Tables and Other Information

- The unit step functions are

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

- Sum of an arithmetic series

$$\sum_{n=N_1}^{N_2} (a + nd) = (N_2 - N_1 + 1)a + \frac{(N_2 - N_1 + 1)(N_2 + N_1)d}{2}$$

- Sum of a geometric series

$$\sum_{n=0}^{N-1} \beta^n = \begin{cases} \frac{1-\beta^N}{1-\beta}, & \beta \neq 1 \\ N, & \beta = 1 \end{cases}$$

- Sine and cosine:

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{j2}$$

- Complex exponentials: $e^{j\theta} = \cos\theta + j\sin\theta$
- More trigonometric identities

$$\begin{aligned}
\sin(\theta_1 \pm \theta_2) &= \sin\theta_1 \, \cos\theta_2 \pm \cos\theta_1 \, \sin\theta_2 \\
\cos(\theta_1 \pm \theta_2) &= \cos\theta_1 \, \cos\theta_2 \mp \sin\theta_1 \, \sin\theta_2 \\
\sin\theta_1 \, \sin\theta_2 &= 1/2\cos(\theta_1 - \theta_2) - 1/2\cos(\theta_1 + \theta_2) \\
\cos\theta_1 \, \cos\theta_2 &= 1/2\cos(\theta_1 - \theta_2) + 1/2\cos(\theta_1 + \theta_2) \\
\sin\theta_1 \, \cos\theta_2 &= 1/2\sin(\theta_1 - \theta_2) + 1/2\sin(\theta_1 + \theta_2)
\end{aligned}$$

- The solutions to the quadratic equation $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

  The Final Value Theorem for Laplace Transforms states that if $x(t) = 0$ for $t < 0$, $x(0)$ is finite, and $x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s)$, then $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$.

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Second-order systems with no finite zeros

Consider a second-order linear time-invariant continuous-time system with transfer function

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The poles of this system are at

$$\begin{cases} s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} & \text{if } 0 \le \zeta \le 1, \\ s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1} & \text{if } \zeta \ge 1. \end{cases}$$

If $\zeta < 1$ the system is said to be underdamped, if $\zeta > 1$ the system is said to be overdamped, and if $\zeta = 1$ the system is said to be critically damped.

The step response of a system, $y_{\text{step}}(t)$, satisfies $y_{\text{step}}(t) \overset{\mathscr{L}}{\longleftrightarrow} \frac{T(s)}{s}$. If the second order system $T(s)$ above is underdamped, then its step response is,

$$y_{\text{step}}(t) = \left(1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\, t + \phi\right)\right) u(t),$$

where $\phi = \operatorname{atan}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) = \operatorname{acos}(\zeta)$. The percentage overshoot of this step response is

$$\text{percentage overshoot} = 100 \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

the 10%–90% rise time is approximately

$$T_r \approx \frac{2.16\zeta + 0.6}{\omega_n}$$

and the δ% settling time is approximately

$$T_{\text{settle},\delta} \approx \frac{\ln\left(\frac{100}{\delta}\right)}{\zeta\omega_n};$$

where $\ln(\cdot)$ denotes the natural logarithm.

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C.1 *Basic Discrete-Time Fourier Series Pairs*

Time Domain	Frequency Domain
$x[n] = \sum_{k=\langle N\rangle} X[k]e^{jk\Omega_o n}$ *Period* $= N$	$X[k] = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\Omega_o n}$ $\Omega_o = \frac{2\pi}{N}$
$x[n] = \begin{cases}1, & \lvert n\rvert \le M \\ 0, & M < \lvert n\rvert \le N/2\end{cases}$ $x[n] = x[n+N]$	$X[k] = \dfrac{\sin\left(k\frac{\Omega_o}{2}(2M+1)\right)}{N\sin\left(k\frac{\Omega_o}{2}\right)}$
$x[n] = e^{jp\Omega_o n}$	$X[k] = \begin{cases}1, & k = p, p\pm N, p\pm 2N, \dots \\ 0, & \text{otherwise}\end{cases}$
$x[n] = \cos(p\Omega_o n)$	$X[k] = \begin{cases}\frac{1}{2}, & k = \pm p, \pm p\pm N, \pm p\pm 2N, \dots \\ 0, & \text{otherwise}\end{cases}$
$x[n] = \sin(p\Omega_o n)$	$X[k] = \begin{cases}\frac{1}{2j}, & k = p, p\pm N, p\pm 2N, \dots \\ \frac{-1}{2j}, & k = -p, -p\pm N, -p\pm 2N, \dots \\ 0, & \text{otherwise}\end{cases}$
$x[n] = 1$	$X[k] = \begin{cases}1, & k = 0, \pm N, \pm 2N, \dots \\ 0, & \text{otherwise}\end{cases}$
$x[n] = \sum_{p=-\infty}^{\infty}\delta[n-pN]$	$X[k] = \frac{1}{N}$

C.2 *Basic Fourier Series Pairs*

Time Domain	Frequency Domain
$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_o t}$ *Period* $= T$	$X[k] = \frac{1}{T}\int_{\langle T\rangle} x(t)e^{-jk\omega_o t}\,dt$ $\omega_o = \frac{2\pi}{T}$
$x(t) = \begin{cases}1, & \lvert t\rvert \le T_s \\ 0, & T_s < \lvert t\rvert \le T/2\end{cases}$	
$x(t) = e^{jp\omega_o t}$	
$x(t) = \sum_{p=-\infty}^{\infty}\delta(t-pT)$	

C.3 *Basic Discrete-Time Fourier Transform Pairs*

Time Domain	Frequency Domain
$x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\Omega})e^{j\Omega n}\,d\Omega$	$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$
$x[n] = \begin{cases}1, & \lvert n\rvert \le M \\ 0, & \text{otherwise}\end{cases}$	$X(e^{j\Omega}) = \dfrac{\sin\left[\Omega\left(\frac{2M+1}{2}\right)\right]}{\sin\left(\frac{\Omega}{2}\right)}$
$x[n] = \alpha^n u[n], \quad \lvert\alpha\rvert < 1$	$X(e^{j\Omega}) = \dfrac{1}{1-\alpha e^{-j\Omega}}$
$x[n] = \delta[n]$	$X(e^{j\Omega}) = 1$
$x[n] = u[n]$	$X(e^{j\Omega}) = \dfrac{1}{1-e^{-j\Omega}} + \pi\sum_{p=-\infty}^{\infty}\delta(\Omega - 2\pi p)$
$x[n] = \frac{1}{\pi n}\sin(Wn), \quad 0 < W \le \pi$	$X(e^{j\Omega}) = \begin{cases}1, & \lvert\Omega\rvert \le W \\ 0, & W < \lvert\Omega\rvert \le \pi\end{cases}$, $X(e^{j\Omega})$ is 2π periodic
$x[n] = (n+1)\alpha^n u[n]$	$X(e^{j\Omega}) = \dfrac{1}{(1-\alpha e^{-j\Omega})^2}$

C.4 *Basic Fourier Transform Pairs*

Time Domain	Frequency Domain
$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$	$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$
$x(t) = \begin{cases}1, & \lvert t\rvert \le T \\ 0, & \text{otherwise}\end{cases}$	$X(j\omega) = \dfrac{2\sin(\omega T)}{\omega}$
$x(t) = \frac{1}{\pi t}\sin(Wt)$	$X(j\omega) = \begin{cases}1, & \lvert\omega\rvert \le W \\ 0, & \text{otherwise}\end{cases}$
$x(t) = \delta(t)$	$X(j\omega) = 1$
$x(t) = 1$	$X(j\omega) = 2\pi\delta(\omega)$
$x(t) = u(t)$	$X(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$
$x(t) = e^{-at}u(t), \quad \text{Re}\{a\} > 0$	$X(j\omega) = \dfrac{1}{a+j\omega}$
$x(t) = te^{-at}u(t), \quad \text{Re}\{a\} > 0$	$X(j\omega) = \dfrac{1}{(a+j\omega)^2}$
$x(t) = e^{-a\lvert t\rvert}, \quad a > 0$	$X(j\omega) = \dfrac{2a}{a^2+\omega^2}$
$x(t) = \frac{1}{\sqrt{2\pi}}e^{-t^2/2}$	$X(j\omega) = e^{-\omega^2/2}$

C.5 *Fourier Transform Pairs for Periodic Signals*

Periodic Time-Domain Signal	Fourier Transform
$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_o t}$	$X(j\omega) = 2\pi\sum_{k=-\infty}^{\infty} X[k]\delta(\omega - k\omega_o)$
$x(t) = \cos(\omega_o t)$	$X(j\omega) = \pi\delta(\omega-\omega_o) + \pi\delta(\omega+\omega_o)$
$x(t) = \sin(\omega_o t)$	$X(j\omega) = \frac{\pi}{j}\delta(\omega-\omega_o) - \frac{\pi}{j}\delta(\omega+\omega_o)$
$x(t) = e^{j\omega_o t}$	$X(j\omega) = 2\pi\delta(\omega-\omega_o)$
$x(t) = \sum_{n=-\infty}^{\infty}\delta(t-n\mathcal{T})$	$X(j\omega) = \frac{2\pi}{\mathcal{T}}\sum_{k=-\infty}^{\infty}\delta\left(\omega - k\frac{2\pi}{\mathcal{T}}\right)$
$x(t) = \begin{cases}1, & \lvert t\rvert \le T_s \\ 0, & T_s < \lvert t\rvert < T/2\end{cases}$ $x(t+T) = x(t)$	$X(j\omega) = \sum_{k=-\infty}^{\infty}\dfrac{2\sin(k\omega_o T_s)}{k}\delta(\omega - k\omega_o)$

C.6 *Discrete-Time Fourier Transform Pairs for Periodic Signals*

Periodic Time-Domain Signal	Discrete-Time Fourier Transform
$x[n] = \sum_{k=\langle N\rangle} X[k]e^{jk\Omega_o n}$	$X(e^{j\Omega}) = 2\pi\sum_{k=-\infty}^{\infty} X[k]\delta(\Omega - k\Omega_o)$
$x[n] = \cos(\Omega_1 n)$	$X(e^{j\Omega}) = \pi\sum_{k=-\infty}^{\infty}\delta(\Omega-\Omega_1-k2\pi) + \delta(\Omega+\Omega_1-k2\pi)$
$x[n] = \sin(\Omega_1 n)$	$X(e^{j\Omega}) = \frac{\pi}{j}\sum_{k=-\infty}^{\infty}\delta(\Omega-\Omega_1-k2\pi) - \delta(\Omega+\Omega_1-k2\pi)$
$x[n] = e^{j\Omega_1 n}$	$X(e^{j\Omega}) = 2\pi\sum_{k=-\infty}^{\infty}\delta(\Omega-\Omega_1-k2\pi)$
$x[n] = \sum_{k=-\infty}^{\infty}\delta(n-kN)$	$X(e^{j\Omega}) = \frac{2\pi}{N}\sum_{k=-\infty}^{\infty}\delta\left(\Omega - \frac{k2\pi}{N}\right)$

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C.7 *Properties of Fourier Representations*

Property	Fourier Transform $x(t) \xleftrightarrow{FT} X(j\omega)$ $y(t) \xleftrightarrow{FT} Y(j\omega)$	Fourier Series $x(t) \xleftrightarrow{FS;\,\omega_o} X[k]$ $y(t) \xleftrightarrow{FS;\,\omega_o} Y[k]$ Period $= T$	Discrete-Time FT $x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $y[n] \xleftrightarrow{DTFT} Y(e^{j\Omega})$	Discrete-Time FS $x[n] \xleftrightarrow{DTFS;\,\Omega_o} X[k]$ $y[n] \xleftrightarrow{DTFS;\,\Omega_o} Y[k]$ Period $= N$
Linearity	$ax(t) + by(t) \xleftrightarrow{FT} aX(j\omega) + bY(j\omega)$	$ax(t) + by(t) \xleftrightarrow{FS;\,\omega_o} aX[k] + bY[k]$	$ax[n] + by[n] \xleftrightarrow{DTFT} aX(e^{j\Omega}) + bY(e^{j\Omega})$	$ax[n] + by[n] \xleftrightarrow{DTFS;\,\Omega_o} aX[k] + bY[k]$
Time shift	$x(t - t_o) \xleftrightarrow{FT} e^{-j\omega t_o}X(j\omega)$	$x(t - t_o) \xleftrightarrow{FS;\,\omega_o} e^{-jk\omega_o t_o}X[k]$	$x[n - n_o] \xleftrightarrow{DTFT} e^{-j\Omega n_o}X(e^{j\Omega})$	$x[n - n_o] \xleftrightarrow{DTFS;\,\Omega_o} e^{-jk\Omega_o n_o}X[k]$
Frequency shift	$e^{j\gamma t}x(t) \xleftrightarrow{FT} X(j(\omega - \gamma))$	$e^{jk_o\omega_o t}x(t) \xleftrightarrow{FS;\,\omega_o} X[k - k_o]$	$e^{j\Gamma n}x[n] \xleftrightarrow{DTFT} X(e^{j(\Omega - \Gamma)})$	$e^{jk_o\Omega_o n}x[n] \xleftrightarrow{DTFS;\,\Omega_o} X[k - k_o]$
Scaling	$x(at) \xleftrightarrow{FT} \frac{1}{\lvert a\rvert} X\left(\frac{j\omega}{a}\right)$	$x(at) \xleftrightarrow{FS;\,a\omega_o} X[k]$	$x_z[n] = 0, \quad n \neq lp$ $x_z[pn] \xleftrightarrow{DTFT} X_z(e^{j\Omega p})$	$x_z[n] = 0, \quad n \neq lp$ $x_z[pn] \xleftrightarrow{DTFS;\,p\Omega_o} pX_z[k]$
Differentiation-time	$\frac{d}{dt}x(t) \xleftrightarrow{FT} j\omega X(j\omega)$	$\frac{d}{dt}x(t) \xleftrightarrow{FS;\,\omega_o} jk\omega_o X[k]$	—	—
	$-jtx(t) \xleftrightarrow{FT} \frac{d}{d\omega}X(j\omega)$		$-jnx[n] \xleftrightarrow{DTFT} \frac{d}{d\Omega}X(e^{j\Omega})$	—
	$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{FT} \frac{X(j\omega)}{j\omega} + \pi X(j0)\delta(\omega)$		$\sum_{k=-\infty}^{n} x[k] \xleftrightarrow{DTFT} \frac{X(e^{j\Omega})}{1 - e^{-j\Omega}} + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\Omega - k2\pi)$	—
Convolution	$\int_{-\infty}^{\infty} x(\tau)y(t - \tau)\,d\tau \xleftrightarrow{FT} X(j\omega)Y(j\omega)$	$\int_{\langle T\rangle} x(\tau)y(t - \tau)\,d\tau \xleftrightarrow{FS;\,\omega_o} TX[k]Y[k]$	$\sum_{l=-\infty}^{\infty} x[l]y[n - l] \xleftrightarrow{DTFT} X(e^{j\Omega})Y(e^{j\Omega})$	$\sum_{l=\langle N\rangle} x[l]y[n - l] \xleftrightarrow{DTFS;\,\Omega_o} NX[k]Y[k]$
Modulation	$x(t)y(t) \xleftrightarrow{FT} \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\nu)Y(j(\omega - \nu))\,d\nu$	$x(t)y(t) \xleftrightarrow{FS;\,\omega_o} \sum_{l=-\infty}^{\infty} X[l]Y[k - l]$	$x[n]y[n] \xleftrightarrow{DTFT} \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\Gamma})Y(e^{j(\Omega - \Gamma)})\,d\Gamma$	$x[n]y[n] \xleftrightarrow{DTFS;\,\Omega_o} \sum_{l=\langle N\rangle} X[l]Y[k - l]$
Parseval's Theorem	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\,d\omega$	$\frac{1}{T}\int_{\langle T\rangle} \lvert x(t)\rvert^2\,dt = \sum_{k=-\infty}^{\infty} \lvert X[k]\rvert^2$	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} \lvert X(e^{j\Omega})\rvert^2\,d\Omega$	$\frac{1}{N}\sum_{n=\langle N\rangle} \lvert x[n]\rvert^2 = \sum_{k=\langle N\rangle} \lvert X[k]\rvert^2$
Duality	$X(jt) \xleftrightarrow{FT} 2\pi x(-\omega)$	$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $X(e^{jt}) \xleftrightarrow{FS;1} x[-k]$	$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $X(e^{jt}) \xleftrightarrow{FS;1} x[-k]$	$X[n] \xleftrightarrow{DTFS;\,\Omega_o} \frac{1}{N}x[-k]$
Symmetry	$x(t)$ real $\xleftrightarrow{FT} X^*(j\omega) = X(-j\omega)$ $x(t)$ imaginary $\xleftrightarrow{FT} X^*(j\omega) = -X(-j\omega)$ $x(t)$ real and even $\xleftrightarrow{FT} \operatorname{Im}\{X(j\omega)\} = 0$ $x(t)$ real and odd $\xleftrightarrow{FT} \operatorname{Re}\{X(j\omega)\} = 0$	$x(t)$ real $\xleftrightarrow{FS;\,\omega_o} X^*[k] = X[-k]$ $x(t)$ imaginary $\xleftrightarrow{FS;\,\omega_o} X^*[k] = -X[-k]$ $x(t)$ real and even $\xleftrightarrow{FS;\,\omega_o} \operatorname{Im}\{X[k]\} = 0$ $x(t)$ real and odd $\xleftrightarrow{FS;\,\omega_o} \operatorname{Re}\{X[k]\} = 0$	$x[n]$ real $\xleftrightarrow{DTFT} X^*(e^{j\Omega}) = X(e^{-j\Omega})$ $x[n]$ imaginary $\xleftrightarrow{DTFT} X^*(e^{j\Omega}) = -X(e^{-j\Omega})$ $x[n]$ real and even $\xleftrightarrow{DTFT} \operatorname{Im}\{X(e^{j\Omega})\} = 0$ $x[n]$ real and odd $\xleftrightarrow{DTFT} \operatorname{Re}\{X(e^{j\Omega})\} = 0$	$x[n]$ real $\xleftrightarrow{DTFS;\,\Omega_o} X^*[k] = X[-k]$ $x[n]$ imaginary $\xleftrightarrow{DTFS;\,\Omega_o} X^*[k] = -X[-k]$ $x[n]$ real and even $\xleftrightarrow{DTFS;\,\Omega_o} \operatorname{Im}\{X[k]\} = 0$ $x[n]$ real and odd $\xleftrightarrow{DTFS;\,\Omega_o} \operatorname{Re}\{X[k]\} = 0$

C.8 *Relating the Four Fourier Representations*

Let

$$g(t) \xleftrightarrow{FS;\,\omega_o = 2\pi/T} G[k]$$
$$v[n] \xleftrightarrow{DTFT} V(e^{j\Omega})$$
$$w[n] \xleftrightarrow{DTFS;\,\Omega_o = 2\pi/N} W[k]$$

■ FT Representation for a Continuous-Time Periodic Signal

$$g(t) \xleftrightarrow{FT} G(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} G[k]\delta(\omega - k\omega_o)$$

DTFT Representation for a Discrete-Time Periodic-Signal

$$w[n] \xleftrightarrow{DTFT} W(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} W[k]\delta(\Omega - k\Omega_o)$$

C.9 *Sampling and Aliasing Relationships*

Let

$$x(t) \xleftrightarrow{FT} X(j\omega)$$
$$v[n] \xleftrightarrow{DTFT} V(e^{j\Omega})$$

■ Impulse Sampling for Continuous-Time Signals

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(n\mathcal{T})\delta(t - n\mathcal{T}) \xleftrightarrow{FT} X_\delta(j\omega) = \frac{1}{\mathcal{T}} \sum_{k=-\infty}^{\infty} X\left(j\left(\omega - k\frac{2\pi}{\mathcal{T}}\right)\right)$$

$X_\delta(j\omega)$ is $2\pi/\mathcal{T}$ periodic.

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D.1 *Basic Laplace Transforms*

Signal	Transform	ROC
$u(t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\} > 0$
$tu(t)$	$\frac{1}{s^2}$	$\mathrm{Re}\{s\} > 0$
$\delta(t-\tau),\quad \tau > 0$	$e^{-s\tau}$	for all s
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\} > -a$
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\mathrm{Re}\{s\} > -a$
$[\cos(\omega_1 t)]u(t)$	$\frac{s}{s^2+\omega_1^2}$	$\mathrm{Re}\{s\} > 0$
$[\sin(\omega_1 t)]u(t)$	$\frac{\omega_1}{s^2+\omega_1^2}$	$\mathrm{Re}\{s\} > 0$
$[e^{-at}\cos(\omega_1 t)]u(t)$	$\frac{s+a}{(s+a)^2+\omega_1^2}$	$\mathrm{Re}\{s\} > -a$
$[e^{-at}\sin(\omega_1 t)]u(t)$	$\frac{\omega_1}{(s+a)^2+\omega_1^2}$	$\mathrm{Re}\{s\} > -a$

■ Bilateral Laplace Transforms for Signals that Are Nonzero for $t \le 0$

Signal	Bilateral Transform	ROC
$\delta(t-\tau),\ \tau \le 0$	$e^{-s\tau}$	for all s
$-u(-t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\} < 0$
$-tu(-t)$	$\frac{1}{s^2}$	$\mathrm{Re}\{s\} < 0$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\} < -a$
$-te^{-at}u(-t)$	$\frac{1}{(s+a)^2}$	$\mathrm{Re}\{s\} < -a$

D.2 *Laplace Transform Properties*

Signal	Unilateral Transform	Bilateral Transform	ROC
$x(t)$	$X(s)$	$X(s)$	R_x
$y(t)$	$Y(s)$	$Y(s)$	R_y
$ax(t) + by(t)$	$aX(s) + bY(s)$	$aX(s) + bY(s)$	At least $R_x \cap R_y$
$x(t-\tau)$	$e^{-s\tau}X(s)$ if $x(t-\tau)u(t) = x(t-\tau)u(t-\tau)$	$e^{-s\tau}X(s)$	R_x
$e^{s_o t}x(t)$	$X(s - s_o)$	$X(s - s_o)$	$R_x - \mathrm{Re}\{s_o\}$
$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	$\frac{R_x}{\lvert a\rvert}$
$x(t) * y(t)$	$X(s)Y(s)$	$X(s)Y(s)$	At least $R_x \cap R_y$
$-tx(t)$	$\frac{d}{ds}X(s)$	$\frac{d}{ds}X(s)$	R_x
$\frac{d}{dt}x(t)$	$sX(s) - x(0^+)$	$sX(s)$	At least R_x
$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{s}\int_{-\infty}^{0^+} x(\tau)\,d\tau + \frac{X(s)}{s}$	$\frac{X(s)}{s}$	At least $R_x \cap \{\mathrm{Re}\{s\} > 0\}$

E.1 *Basic z-Transforms*

Signal	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert > \lvert\alpha\rvert$
$n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$[\cos(\Omega_1 n)]u[n]$	$\frac{1 - z^{-1}\cos\Omega_1}{1 - z^{-1}2\cos\Omega_1 + z^{-2}}$	$\lvert z\rvert > 1$
$[\sin(\Omega_1 n)]u[n]$	$\frac{z^{-1}\sin\Omega_1}{1 - z^{-1}2\cos\Omega_1 + z^{-2}}$	$\lvert z\rvert > 1$
$[r^n\cos(\Omega_1 n)]u[n]$	$\frac{1 - z^{-1}r\cos\Omega_1}{1 - z^{-1}2r\cos\Omega_1 + r^2z^{-2}}$	$\lvert z\rvert > r$
$[r^n\sin(\Omega_1 n)]u[n]$	$\frac{z^{-1}r\sin\Omega_1}{1 - z^{-1}2r\cos\Omega_1 + r^2z^{-2}}$	$\lvert z\rvert > r$

■ Bilateral Transforms for Signals that Are Nonzero for $n < 0$

Signal	Bilateral Transform	ROC
$u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert < \lvert\alpha\rvert$
$-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert < \lvert\alpha\rvert$

E.2 *z-Transform Properties*

Signal	Unilateral Transform	Bilateral Transform	ROC
$x[n]$	$X(z)$	$X(z)$	R_x
$y[n]$	$Y(z)$	$Y(z)$	R_y
$ax[n] + by[n]$	$aX(z) + bY(z)$	$aX(z) + bY(z)$	At least $R_x \cap R_y$
$x[n-k]$	See below	$z^{-k}X(z)$	R_x except possibly $\lvert z\rvert = 0, \infty$
$\alpha^n x[n]$	$X\left(\frac{z}{\alpha}\right)$	$X\left(\frac{z}{\alpha}\right)$	$\lvert\alpha\rvert R_x$
$x[-n]$	—	$X\left(\frac{1}{z}\right)$	$\frac{1}{R_x}$
$x[n] * y[n]$	$X(z)Y(z)$	$X(z)Y(z)$	At least $R_x \cap R_y$
$nx[n]$	$-z\frac{d}{dz}X(z)$	$-z\frac{d}{dz}X(z)$	R_x except possibly addition or deletion of $z = 0$

The End.