Efficient Design of FMT Systems

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***Abstract*—An efficient channel-independent design method for the prototype filter of a filtered multitone (FMT) transceiver is proposed, along with an iterative power-loading algorithm for FMT. The insight gained from this design is used to choose the number of subchannels in an FMT system.**

***Index Terms*—Convex optimization, filtered multitone (FMT) modulation, multicarrier communications, power loading.**

I. Introduction

FILTERED multitone (FMT) is a multichannel modulation scheme in which the subchannels are synthesized using a modulated filterbank structure [1]. The high level of spectral containment achieved by FMT enables independent processing of the subchannels by the receiver, and makes it a potential candidate for broadband communication systems in which there is a need to mitigate the impact of frequency selectivity, narrowband interference, crosstalk, or regulatory spectral masks; e.g., digital subscriber lines (DSL) [2]. Indeed, it has been shown [1] that for long DSL cables, the achievable rate of FMT is larger than that of discrete multitone (DMT) modulation. FMT is also attractive for certain wireless communication applications [3], [4].

The prototype filter for the FMT filterbank is a key design parameter, and some preliminary design methods have been proposed [1], [5], [6]. We develop herein a more comprehensive approach, in which we quantify an inherent tradeoff between channel-independent measures of the intersubchannel interference (ICI) and the intrasubchannel intersymbol interference (ISI). We provide an efficient algorithm that provides a filter that achieves a desired point on the tradeoff curve, and demonstrate the significant impact that the resulting design can have on the achievable rate of an FMT system.

The achievable rate of a multichannel system depends on the power "loaded" on to each subchannel. While conventional waterfilling provides the optimal power-loading method for DMT systems [2], the presence of nonnegligible ISI in FMT systems renders waterfilling suboptimal. We propose an iterative power-loading algorithm for FMT systems that incorporates the effects of the residual ISI.

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II. System Analysis and Design

The block diagram of an FMT system with M subchannels and up- and down-samplers of rate N is shown in Fig. 1. The filterbank is constructed from a single real finite impulse response (FIR) prototype filter with impulse response $h[n]$ and frequency response $H(e^{j\omega})$. The transmission filter for each subchannel is a frequency-shifted version of the prototype filter, and the first step performed by the receiver is matched filtering of each subchannel. The additive noise in Fig. 1 represents all the external noise and interference, and $c[n]$ denotes the impulse response of the equivalent discrete-time channel. In FMT systems, ISI is usually present in the subchannel outputs $y_k[n]$, and this fact must be incorporated into the design of the detectors [3]. Often, a symbol-rate minimum mean-square error (MMSE) decision-feedback equalizer (DFE) is used [1].

In many multichannel schemes, including DMT, the subchannels are synthesized in a channel-independent manner, and the scheme is adapted to the channel and interference environment by allocating power to the subchannels. The design of FMT systems usually follows the same strategy. In the following subsection, we provide a channel-independent design method for the prototype filter, and in Section II-B, we provide a power-loading algorithm for FMT.

A. Prototype Filter Design

Our approach to the design of the prototype filter is to quantify the inherent tradeoff between channel-independent measures of the ICI and ISI, and to provide an efficient algorithm for obtaining filters that achieve that tradeoff.

In order to bound the ICI, we observe that the signal at the input to the detector of the kth subchannel in Fig. 1 can be written as [7]

$$y_k[n] = \sum_{i=1}^{M} x_i[n] \star f_{ki}[n] + \eta_k[n] \tag{1}$$

where $\star$ denotes convolution, $\eta_k[n]$ represents the additive noise of Fig. 1 after being filtered and down-sampled by a factor of N, and $f_{ki}[n] = g_{ki}[Nn]$, where $g_{ki}[n] = h_i[n] \star c[n] \star h_k^*[-n]$, and $h_i[n] = h[n]e^{j\omega_i n}$. (For notational convenience, we have allowed noncausal filtering at the receiver.) We assume that the sequences to be transmitted satisfy

$$E\left\{x_i[n_1]x_\ell^*[n_2]\right\} = P_i\delta[i-\ell]\delta[n_1-n_2] \tag{2}$$

where $E\{\cdot\}$ denotes statistical expectation, $\delta[\cdot]$ is the Kronecker delta, and P_i is the power of the ith input sequence $x_i[n]$. By computing the power spectral density (PSD) of y_k and by using

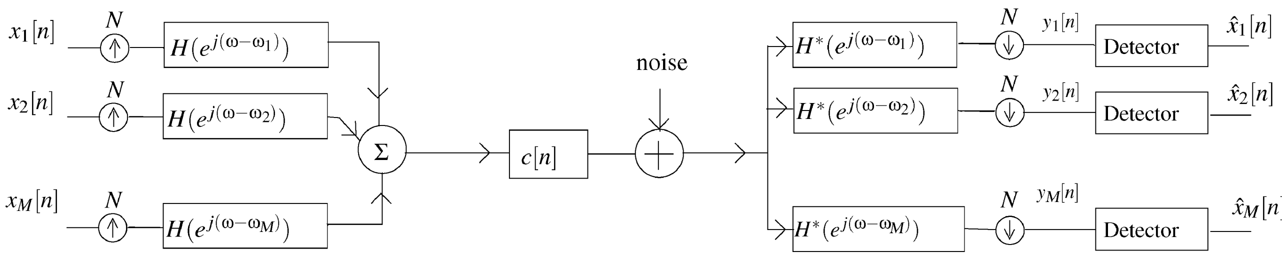

Fig. 1. An M-subchannel FMT communication system with $\omega_i = (i-1)2\pi/M$. We will focus on systems in which the detector incorporates a DFE.

techniques from [8], it can be shown [9] that the power of the ICI present in y_k can be bounded as

$$P_{\text{ICI}} \leq P_I U_c^2 \left(U_p^2 E_{\text{sb}} + U_{\text{sb}}^2 E_h + U_{\text{sb}}^2 E_{\text{sb}}\right) \tag{3}$$

where $P_I = \max_i\{P_i\}$, E_h is the energy of the prototype filter, $E_h = (1/2\pi)\int_0^{2\pi} \left|H(e^{j\omega})\right|^2 d\omega$, E_{sb} is the stopband energy of the prototype filter, $E_{\text{sb}} = (1/2\pi)\int_{\pi/M}^{2\pi-(\pi/M)} \left|H(e^{j\omega})\right|^2 d\omega$, and U_p and U_{sb} are the maximum values of $|H(e^{j\omega})|$ in the passband ($\omega \in (-\pi/M, \pi/M)$) and stopband ($\omega \in [\pi/M, 2\pi - \pi/M]$), respectively. In (3), U_c is an upper bound for $|C(e^{j\omega})|$ $\forall\omega$. It can also be shown [9] that the power of the desired component in y_k can be bounded as

$$P_{\text{desired signal}} \leq P_I U_c^2 U_p^2 E_h. \tag{4}$$

In an FMT system, ISI is introduced both by the filterbank and the channel. To study the ISI that is introduced by the filterbank alone, we assume that the frequency response of the channel is ideal over the subchannel bandwidth. With this assumption, using (1) and (2), it can be shown [9] that the signal-to-ISI power ratio in each subchannel is

$$\frac{\text{Signal power}}{\text{ISI power}} = \frac{E_h^2}{\sum_{n\neq 0}\left(\sum_\ell h[\ell]h[\ell - Nn]\right)^2}. \tag{5}$$

By examining (3)–(5), it can be concluded that in order to suppress the ICI, E_{sb} must be kept small,[1] while ISI suppression requires that the "ISI factor" $\sum_{n\neq 0}(\sum_\ell h[\ell]h[\ell - Nn])^2$ be kept small. These are conflicting objectives, and hence, there is an inherent tradeoff between ICI suppression and ISI suppression. In particular, for filters of a given length and a given energy E_h, if the ISI factor is required to be less than t_d^2, then a filter that achieves the smallest possible value of E_{sb} can be found by solving the following optimization problem:

$$\text{minimize} \quad \frac{1}{2\pi}\int_{\pi/M}^{2\pi-\pi/M} \left|H(e^{j\omega})\right|^2 d\omega \tag{6a}$$

$$\text{subject to} \sum_{n, n\neq 0}\left(\sum_\ell h[\ell]h[\ell - Nn]\right)^2 \leq t_d^2 \tag{6b}$$

$$\sum_\ell (h[\ell])^2 = E_h. \tag{6c}$$

The problem in (6) is not convex, but as shown in the Appendix, it can be precisely transformed into a convex optimization problem in the autocorrelation sequence of $h[n]$, and hence, a globally optimal filter can be efficiently computed. Furthermore, by solving the convex transformation of (6) for different values of t_d, we can efficiently quantify the tradeoff between ICI and the ISI factor.[2] In Section III, we will explain how to choose t_d, and we will see that appropriate selection of t_d can significantly improve the achievable rate of the system.

B. Power-Loading Algorithm

As we will demonstrate in Section III, the prototype filter in an FMT system is usually designed so that the ICI is negligible. Therefore, the ISI in each subchannel can be quite substantial. Even though an MMSE-DFE is typically employed on each subchannel to mitigate the effects of ISI, the residual ISI at the symbol detector within the DFE may not be negligible, especially if low complexity is required. For such systems, the conventional waterfilling power-loading strategy can be significantly suboptimal. In this section, we develop a power-loading algorithm for FMT that incorporates nonnegligible ISI.

Since the ICI generated by a well-designed prototype is negligible, compared with the other sources of interference (such as ISI, additive noise, and crosstalk), the signal-to-interference-plus-noise ratio (SINR) at the input to the symbol detector of the MMSE-DFE in the ith subchannel can be approximated as

$$\text{SINR}_i \approx \frac{P_i\left|\tilde{f}_{ii}[d_i]\right|^2}{P_i\sum_{\ell<d_i}\left|\tilde{f}_{ii}[\ell]\right|^2 + P_{\text{ext}}} \tag{7}$$

where $\tilde{f}_{ii}[\ell]$ denotes the convolution of $f_{ii}[\ell]$ in (1) with the impulse response of the feedforward filter of the MMSE-DFE for the ith subchannel, d_i denotes the "cursor position" of the DFE, and P_{ext} denotes the power of all external noise and interference at the input to the symbol detector in the DFE in the ith

[1] U_{sb} must also be kept small and U_p controlled, but using the fact that $U_{\text{sb}} \ll U_p$, it can be shown that E_{sb} is the dominant component of the bound on the right-hand side of (3). A more general design that includes explicit control of U_{sb} and U_p appears in [9].

[2] The tradeoff can also be efficiently quantified by solving the (precise convex transformation of the) problem $\min(1-\lambda)E_{\text{sb}} + \lambda\sum_{n,n\neq 0}(\sum_\ell h[\ell]h[\ell - Nn])^2$, subject to (6c), for different values of $\lambda \in [0, 1]$. However, since t_d is directly related to the equalization effort required at the receiver, the form in (6) is usually the more convenient one.

subchannel. Note that we have assumed that there is no error propagation in the DFEs, and hence, that the ISI from previously detected symbols is eliminated. A key observation from (7) is that due to the nonnegligible ISI term, both the numerator and denominator depend on P_i.

For an FMT scheme, the conventional power-loading problem can be stated as

$$\text{maximize} \quad \sum_{i=1}^{M} \log_2(1 + \mathrm{SINR}_i/\Gamma) \tag{8a}$$

$$\text{subject to} \quad \sum_{i=1}^{M} P_i = P_{\mathrm{total}}, \quad P_i \geq 0\, \forall i \tag{8b}$$

where SINR_i was given in (7), and Γ is the SINR gap [2]. It can be shown [9] that for a given set of tap values for the DFEs, the optimization problem in (8) is convex, and hence, can be efficiently solved to find the optimal subchannel power allocation. On the other hand, once the subchannel powers have been allocated, the DFE coefficients have to be readjusted. When the DFE coefficients change, $\tilde{f}_{ii}[\ell]$ and P_{ext} will change, and thus each SINR_i will change. Therefore, we propose the following iterative algorithm for power loading in an FMT system. Let $\mathbf{p}_k$ denote the vector of allocated powers at iteration k.

1) Set $k = 0$ and set each element of $\mathbf{p}_0$ to P_{total}/M. Choose the stopping parameter ε.
2) Set $k = k + 1$ and calculate the MMSE-DFE coefficients for each subchannel.
3) Compute $\mathbf{p}_k$ by efficiently solving (8).
4) If $\|\mathbf{p}_k - \mathbf{p}_{k-1}\|_2 < \varepsilon P_{\mathrm{total}}$, stop, else return to step 2.

It can be analytically shown [9] that this algorithm converges, and our examples in Section III indicate that it converges quickly.

III. Numerical Results

We first consider an FMT system with $M = 32$, $N = 36$ (i.e., an excess bandwidth of 12.5%), and a prototype filter of length $L = 320$. For different values of the ISI factor t_d, we found the corresponding prototype filter with minimum stopband energy using the method of Section II. The resulting inherent tradeoff is shown in Fig. 2, where the normalized minimum stopband energy is plotted against the normalized ISI factor t_d/E_h. This is the inherent tradeoff, in the sense that points on the curve can be achieved, but no point below this curve can be achieved by a filter of length ≤ 320. An important engineering design question is: At which point on this tradeoff should the system operate? The answer depends on the equalization effort that one is willing to expend at the receiver in order to mitigate the ISI. An appropriate point on the curve would be the one that maximizes the achievable rate for a given DFE structure. As a practical example, we computed the achievable rates for filters on the tradeoff curve for a voice-grade unshielded twisted pair cable (UTP-3) model [1] for the DSL channel. The PSDs of the near-end and far-end crosstalk (NEXT and FEXT) were computed based on the model for a 50-pair binder [1], [2]. The results were obtained for symmetric transmission with a sampling rate of 11 Msample/s, a transmit signal power of 10 dBm, and an AWGN PSD equal to -140 dBm/Hz. In each subchannel,

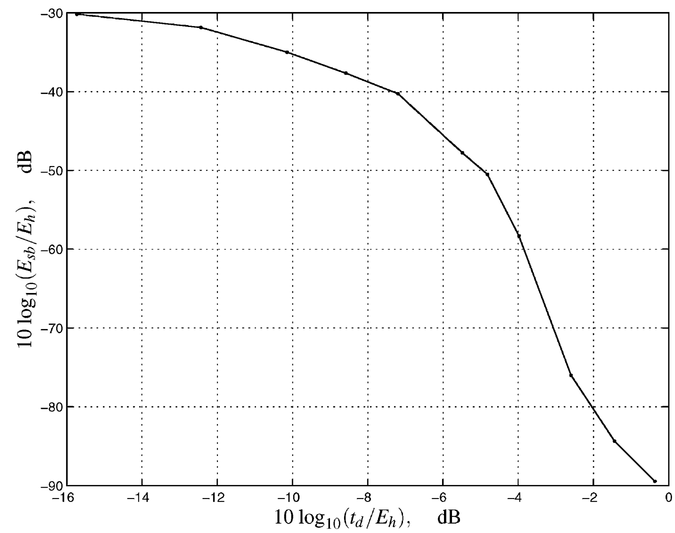

Fig. 2. Tradeoff curve for the normalized stopband energy against the normalized ISI factor of the prototype filter for an FMT system with $M = 32$ subchannels, up-/down-sampling factor $N = 36$, and a prototype filter of length 320.

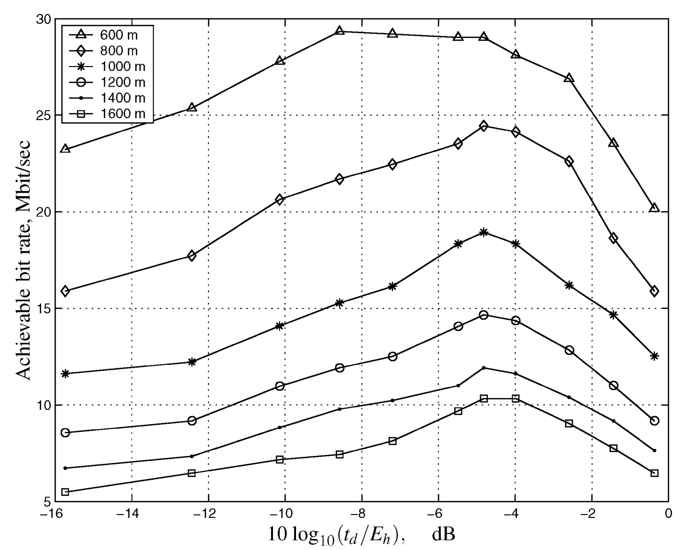

Fig. 3. Achievable bit rate versus normalized ISI factor of the prototype filter for different cable lengths. The FMT system has the characteristics of $M = 32$ subchannels, up-/down-sampling factor $N = 36$, a prototype filter of length 320, and a sampling rate of 11 Msample/s.

a symbol-spaced MMSE-DFE with 20 feedforward taps and 15 feedback taps was employed. The equalizer was assumed to be free of error propagation. An SINR gap of $\Gamma = 9.8$ dB was used in the computations, which is appropriate for quadrature amplitude modulation at a symbol-error probability of 10^{-7} [2]. For the allocation of the power among the subchannels, the power-loading algorithm of Section II was used, with a stopping criterion of $\varepsilon = 0.001$. It was observed that the loading algorithm converged in four or five iterations. Fig. 3 contains the plot of the achievable rate against the normalized ISI factor for various cable lengths between 600 and 1600 m. (These achievable rates are up to 5% higher than those achieved via waterfilling [9].) It is observed that the achievable rate varies substantially as the designed filter traverses the tradeoff of Fig. 2, and it can be seen that a balance between the ISI factor and the stopband

TABLE I
MAXIMUM ACHIEVABLE BIT RATE VERSUS THE NUMBER OF SUBCHANNELS FOR A CABLE LENGTH OF 1600 M

M	N	Maximum achievable bit rate Mbit/sec
8	9	2.07
16	18	4.89
32	36	10.33
64	72	8.96
128	144	3.68

energy of the prototype filter provides the highest achievable rate in Fig. 3. Reducing the ISI factor from that point would reduce the impact of ISI on the achievable rate, but this reduction in ISI would come at the price of an increase in the stopband energy of the filter. The resulting increase in the power of ICI and NEXT would cause a reduction in the achievable rate. From Fig. 2, it can be seen that the prototype filter corresponding to $t_d/E_h = -4.8$ dB provides close to optimal performance for all the cable lengths shown in Fig. 2.

The tradeoff between ISI and ICI also impacts the choice of the number of subchannels in an FMT system. If the number of subchannels is small, the wide-bandwidth subchannels will expose the system to a high level of ISI generated by the variation of the frequency response of the channel over each subchannel. On the other hand, as the number of subchannels grows, the spectral containment required from the prototype filter in order to keep ICI and crosstalk to a manageable level increases. According to the tradeoff curve obtained above, the system will, therefore, suffer from an increasing amount of ISI generated by the prototype filter. As one might expect, the optimal choice for the number of subchannels requires a balance between these two sources of ISI. As an example, we considered the same system as in the above example, but with different numbers of subchannels. (The excess bandwidth was maintained at 12.5%; i.e., $N/M = 1.125$.) We found the optimum filters of length 320 for a cable length of 1600 m and $M = 8, 16, 32, 64, 128$ subchannels. Table I summarizes the resulting maximum achievable rates, and as expected, the highest rate is achieved by using a moderate number of subchannels.

APPENDIX
CONVEX FORMULATION OF (6)

Let L denote the length of the prototype filter and $r_h[n]$ denote the autocorrelation function $r_h[n] = \sum_\ell h[\ell]h[\ell - n]$. Let $R_h(e^{j\omega})$ denote the Fourier transform of $r_h[n]$, and define $b[0] = 1 - 1/M$ and $b[n] = -2/(\pi n)\sin(\pi n/M)$, $n = 1, 2, \ldots, L-1$. Using techniques from [10], the problem in (6) can be precisely transformed into the following convex optimization problem:

$$\text{minimize} \quad \sum_{n=0}^{L-1} r_h[n]b[n] \tag{9a}$$

$$\text{subject to} \quad \sum_{n\geq 1} r_h^2[Nn] \leq \frac{t_d^2}{2} \tag{9b}$$

$$r_h[0] = E_h \tag{9c}$$

$$R_h(e^{j\omega}) \geq 0 \quad \forall\omega. \tag{9d}$$

While the problem in (9) is convex, (9d) generates an infinite number of constraints. These constraints can be approximated by discretization or precisely enforced using a linear matrix inequality; e.g., [11]. Once formulated in one of those forms, the problem in (9) can be efficiently solved for the optimal $r_h[n]$ using general-purpose implementations of interior point methods, such as SeDuMi [12]. We can then extract a corresponding $h[n]$ using standard spectral factorization techniques; e.g., [10].

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