OBTAINING THE FREQUENCY RESPONSE FROM POLES AND ZEROS.

- Recall that:

  if jw-axis is in the ROC,
  frequency response is a special case of transfer function

  $$H(jw) = H(s)\big|_{s=jw}$$

  if jw-axis outside ROC, system is unstable and frequency response is undefined

  c) if jw-axis on the boundary, system is still unstable, but the frequency response will be finite, except at the poles

If the $j\omega$-axis is in the RoC how can we sketch $|H(j\omega)|$ and $\angle H(j\omega)$?

$$H(s) = \frac{b_M/a_N \; \prod_k (s - c_k)}{\prod_k (s - d_k)}$$

$$H(j\omega) = \frac{b_M/a_N \; \prod_k (j\omega - c_k)}{\prod_k (j\omega - d_k)}$$

what is $|H(j\omega)|$?

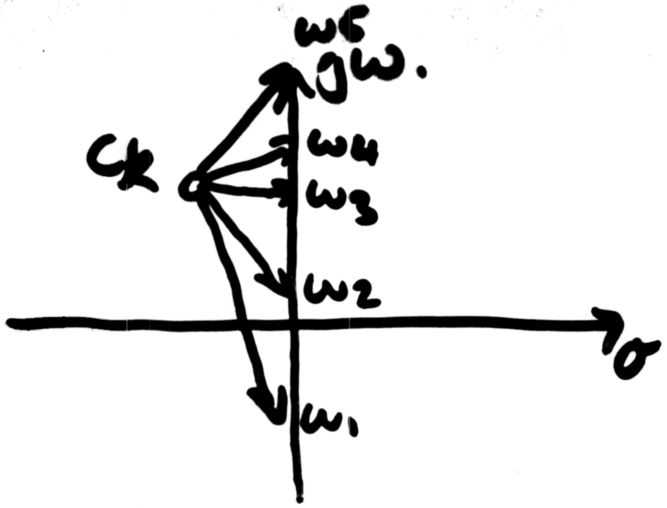
Recall $|ab| = |a||b|$, $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

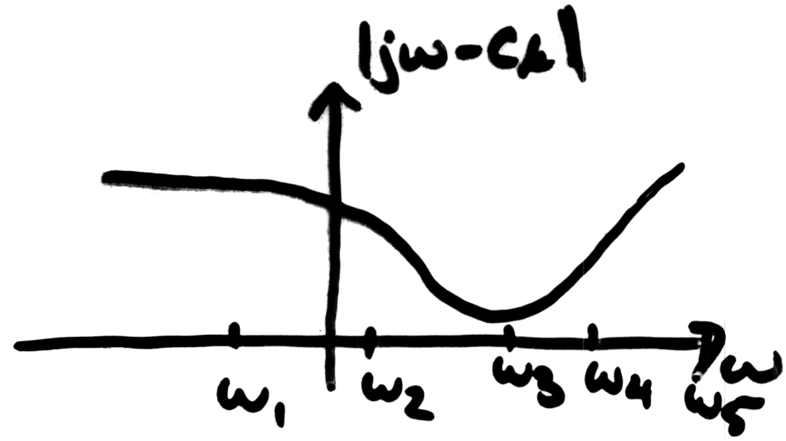
Hence $|H(j\omega)| \quad \dfrac{|b_M/a_N| \; \prod_k |j\omega - c_k|}{\prod_k |j\omega - d_k|}$

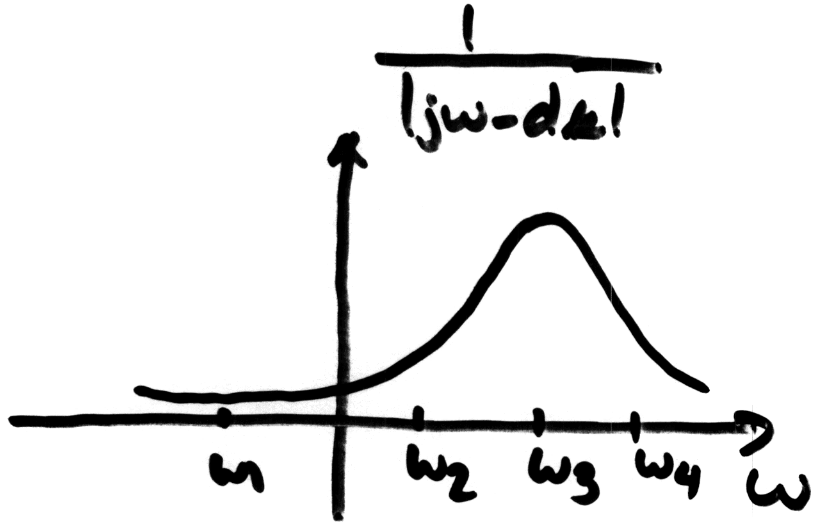
what is $|j\omega - c_k|$? = distance from point $j\omega$ to c_k (zero)

$|j\omega - d_k|$ = distance from point $j\omega$ to pole d_k

$$\Rightarrow |H(j\omega)| = \frac{|b_M/a_M| \cdot \text{product of distances to zeros}}{\text{product of distances to poles}}$$







If poles and zeros are close to the axis, peaks are higher, valleys deeper

If poles and zeros are far from the axis, they have less effect

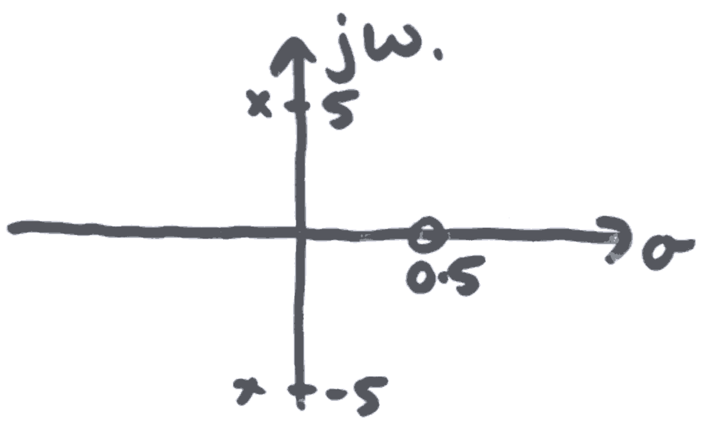
EXAMPLE 6.21

Sketch the magnitude frequency response of a stable LTI system with transfer function

$$H(s) = \frac{s-0.5}{(s+0.1-j5)(s+0.1+j5)}$$

ANS: - System is stable, hence $H(j\omega)$ is finite

- Pole zero plot.



For low frequencies

distance to zero is small

dist to poles is large

$\Rightarrow |H(j\omega)|$ is small

Around $\omega=5$, $\omega=-5$

- distance to one pole is small
- other terms large

$\Rightarrow |H(j\omega)|$ is large.

For large ω, or large -ve ω

all distances approx the same.

$$|H(j\omega)| \sim \frac{\omega}{\omega^2} = \frac{1}{\omega}$$

$\Rightarrow$ small

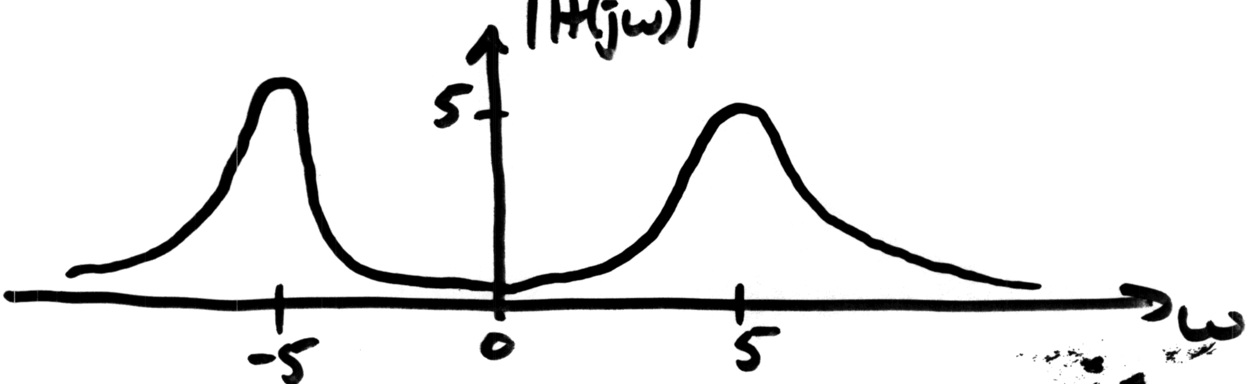
Lets get some numbers

$$H(j0) = \frac{0.5}{|0.1+j5|\ |0.1-j5|}$$

$$\approx \frac{0.5}{5^2} = 0.02$$

$$H(j5) = \frac{|j5-0.5|}{0.1\ |j10+0.1|}$$

$$\approx \frac{5}{0.1+10} \approx 5$$



WHAT ABOUT PHASE?

Recall that for complex numbers, v and z,

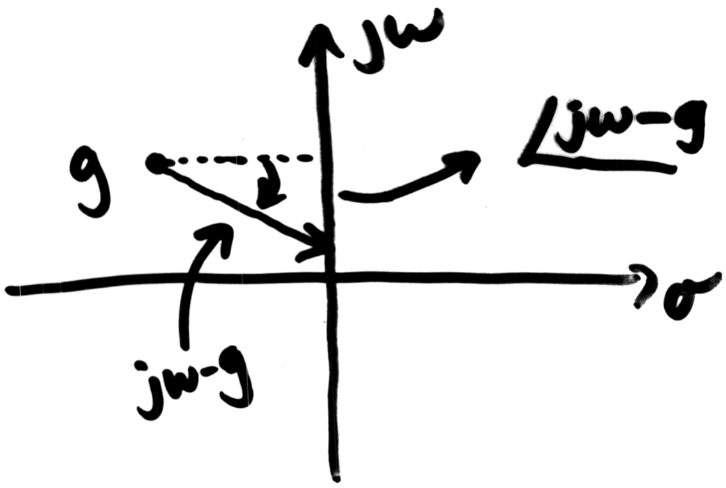
$$\angle \frac{v}{z} = \angle v \quad \angle z$$

$$\angle vz \quad \angle v + \angle z$$

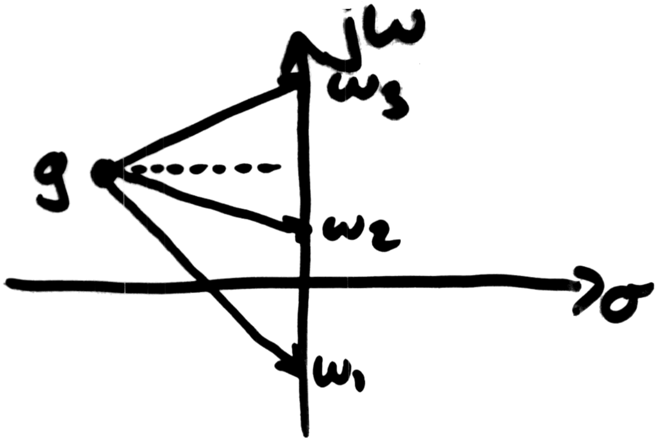
Hence

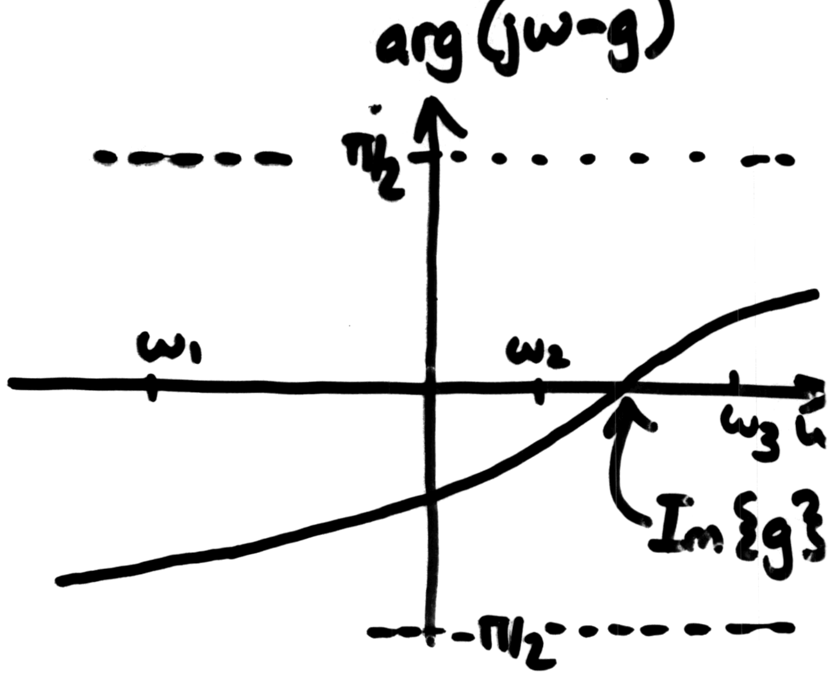
$$\angle H(j\omega) = \angle \frac{b_M}{a_N} + \sum_k \angle j\omega - c_k - \sum_k \angle j\omega - d_k$$

phase of gain + sum of angles subtended to zeros − sum of angles subtended to poles.



Recall angles measured anticlockwise





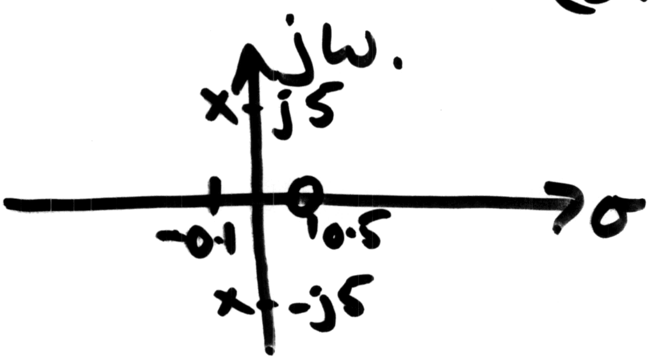
Note f g is close to the axis, phase changes rapidly

if g is far away, phase changes are slower

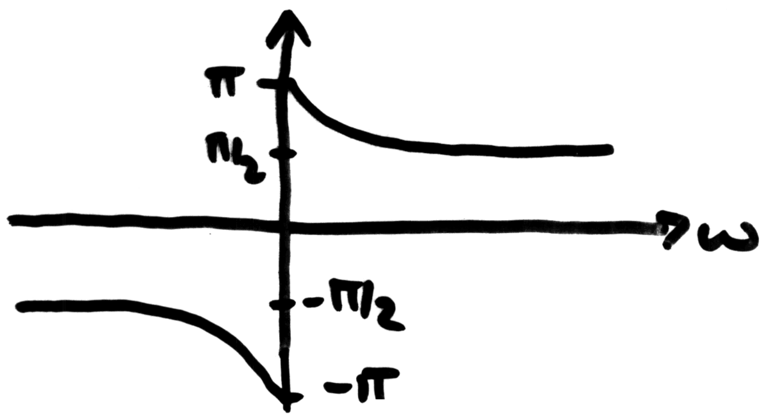
EXAMPLE 6.22.

Sketch the phase response of a stable system with

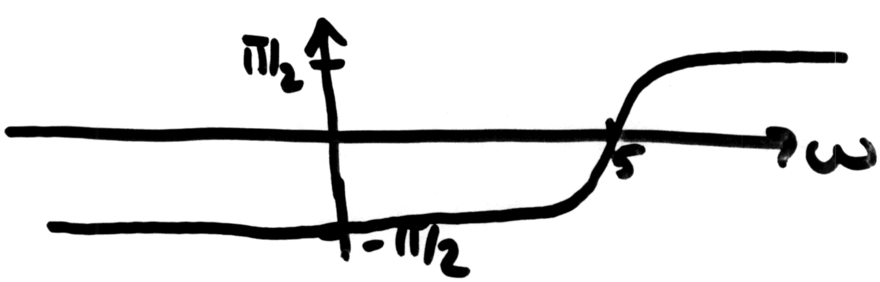
$$H(s) = \frac{s-0.5}{(s+0.1-j5)(s+0.1+j5)}$$



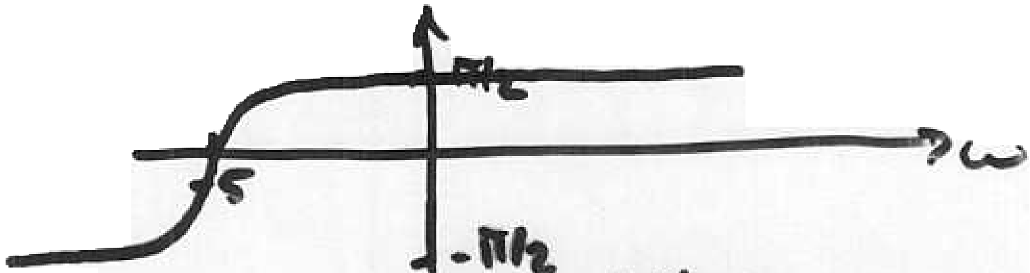
zero only.



upper pole



lower pole



Total