EE3CL4: Introduction to Linear Control Systems

Post-Reading-Week Conceptual Review

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Outline

1. Instinct-based Design
2. Models
3. Feedback systems
4. Control System Design

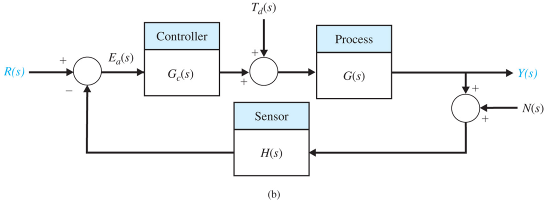
Informal Review

- So what have we done so far?
- Instinct-based design
  - Proportional control of walking to the half-way line; worked quite well, if a bit slow
  - Proportional control of drone hover; did not work so well
  - Weaknesses and lack of reliability suggested model-based design

Models

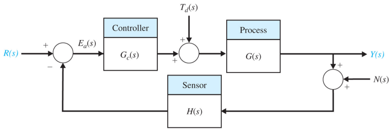
- Simplified models for mechanical systems
  - Free body diagrams; akin to node/mesh analysis
  - Force = mass x acceleration
  - If model is linear and does not change in time;
    $\Longrightarrow$ linear time-invariant (LTI) differential equations
  - Analysis can be simplified using Laplace Transforms
    - Can learn a lot about system behaviour from poles and zeros
    - Also simplifies analysis of interconnections of systems (block diagram models)

Control of LTI systems



- $G(s)$: system to be controlled
- $H(s)$: chosen sensor
  We will focus on "good" sensors that can be approximated by $H(s) \approx 1$
- $G_c(s)$: controller that we will design

Control of LTI systems



(b)

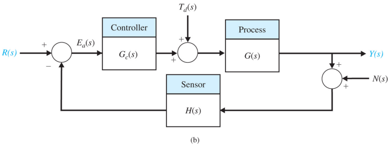
With $H(s) = 1$ and $E(s) = R(s) - Y(s)$,

$$E(s) = \frac{1}{1 + G_c(s)G(s)} R(s) - \frac{G(s)}{1 + G_c(s)G(s)} T_d(s) + \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} N(s)$$

Many properties of these closed-loop transfer functions depend strongly on the open-loop transfer function $G_c(s)G(s)$.

- for good tracking, want $G_c(s)G(s)$ to be large when $R(s)$ is dominant
- for good disturbance rejection, want $G_c(s)G(s)$ to be large when $T_d(s)$ is significant
- for good noise suppression, want $G_c(s)G(s)$ to be small when $N(s)$ is dominant

Control of LTI Systems



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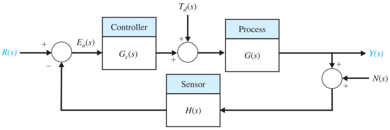
- Also, the steady-state error due to step and ramp inputs
  - depends strongly on the number of integrators in the open loop
  - when finite (and not zero) depends on open-loop pole and zero positions

Control of LTI Systems



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With $H(s) = 1$ and $E(s) = R(s) - Y(s)$,

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- Transient input-output properties depend quite strongly on closed-loop pole and zero positions
  - For second-order systems with no zeros, we can quantify relationships between closed-loop pole positions and settling time, and between closed-loop pole positions and overshoot/damping

Design of LTI Control Systems

- How do we start to quantify our insights?
- We can use the Routh-Hurwitz technique to determine choices of the controller parameters that lead to a stable closed loop
- We can also do this for settling time, but that is algebraically quite complicated
- We can handle steady-state error constraints with simple equations
- We combined these ideas for a two-parameter design approach with stability, steady-state error and settling time constraints
- Enabled us to bound the areas in the design parameter space that gave us the desired performance.
- Extension to three controller parameters makes visualization more difficult; extension to four parameters really difficult.

Design of LTI Control Systems

- Looks like we might need a more flexible design technique
- So many things depend on closed-loop pole (and zero) positions,
  - stability, transient response (including settling time, damping), steady-state errors due to step and ramp,
- let's try to get an idea of the path that the closed-loop poles take as we change one of our design parameters.
- Let's also try to get an idea about how to choose the other design parameters so that that path goes were we would like it to go.
- Finally, let's try to get an idea of how to choose the value of the design parameter so that we are able to place the closed-loop poles at specified points on the path