INTRODUCTION

What is a signal?

- a function of one or more variables which conveys information about a physical phenomenon
  - Sometimes we calculate the function ourselves. e.g., GDP
  - Sometimes the function is computed by others (or nature) and only the signal itself can be observed.

What is a system?

An entity which manipulates one or more signals to produce a new signal

e.g., voice recognition,
communications systems,
Automated aircraft landing systems
Canada Arm.
PLAY STATION 2

FUNDAMENTALS

After excitement of potential applications we now must make sure we understand what happens in simple systems

We begin by looking at classes of signals.

1. Real-valued and complex-valued signals
2. Continuous-time and discrete-time signals

Continuous-time: $x(t)$ defined for all real t, e.g. microphone output

Discrete-time: $x[n]$, only defined for integer n

Note notation ( ) encloses real arguments
[ ] encloses integer arguments

Often $x[n]$ is a sampled version of $x(t)$

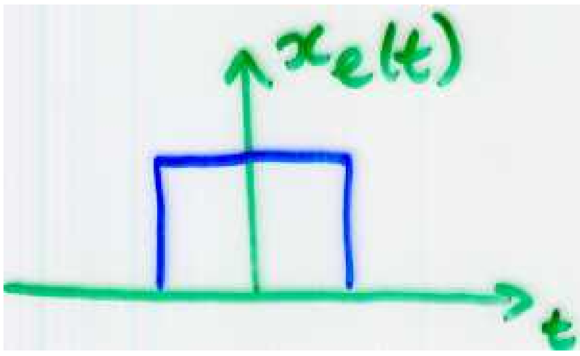
$x[n] = x(nT)$, where T is the sampling period

3. Even symmetric and odd-symmetric signals

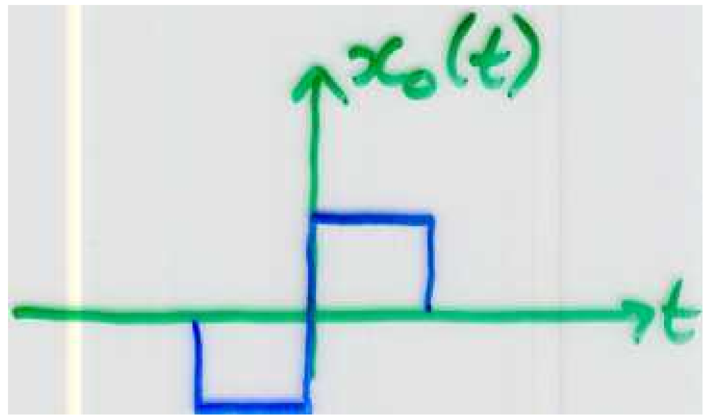
even: $x(-t) = x(t)$, eg cosine

odd: $x(-t) = -x(t)$, e.g. sine

conjugate symmetric: $x(-t) = x^*(t)$



even



odd.

4 Periodic and non-periodic signals

$x(t)$ is periodic with period T if

$$x(t+T) = x(t) \quad \text{for all } T$$

- The smallest such T is the fundamental period
- Fundamental frequency is $f = \frac{1}{T}$ Hz
- Fundamental angular frequency is $\omega = \frac{2\pi}{T}$ rad s^{-1}
- Examples, sine, cosine, squarewave, sawtooth
- If there is no such T, $x(t)$ is non-periodic

$x[n]$ is periodic with period N (integer) if

$$x[n+N] = x[n] \quad \text{for all } n.$$

and is non-periodic if there is no such N.

- The smallest N is the fundamental periodic
- fundamental frequency is $F = \frac{1}{N}$ cycles-per-sample
- fundamental angular freq is $\Omega = \frac{2\pi}{N}$ radians per sample

Note (i) Differences in units of frequency

(ii) we use f, ω for frequency of CT signals

F, Ω for frequency of DT signals

(iii) Not all discrete-time sinusoids are periodic. For example,

$$x[n] = \cos\left(\frac{2\pi}{\sqrt{3}} n\right)$$

has $F = \frac{1}{\sqrt{3}}$

but there is no integer N such that

$$x[n+N] = x[n] \quad \text{for all } n.$$

Deterministic and random signals

Deterministic signals can be modelled by a completely specified function of time

Random signals, such as noise, can only be described in terms of their statistics

We will only deal with deterministic signals in this course. You will deal with random signals in EE3TQ4

Energy and Power Signals

- For a resistor with resistance R + voltage drop $v(t)$, the instantaneous power dissipation is

$$p(t) = \frac{v^2(t)}{R} = i^2(t) R$$

- If the resistor is 1Ω, then these have the form

$$p(t) = x^2(t)$$

which we call the instantaneous power of $x(t)$

The total energy dissipated is

$$E = \lim_{T \to \infty} \int_{-T/2}^{T/2} x^2(t)\,dt$$

and the average power is

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

Note that if $x(t)$ is periodic with period T_0, the average power is

$$P = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t)\,dt$$

In the discrete-time case, sums replace the integrals:

$$E = \sum_{n=-\infty}^{\infty} x^2[n]$$

$$P = \lim_{N \to \infty} \frac{1}{2N} \sum_{n=-N}^{N} x^2[n]$$

For a periodic signal with period N_0,

$$P = \frac{1}{N_0} \sum_{n=0}^{N_0-1} x^2[n]$$

It will be convenient to distinguish between signals of finite energy and finite power:

Energy signals have $0 < E < \infty$

Power signals have $0 < P < \infty$

Note that an energy signal has zero power and a power signal has infinite energy

Deterministic signals which have finite length or decay in time are energy signals

Periodic signals are generally power signals

BASIC OPERATIONS ON SIGNALS.

What might a system do to a signal?

$$x(t) \longrightarrow \boxed{\text{CT System}} \longrightarrow y(t) \qquad\qquad x[n] \longrightarrow \boxed{\text{DT System}} \longrightarrow y[n]$$

Amplitude Scaling Amplification / Attenuation

$$y(t) = c\,x(t) \qquad\qquad y[n] = c\,x[n]$$

Addition, e.g., mixing

$$y(t) = x_1(t) + x_2(t) \quad , \quad y[n] = x_1[n] + x_2[n]$$

Multiplication, e.g., modulation in AM radio and cell phones

$$y(t) = x_1(t)\,x_2(t) \quad ; \quad y[n] = x_1[n]\,x_2[n]$$

Differentiation

$$y(t) = \frac{d}{dt}\,x(t)$$

e.g., inductor, $v(t) = L\,\frac{di(t)}{dt}$

Integration

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

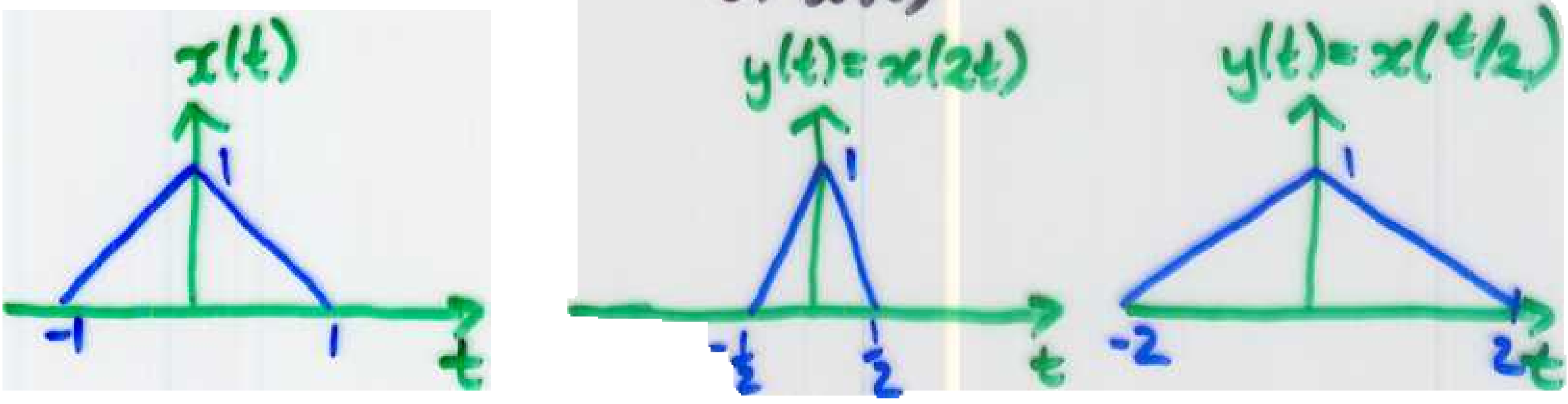
e.g., capacitor, $v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$

Time Scaling

$y(t) = x(at)$

if $a > 1$ $y(t)$ is a compressed/shrunk version of $x(t)$

if $0 < a < 1$ $y(t)$ is an expanded/stretched version of $x(t)$



Time scaling can be done in discrete-time systems, but we'll save that for graduate courses

Reflection

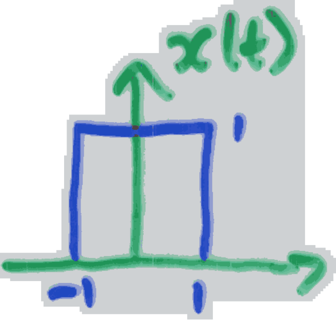
$y(t) = x(-t)$ $y[n] = x[-n]$

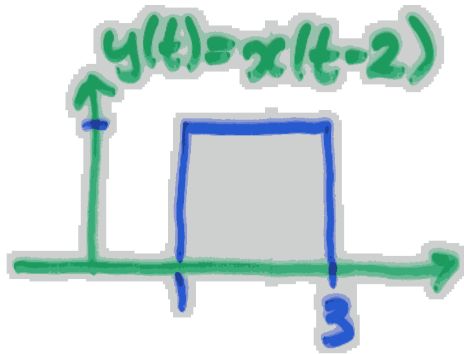
Time Shifting

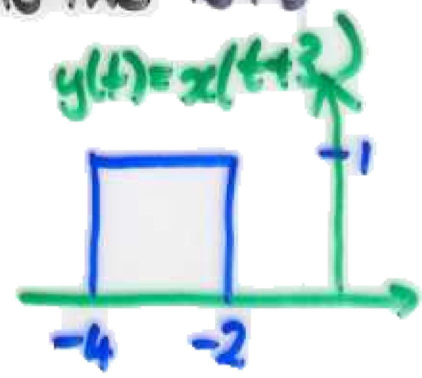
$$y(t) = x(t - t_0) \quad ; \quad y[n] = x[n - n_0]$$

if $t_0, n_0 > 0$ $\quad x$ is shifted to the right

if $t_0, n_0 < 0$ $\quad x$ is shifted to the left







Time Scaling and Time Shifting combined

Many of you will have difficulty sketching

$$y(t) = x(at - b)$$

To help you get it right, first sketch

$$v(t) = x(t - b)$$

Then $y(t) = v(at)$

Sanity checks

$$y(0) = x(-b)$$
$$y(b/a) = x(0)$$

You will need to practice this; eg Ex 1.4