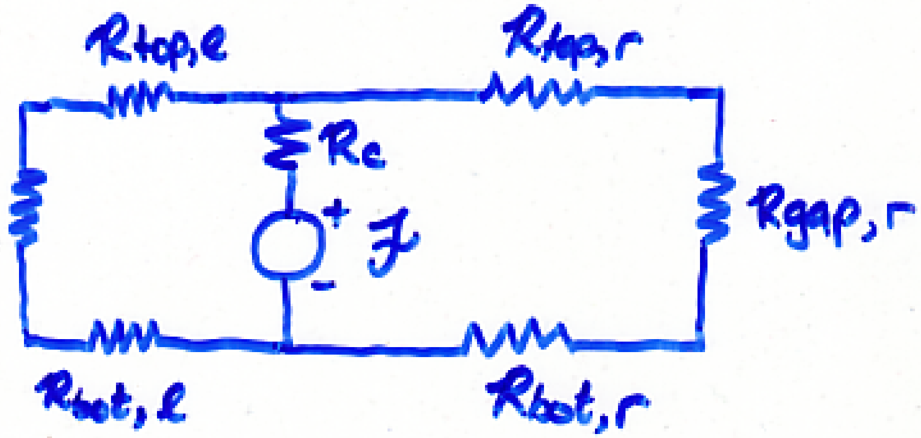
① Find magnetic equiv. circuit, with MMF and reluctances.

② Find ϕ via $\phi = \mathcal{F}/\mathcal{R}$

③ Find B via $B = \phi/A$.

Circuit structure

- Centre strut contains MMF and a common reluctance.
- Left + right loops have different reluctances
- Equiv model:



MMF: $\mathcal{F} = Ni = 1000$ Aturns.

$$\mathcal{R}_c = \frac{l_c}{\mu A_{core}} = \frac{10\times10^{-2}}{1000\mu_0 \times 4\times10^{-4}} = 1.989\times10^{5}$$

$$\mathcal{R}_{top,l} = \frac{(10+4.5)\times10^{-2}}{1000\mu_0 \times 4\times10^{-4}} =$$

$$\mathcal{R}_{bot,l} = \frac{(10+4.5)\times10^{-2}}{1000\mu_0 \times 4\times10^{-4}}.$$

$$\mathcal{R}_{top,r} = \mathcal{R}_{bot,r} = \frac{(10+4.75)\times10^{-2}}{1000\mu_0 \times 4\times10^{-4}}$$

$$\mathcal{R}_{gap,l} = \frac{10^{-2}}{\mu_0\left([2+1]\times[2+1]\right)\times10^{-4}}$$

add length of gap to each dimension to account for fringing

$$\mathcal{R}_{gap,r} = \frac{0.5\times10^{-2}}{\mu_0\left([2+0.5]\times[2+0.5]\right)\times10^{-4}}.$$

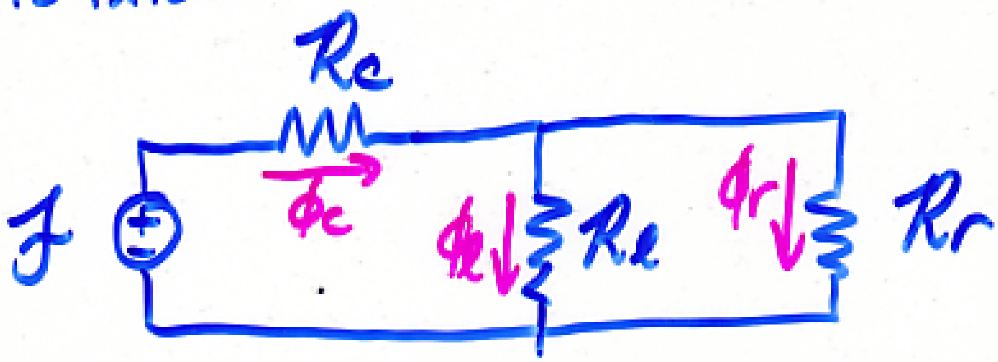
Now combine reluctances in series

$\mathcal{R}_\ell = \mathcal{R}_{top,\ell} + \mathcal{R}_{gap,\ell} + \mathcal{R}_{bot,\ell} = 9.42 \times 10^6.$

$\mathcal{R}_r = \mathcal{R}_{top,r} + \mathcal{R}_{gap,r} + \mathcal{R}_{bot,r} = 6.953 \times 10^6$

$\mathcal{R}_c = 1.989 \times 10^5$

Redraw



$\Rightarrow$ Solve for ϕs using current division ideas

Total reluctance, $\mathcal{R}_t = \mathcal{R}_c + \dfrac{1}{1/\mathcal{R}_\ell + 1/\mathcal{R}_r}$

$= 4.199 \times 10^6$

$\Rightarrow \phi_c = \mathcal{F}/\mathcal{R}_t = 238.1\ \mu\text{Wb}.$

Flux division $\phi_\ell = \phi_c \dfrac{\mathcal{R}_r}{\mathcal{R}_\ell + \mathcal{R}_r} = 101.1\ \mu\text{Wb}$

$\phi_r = \phi_c \dfrac{\mathcal{R}_\ell}{\mathcal{R}_\ell + \mathcal{R}_r} = 137.0\ \mu\text{Wb}$

$$B_{gap,l} = \frac{\phi_l}{([2+1] \times [2+1]) \times 10^{-4}} = 0.1123\,T$$

$$B_{gap,r} = \frac{\phi_r}{([2+0.5] \times [2+0.5]) \times 10^{-4}} = 0.2192\,T$$

Notes

- Core reluctances usually small with respect to gaps.
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