POWER CONTROL FOR MULTIPLE SPECTRUM-SHARING NETWORKS UNDER RANDOM GEOMETRIC TOPOLOGIES

Ran Cai⋆*, Jian-Kang Zhang*†*, Timothy N. Davidson*†*, Kon Max Wong*†*, and P. C. Ching*⋆

⋆ Department of Electronic Engineering, The Chinese University of Hong Kong, Hong Kong
† Department of Electrical and Computer Engineering, McMaster University, Hamilton, ON, Canada

ABSTRACT

This paper develops a power control strategy for multiple spectrum-sharing networks of single antenna nodes in a spectrum underlay scenario. A distinguishing feature of the proposed strategy is that it requires only knowledge of the spatial distribution of the nodes, rather than instantaneous channel state information. The strategy seeks to maximize a weighted sum of the throughput of each network while guaranteeing specified successful transmission probabilities. In its native form, this joint power allocation problem is difficult to solve. However, we show that the problem can be transformed into a convex optimization formulation that can be efficiently solved using general purpose tools. Furthermore, we analyze the optimality conditions and obtain a quasi-closed form solution reminiscent of waterfilling. Numerical results demonstrate that spectrum sharing employing the proposed optimal power yields a substantial throughput gain over allocating the spectrum to a single network.

Index Terms— Spectrum sharing, power control, convex optimization, stochastic geometry, wireless networks.

1. INTRODUCTION

A compelling approach to the enhancement of spectral utilization is to permit two or more networks that occupy a given geographical region to operate in the same frequency band; e.g. [1,2]. This spectrum sharing mode is often referred to as spectrum underlay. In many such scenarios there is a licensed primary network whose access to the spectrum is to be guaranteed, and a secondary network whose nodes can access the band so long as the interference that they impose on the primary network lies below a tolerable limit. Recent works (e.g., [3–13]) have demonstrated that judicious power control schemes can indeed yield a substantial increase in the overall spectral utilization while maintaining specified levels of quality-of-service (QoS) in the individual networks. However, most of the existing schemes have been developed for deterministic network topologies in which either channel state information (CSI) or user location is required. In practice, obtaining sufficiently accurate channel or position information may be difficult, or may consume an unreasonably large fraction of the spectrum resources provided by the channel that would otherwise be assigned to communication. Even if robustness to inaccuracies in the available information is explicitly incorporated into the designs (e.g., [10–13]), substantial resources are still required to exchange the necessary information.

In this work, we take a different approach and devise a power control strategy based on a stochastic model for the geometry of the networks. In particular, we employ independent two-dimensional Poisson point process models [14] for the stochastic geometry of each network. This model has been adopted in some analyses of spectrum-sharing networks [15–22], and has been employed in the development of power control schemes for a pair of spectrum-sharing networks [21, 22]. In this paper we address the problem of power control for multiple spectrum-sharing networks. This is an important problem because the spectrum utilization efficiency is conditioned on the number of secondary networks [19, 23]. Furthermore, our work not only provides an algorithm for dynamic spectrum access in cognitive radio networks [2] with low signaling overhead, but also provides guidance in the design of conventional networks with concurrent transmission, including Wi-Fi, Bluetooth, sensors and other cordless devices operating in the ISM band of 2.4 GHz [24, 25], and even for the emerging co-existence problem of macro-cells and micro-cells [26].

In formulating the power control problem for multiple spectrum-sharing networks using only stochastic models for the network geometries, we consider the increase in utility that can be achieved and the need to control the total interference imposed on each network. We formulate the power control problem so as to maximize a weighted sum of the throughput of each network while guaranteeing that the increase in the outage probability in each network incurred by spectrum sharing is bounded by a pre-specified level (Actually, that constraint is specified in terms of the decrease in the probability of successful transmission.). While that problem initially appears to be difficult to solve, we show that it can be transformed into a convex optimization problem that can be efficiently solved. This transformation decouples the variables in the objective and all but one of the constraints, and that decoupling is exploited to obtain a quasi-closed form solution to the problem.

2. SPECTRUM-SHARING NETWORK MODEL

We consider a scenario in which one primary network (PN), referred to as network 0, and multiple secondary networks (SNs), referred to as networks $1, 2, \ldots, M$, coexist in the same region and share the same spectrum. Fig. 1 illustrates an example of this type of spectrum sharing between three networks.

The proposed power control strategy is based on a model for the spatial distribution of the nodes in the networks. The primary transmitters (PTs) are modeled as being distributed according to a homogeneous two-dimensional Poisson point process (PPP) of density λ_0, denoted by $\Phi_0 = \left\{X_0^{(j)}\right\}$, where $X_0^{(j)} \in \mathbb{R}^2$ is the coordinate of the PT j. A reference primary receiver (PR) is assumed to be located at O_0, which is a distance R_0 away from its associated PT located at $X_0^{(0)}$. Based on the Palm distribution and Slivnyak's theory of PPP in stochastic geometry [14, 27], all the receivers in a PPP network have the same statistics for signal reception and an

This work was partially supported by a grant from the Natural Science and Engineering Research Council of Canada.

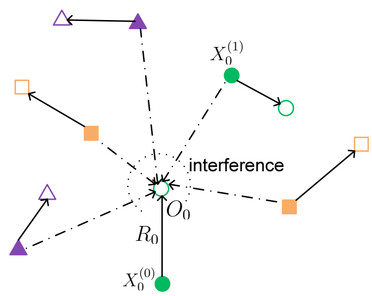

Fig. 1: An example of spectrum sharing between one PN (circles) and two SNs (triangles and squares). The solid nodes denote transmitters, the hollow nodes denote receivers, solid arrows represent transmission links, and dashed arrows represent the interference at the reference receiver O_0 of the PN.

additional node does not change the distribution of the others. Therefore, the performance of the PN can be evaluated through a reference transmitter-receiver location pair $\left(X_0^{(0)}, O_0\right)$.

The distribution of the transmitters in the SN m is modeled by an independent homogeneous PPP of density λ_m, denoted by $\Phi_m = \left\{X_m^{(j)}\right\}$. The reference transmitter-receiver location pair for SN m is denoted by $\left(X_m^{(0)}, O_m\right)$, and the associated reference transmission distance is denoted by R_m.

Each node in each network is equipped with a single antenna and performs omnidirectional single-hop transmission. Considering both path-loss and the Rayleigh fading effects, the received power $P_{a,b}$ at receiver b from transmitter a can be modeled as

$$P_{a,b} = P_a \cdot H_{a,b} \cdot D_{a,b}^{-\alpha},$$

where P_a is the transmitting power at node a and $H_{a,b}$ is the Rayleigh fading factor from node a to node b, which is exponentially distributed with unit mean. All the Rayleigh fading components are modeled as being independent and identically distributed (i.i.d.). The term $D_{a,b}$ is the distance between nodes a and b, and α is the path-loss exponent which is assumed to be constant over the region of interest. All the transmitters in network m employ the same transmission power P_m. The sum interference $I_{n,m}$ from network n to the reference receiver O_m in network m is $I_{n,m} = P_n \cdot \sum_{X_n^{(j)} \in \Phi_n} H_{X_n^{(j)}, O_m} \cdot D_{X_n^{(j)}, O_m}^{-\alpha}$.

We consider interference-limited spectrum sharing, in the sense that the impact of the thermal noise is negligible in comparison to the interference. One of the network performance criteria that we will consider is a QoS requirement; that is, a specified signal-to-interference ratio (SIR) should be achieved at the reference receiver with high probability. Given an SIR target for network m, denoted by β_m, the probability that the SIR is satisfied is

$$p_m = \mathbb{P}\left[\mathrm{SIR}_m > \beta_m\right] = \exp\Big[-b_m \sum_{n=0}^{M} \lambda_n (P_n/P_m)^{2/\alpha}\Big], \quad (1)$$

where $b_m = 2\pi^2\alpha^{-1}\csc(2\pi\alpha^{-1})\beta_m^{2/\alpha}R_m^2 > 0$ and $\csc(\cdot)$ represents the cosecant function. The expression in (1) can be obtained by extending the results in [17, 22] in a straightforward way. As expected, p_m is an increasing function of P_m, due to stronger signal reception, and a decreasing function of the transmission power of the other networks. Typically, the target SIR β_m is selected so that if this SIR is achieved then the chosen coding scheme for network m is able to communicate successfully with high probability. For that reason, we will refer to p_m as the successful transmission probability. To form a baseline for our evaluation of the performance of spectrum sharing, we define exclusive access as the policy that only network m uses the spectrum, without the appearance of any other network. Since successful transmission probability is defined in terms of an SIR target, in the exclusive access case, it is independent of transmission power. If network m is granted exclusive access, the expression in (1) reduces to

$$\widetilde{p}_m = \exp(-b_m\lambda_m). \quad (2)$$

The performance of the spectrum-sharing networks will be assessed using a utility function that is defined as a weighted sum of the throughout of each network, i.e.,

$$U = \sum_{m=0}^{M} \omega_m \cdot B \cdot \log_2(1+\beta_m) \cdot p_m, \quad (3)$$

where B represents the bandwidth, $B \cdot \log_2(1+\beta_m)$ is the maximum achievable rate at a target SIR of β_m, and p_m in (1) is the probability that idealized transmission at that rate is successful. The positive weight ω_m may be selected to signify the relative access priorities of the networks [13], fairness consideration [19], and the probability that the network is allowed to transmit [28]. This spectrum-sharing throughput utility function is also consistent with transmission capacity [29]. The baseline value for the utility is the value obtained when the PN is granted exclusive access to the frequency band: $U_0 = \omega_0 \cdot B \cdot \log_2(1+\beta_0) \cdot \exp(-b_0\lambda_0)$.

3. OPTIMAL POWER CONTROL STRATEGY

The power control problem of interest is the maximization of the utility of the multiple spectrum-sharing networks, while guaranteeing that the decrement in the successful transmission probability of each network m caused by spectrum sharing from what would have been achieved under exclusive access is less than a prescribed limit, denoted by δ_m. If we let $\mathbf{p} = [P_1, P_2 \ldots, P_M]^T$ denote the vector of secondary transmission powers, then the problem can be formulated as

$$\begin{aligned} (\mathbf{P0}) \ \max_{\mathbf{p}} \quad & U \\ \text{s.t.} \quad & \widetilde{p}_m - p_m \le \delta_m, \quad \text{for } m = 0, \ldots, M \\ & P_m \ge 0, \quad \text{for } m = 1, \ldots, M. \end{aligned}$$

where p_m was defined in (1), $\widetilde{p}_m$ in (2) and U in (3). In order to obtain a simplified formulation, let $q_m = P_m^{2/\alpha}$ be the scaled transmission power of network m, and $\mathbf{q} = [q_1, \ldots, q_M]$ be the scaled secondary transmission power vector. Then the above power control problem can be expressed explicitly as

$$(\mathbf{P1}) \ \max_{\mathbf{q}} \ \sum_{m=0}^{M} a_m \exp\Big[-b_m \sum_{n=0}^{M} \lambda_n (q_n/q_m)\Big] \quad (4a)$$

$$\text{s.t.} \ \sum_{n=0}^{M} \lambda_n q_n/q_m \le \eta_m, \quad \text{for } m = 0, \ldots, M \quad (4b)$$

$$q_m \ge 0, \quad \text{for } m = 1, \ldots, M. \quad (4c)$$

where $a_m = \omega_m\lambda_m B\log_2(1+\beta_m) > 0$ is the positive weighted data rate, and $\eta_m = -\ln[\exp(-b_m\lambda_m) - \delta_m]/b_m > 0$ is proportional to λ_m. Note $\lambda_m/\eta_m = -b_m\lambda_m/\ln[\exp(-b_m\lambda_m) - \delta_m] < -b_m\lambda_m/\ln[\exp(-b_m\lambda_m)] = 1$.

We begin our analysis of $(\mathbf{P1})$ with the following theorem.

Theorem 1. *Problem* $(\mathbf{P1})$ *is feasible iff* $\sum_{n=0}^{M}(\lambda_n/\eta_n) \le 1$.

Proof: Necessary condition: Let $t = \sum_{n=0}^{M}\lambda_n q_n > \lambda_0 q_0 = t^{(0)}$. From (4b) and (4c), we have that $q_n \ge t^{(0)}/\eta_n = q_n^{(0)}$. Then we can substitute $q_n^{(0)}$ to (4b) and derive a tighter lower bound $t \ge t^{(1)} = t^{(0)}\big[1 + \sum_{n=1}^{M}(\lambda_n/\eta_n)\big]$. After k iterations, $t \ge t^{(k)} = \lambda_0 q_0 + t^{(k-1)}\sum_{n=1}^{M}(\lambda_n/\eta_n) = t^{(0)}\big\{1+\sum_{n=1}^{M}(\lambda_n/\eta_n) + \ldots + \big[\sum_{n=1}^{M}(\lambda_n/\eta_n)\big]^k\big\}$. Given that $t^{(k)}$ is bounded, we have $\sum_{n=1}^{M}(\lambda_n/\eta_n) < 1$ for all the SNs, and hence the lower bound of t converges to $t_{\min} = t^{(0)}\big[1 - \sum_{n=1}^{M}(\lambda_n/\eta_n)\big]^{-1}$. Also, by substituting $m = 0$ into (4b), we infer that $t \le t_{\max} = \eta_0 q_0$. Given $t_{\min} \le t_{\max}$, we further have $\sum_{n=0}^{M}(\lambda_n/\eta_n) \le 1$ where this sum includes the PN. Consequently, a necessary condition for the feasibility of $(\mathbf{P1})$ is $\sum_{n=0}^{M}(\lambda_n/\eta_n) \le 1$.

Sufficient condition: Given $\sum_{n=0}^{M}(\lambda_n/\eta_n) \le 1$, we have $\sum_{n=1}^{M}(\lambda_n/\eta_n) < 1$, and $t_{\min} = \lambda_0 q_0[1-\sum_{n=1}^{M}(\lambda_n/\eta_n)]^{-1} \le t \le t_{\max}$, then there exists a feasible point $[t/\eta_1, \ldots, t/\eta_m, \ldots, t/\eta_M]^T$. Thus, $(\mathbf{P1})$ is feasible if $\sum_{n=0}^{M}(\lambda_n/\eta_n) \le 1$. ■

The feasibility condition in Theorem 1 guides the selection of the network density λ_m, SIR threshold β_m, and the limit on the decrement of the successful transmission probability, δ_m. For an individual network we always have $\lambda_m/\eta_m < 1$. Theorem 1 indicates how these network properties must be related in order for multiple networks to coexist.

Corollary 1. *If problem* $(\mathbf{P1})$ *is feasible, then the optimal solution exists, and the optimal power allocated for each SN is bounded between two network-dependent multiples of the transmission power of the PN, i.e.,* $\Big\{\lambda_0\big[1 - \sum_{n=1}^{M}(\lambda_n/\eta_n)\big]^{-1}\eta_m^{-1}\Big\}^{\alpha/2}P_0 \le P_m \le \big[(\eta_0 - \lambda_0)\lambda_m^{-1}\big]^{\alpha/2}P_0$, $\forall m = 1, 2 \ldots, M$.

Proof: From (4b) and (4c), $t_{\min}\eta_m^{-1} \le q_m \le (\eta_0 - \lambda_0)\lambda_m^{-1}q_0$. Thus, the feasible region of $(\mathbf{P1})$ is closed and bounded. Since the objective function of $(\mathbf{P1})$ in (4a) is continuous and differentiable, the solution exists according to the Theorem of Weierstrass [30]. The bounds arise from (4b), (4c) and the definition of q_m. ■

Corollary 1 indicates the relation between the primary and secondary transmission powers, also implies that under the feasibility condition, every SN obtains access to the spectrum and contributes to the total spectrum-sharing throughput. None of the spectrum-sharing networks will monopolize or vanish, though the contribution of network m can be controlled through the weight ω_m.

Problem $(\mathbf{P1})$ can be solved by using an exhaustive M dimensional search in the feasible region. However, that search rapidly becomes costly as M increases. Hence, we are encouraged to develop more efficient algorithms. Let $r_m = q_m b_m^{-1} t^{-1}$ be the scaled power allocation ratio of network m. With that notation, we can formulate an equivalent optimization problem $(\mathbf{P2})$ with respect to the $M+1$ dimensional parameter vector $\mathbf{r} = [r_0, r_1, \ldots, r_M]^T$:

$$(\mathbf{P2})\ \max_{\mathbf{r}} \sum_{m=0}^{M} a_m \exp(-r_m^{-1}) \tag{5a}$$

$$\text{s.t.}\ (\eta_m b_m)^{-1} - r_m \le 0, \quad \text{for } m = 0, \ldots, M \tag{5b}$$

$$\sum_{m=0}^{M} \lambda_m b_m r_m - 1 = 0. \tag{5c}$$

As formalized in the following theorem, under the assumption that the successful transmission probability of each SN in exclusive access mode is not unreasonably small, problem $(\mathbf{P2})$ is convex.

Theorem 2. *If* $\widetilde{p}_m > \exp(-2) + \delta_m$, $\forall m = 0, 1 \ldots, M$, *problem* $(\mathbf{P2})$ *is strictly convex.*

Proof: Let $f_m(r_m) = \exp(-r_m^{-1})$, then $f_m''(r_m) = r_m^{-3}\exp(-r_m^{-1})(r_m^{-1} - 2)$. From (5b), $r_m^{-1} \le \eta_m b_m = -\ln[\exp(-b_m\lambda_m) - \delta_m]$. If $\widetilde{p}_m = \exp(-b_m\lambda_m) > \exp(-2) + \delta_m \approx 0.1353 + \delta_m$, then $r_m^{-1} < 2$, and hence $f_m''(r_m) < 0$. Since the objective function in $(\mathbf{P2})$ is $\sum_{m=0}^{M} a_m f_m(r_m)$ with $a_m > 0$, it is sufficient to show that the objective function is strictly concave. The proof follows by observing that the constraints are linear. ■

The key to the convexity of $(\mathbf{P2})$ is that by defining $r_m = q_m b_m^{-1} t^{-1}$, the M variables that were coupled in $(\mathbf{P1})$, q_m, are replaced by $M+1$ decoupled variables in $(\mathbf{P2})$, r_m. Furthermore, by strict convexity, Theorem 2 guarantees the uniqueness of the global optimum. Problem $(\mathbf{P2})$ can be efficiently solved using general purpose tools, such as interior-point methods [31]. The global optimal transmission power of SN m can then be extracted via the optimal solution $\mathbf{r}^* = [r_0^*, r_1^*, \ldots, r_M^*]^T$ of $(\mathbf{P2})$ and the PN power P_0, based on the expressions for r_m and q_m. The proposed approach can be summarized by Algorithm 1.

Algorithm 1 Using general purpose tools

1: Solve $(\mathbf{P2})$ using a general purpose tool like an interior-point method, and obtain the maximum spectrum-sharing throughput utility U^* and the associated globally optimal $\mathbf{r}^*$.
2: The optimal secondary transmission power is $P_m^* = [r_m^* b_m (r_0^* b_0)^{-1}]^{\alpha/2} P_0$, for $m = 1, \ldots, M$.

Algorithm 1 efficiently solves the QoS-constrained weighted sum rate problem $(\mathbf{P2})$. However, employing a general purpose solver is not necessarily the most efficient approach. By carefully inspecting (5b), we observe that some components of the optimal solution lie on the boundary of $r_m^* = (\eta_m b_m)^{-1}$, whereas the others satisfy $r_m^* > (\eta_m b_m)^{-1}$. This suggests that it may be possible to derive a specialized algorithm, or even obtain a quasi-closed form solution. We now take steps in that direction with some formal results on the structure of the optimal solution.

Theorem 3. *The optimal secondary transmission power is* $r_m^* = (\eta_m b_m)^{-1}$, $\forall m = 0, 1, \ldots, M$, *iff* $\sum_{n=0}^{M}(\lambda_n/\eta_n) = 1$.

Proof: The necessary condition is established by substituting $r_m^* = (\eta_m b_m)^{-1}$ into (5c). Sufficiency is shown as follows: Suppose $\exists\, \breve{m} \in \{0, 1, \ldots, M\}$, such that $r_{\breve{m}} > (\eta_{\breve{m}} b_{\breve{m}})^{-1}$. Then $1 \ge \sum_{n\ne\breve{m}} \lambda_n/\eta_n + \lambda_{\breve{m}} b_{\breve{m}} r_{\breve{m}} > \sum_{n=0}^{M}(\lambda_n/\eta_n)$, which contradicts the assumption. ■

To analyze the case when $\sum_{n=0}^{M}(\lambda_n/\eta_n) < 1$, let us define $F(x) = x^{-2}\exp(-x^{-1})$, $c_m = a_m b_m \eta_m^2 \lambda_m^{-1}\exp(-\eta_m b_m)$, and re-index the networks using $\{m_k\}$ rather than $\{m\}$, so that $c_{m_k} \ge c_{m_{k+1}}$, for $k = 0, 1, \ldots, M-1$.

Lemma 1. *If there are* i *optimal components satisfying* $r_m^* > (\eta_m b_m)^{-1}$ *and* $M - i$ *optimal components satisfying* $r_m^* = (\eta_m b_m)^{-1}$, *then* $r_{m_k}^* > (\eta_{m_k} b_{m_k})^{-1}$, *for* $k = 0, \ldots, i-1$, *and* $r_{m_k}^* = (\eta_{m_k} b_{m_k})^{-1}$, *for* $k = i, \ldots, M$.

The proof is based on the analysis of the Karush-Kuhn-Tucker (KKT) optimality conditions [31] and the monotonicity of the function $F(r_m)$. The detailed proof is omitted due to space limitations.

Table 1: Spectrum sharing between two networks

Weights	Nature of P_1^*	Value of P_1^* (W)	U (bits/s/Hz)	U_0 (bits/s/Hz)	Utility improvement (%)
$\omega_0 = \omega_1 = 0.5$	interior point	0.21	2.87	1.71	68%
$\omega_0 = 0.1, \omega_1 = 0.9$	at the upper bound	0.79	2.52	0.34	640%
$\omega_0 = 0.95, \omega_1 = 0.05$	at the lower bound	0.03	3.33	3.24	3%

Lemma 1 indicates that if we know the number of optimal components that are on the boundary, then we can determine which components are on the boundary simply by re-indexing the networks. The remaining question is whether i is the largest number of components that satisfy $r_m > (\eta_m b_m)^{-1}$. The following lemma, whose proof is also omitted for brevity, will help us determine the value of i. Let $U^{(j)^*}$ denote the maximal value of objective function of $(\mathbf{P2})$ when $r_{m_k} = (\eta_{m_k} b_{m_k})^{-1}$, for $k = j+1, \dots, M$, in addition to (5b) and (5c) holding for r_{m_k}, for $k = 0, \dots, j$.

Lemma 2. $U^{(j+1)^*} \geq U^{(j)^*}$*, for* $j = 0, 1, \dots, M-1$.

Lemma 2 indicates that we should seek the largest value for i for which the KKT conditions admit a solution. If we re-index the networks, based on Lemma 1, that largest value for i can be found in a straightforward manner. In particular, if we define

$$d_m = a_m^{-1} b_m \lambda_m,$$

$$G_i(x) = \sum_{k=0}^{i} \lambda_{m_k} b_{m_k} F^{-1}(x d_{m_k}) - 1 + \sum_{k=i+1}^{M} (\lambda_{m_k}/\eta_{m_k}),$$

then we have the following theorem.

Theorem 4. *If* $\sum_{n=0}^{M}(\lambda_n/\eta_n) < 1$*, a quasi-closed form solution to (*$\mathbf{P2}$*) and hence to (*$\mathbf{P0}$*) can be obtained efficiently by Algorithm 2.*

Algorithm 2 A quasi-closed form solution

1: Re-index the networks using $\{m_k\}$ rather than $\{m\}$, so that $c_{m_k} \geq c_{m_{k+1}}$, for $k = 0, 1, \dots, M$.
2: Initialize $i = 0$. Set $\mu^{(0)} = 0$.
3: **while** $\mu^{(i)} < c_{m_i}$ **do**
4: $\quad i := i + 1$;
5: $\quad$ compute $\mu^{(i)}$, the root of $G_i(\mu) = 0$;
6: **end while**.
7: $r_{m_k}^* = \begin{cases} F^{-1}(\mu^{(i-1)} d_{m_k}), & \text{for } k = 0, \dots, i-1 \\ (\eta_{m_k} b_{m_k})^{-1}, & \text{for } k = i, \dots, M. \end{cases}$
8: $P_{m_k}^* = [r_{m_k}^* b_{m_k} (r_0^* b_0)^{-1}]^{\alpha/2} P_0$, for $k = 0, 1, \dots, M$.
9: $U^* = \sum_{k=0}^{M} a_{m_k} \exp[-(r_{m_k}^*)^{-1}]$.

Proof: Using the KKT conditions, if $r_{m_k} > (\eta_{m_k} b_{m_k})^{-1}$, then $r_{m_k} = F^{-1}(\mu d_{m_k})$, where μ is the Lagrange multiplier of (5c). If $r_{m_k} > (\eta_{m_k} b_{m_k})^{-1}$ for $k = 0, \dots, i-1$ and $r_{m_k}^* = (\eta_{m_k} b_{m_k})^{-1}$ for $k = i, \dots, M$, then $G_{i-1}(\mu) = 0$. The existence of the solution $\mu^{(i-1)}$ to that equation is a consequence of the feasibility of $(\mathbf{P2})$. The monotonicity of $G_i(x)$ leads to the uniqueness of $\mu^{(i-1)}$. Due to the monotonic decreasing nature of $F(x)$, $\mu^{(i-1)} < c_{m_{i-1}}$. ■

After sorting the network index, Algorithm 2 only requires no more than $M+1$ iterations to find the critical number i. Moreover, at each iteration, we only need to find the root of a one-dimensional function, and it can be expressed in a quasi-closed form reminiscent of waterfilling. In contrast, the original formulation $(\mathbf{P2})$ has $M+1$ variables.

4. CASE STUDIES AND NUMERICAL RESULTS

In this section, we demonstrate the potential of the proposed power control strategy using three simple case studies involving a PN and a single SN ($M = 1$). In these cases, the quasi-closed form solutions in Algorithm 2 take explicit closed forms.

The following parameters are used: network densities are $\lambda_0 = 10^{-6}\text{m}^{-2}$, $\lambda_1 = 2 \times 10^{-5}\text{m}^{-2}$; path-loss exponent is $\alpha = 3$; reference transmission distances are $R_0 = 20$m, $R_1 = 10$m; primary transmission power is $P_0 = 10$ dB; target SIR thresholds are $\beta_0 = 10, \beta_1 = 5$; successful transmission probability decrement bounds are $\delta_0 = 0.05, \delta_1 = 0.1$; and the spectrum bandwidth is normalized to be unity. These parameter settings satisfy $\sum_{n=0}^{M}(\lambda_n/\eta_n) < 1$, so the feasibility of the power control problem is ensured (cf. Theorem 1), and Theorem 4 is applicable. Also, Corollary 1 indicates that P_1 is bounded between $P_{1,\text{min}} = \left[\lambda_0(\eta_1 - \lambda_1)^{-1}\right]^{\alpha/2} P_0$ and $P_{1,\text{max}} = \left[(\eta_0 - \lambda_0)\lambda_1^{-1}\right]^{\alpha/2} P_0$.

We first consider the case of equal weights, i.e., $\omega_0 = \omega_1 = 0.5$. These weights can be interpreted as two access fairness factors that avoid domination in the spectrum usage. For this setting, it can be shown that the optimal P_1^* is an interior point lying strictly between $P_{1,\text{min}}$ and $P_{1,\text{max}}$, $P_1^* = \left[F^{-1}(\mu^{(1)} d_1)\, b_1 b_0^{-1} / F^{-1}(\mu^{(1)} d_0)\right]^{\alpha/2} P_0$, where $\mu^{(1)}$ is the solution of $\lambda_0 b_0 F^{-1}(\mu d_0) + \lambda_1 b_1 F^{-1}(\mu d_1) = 1$. As shown in Table 1, allowing the SN to access the spectrum results in a 68% improvement in the throughput utility.

Secondly, we consider the case of $\omega_0 = 0.1$ and $\omega_1 = 0.9$, representing a scenario where the PN accesses the spectrum with low frequency, whereas the SN is quite active. It can be shown that U is an increasing function over the feasible region of P_1. As a result, P_1^* is at the upper bound, i.e., $P_1^* = P_{1,\text{max}}$. Table 1 demonstrates that the proposed power strategy provides a dramatic increase in the throughput utility when compared with exclusive use assigned to the PN.

Thirdly, we consider the case of $\omega_0 = 0.95$ and $\omega_1 = 0.05$, representing a scenario where PN accesses the spectrum with high frequency or allows little performance degradation. Consequently, there is little chance for SN to coexist. For this setting, U is a decreasing function in the feasible region of P_1, and hence P_1^* is at the lower bound, i.e., $P_1^* = P_{1,\text{min}}$. Table 1 illustrates that in this case the improvement in throughput utility is 3%, which is consistent with the intuition.

5. CONCLUSION

In this paper, we developed a power control strategy for multiple spectrum-sharing networks that depends only on a model for the spatial distribution of the nodes. We developed a quasi-closed form expression for the power allocation that maximizes a weighted sum of the throughput of the networks, subject to QoS constraints in each network. Case studies demonstrated that in a variety of settings substantial gains in the throughput can be obtained using proposed strategy, with dramatic gains being obtained when the PN accesses the spectrum infrequently.

6. REFERENCES

[1] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Computer Networks*, vol. 50, pp. 2127-2159, Sept. 2006.

[2] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 79-89, May 2007.

[3] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649-658, Feb. 2007.

[4] R. Zhang and Y.-C. Liang,"Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 88-102, Feb. 2008.

[5] L. B. Le and E. Hossain, "Resource allocation for spectrum underlay in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5306-5315, Dec. 2008.

[6] X. Kang, Y.-C. Liang, A. Nallanathan, H. K. Garg and R. Zhang, "Optimal power allocation for fading channels in cognitive radio networks: Ergodic capacity and outage capacity," *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 940-950, Feb. 2009.

[7] J.-S. Pang, G. Scutari, D. P. Palomar, and F. Facchinei, "Design of cognitive radio systems under temperature-interference constraints: A variational inequality approach," *IEEE Trans. Signal Process.*, vol. 58, no. 6, pp. 3251-3271, Jun. 2010.

[8] I. Mitliagkas, N. D. Sidiropoulos, and A. Swami,"Joint power and admission control for ad-hoc and cognitive underlay networks: Convex approximation and distributed implementation," *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 4110-4121, Dec. 2011.

[9] B. L. Mark and A. O. Nasif, "Estimation of maximum interference-free power level for opportunistic spectrum access," *IEEE Trans. Wireless Commun.*, vol. 8, no. 5, pp. 2505-2513, May 2009.

[10] A. G. Marques, X. Wang, and G. B. Giannakis, "Dynamic resource management for cognitive radios using limited-rate feedback," *IEEE Trans. Signal Process.*, vol. 57, no. 9, pp. 3651-3666, Sept. 2009.

[11] G. Zheng, K.-K. Wong, and B. Ottersten, "Robust cognitive beamforming with bounded channel uncertainties," *IEEE Trans. Signal Process.*, vol. 57, no. 12, pp. 4871-4881, Dec. 2009.

[12] H. A. Suraweera, P. J. Smith, and M. Shafi, "Capacity limits and performance analysis of cognitive radio with imperfect channel knowledge," *IEEE Trans. Veh. Technol.*, vol. 59, no. 4, pp. 1811-1822, May 2010.

[13] E. Dall'Anese, S.-J. Kim, G. B. Giannakis, and S. Pupolin, "Power control for cognitive radio networks under channel uncertainty," *IEEE Trans. Wireless Commun.*, vol. 10, no. 10, pp. 3541-3551, Oct. 2011.

[14] M. Haenggi, J. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1029-1046, Sept. 2009.

[15] A. Ghasemi and E. S. Sousa, "Interference aggregation in spectrum sensing cognitive wireless networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 41-56, Feb. 2008.

[16] R. Menon, R. Buehrer, and J. Reed, "On the impact of dynamic spectrum sharing techniques on legacy radio systems," *IEEE Trans. Wireless Commun.,* vol. 7, no. 11, pp. 4198-4207, Nov. 2008.

[17] C. Yin, C. Chen, T. Liu, and S. Cui, "Generalized results of transmission capacities for overlaid wireless networks," in *Proc. IEEE Int. Symp. Inf. Theory*, pp. 1774-1778, Seoul, South Korea, Jul. 2009.

[18] K. W. Choi, E. Hossain, and D. I. Kim, "Cooperative spectrum sensing under a random geometric primary user network model," *IEEE Trans. Wireless Commun.*, vol. 10, no. 6, pp. 1932-1944, Jun. 2011.

[19] J. Lee, J. G. Andrews, and D. Hong,"Spectrum-sharing transmission capacity," *IEEE Trans. Wireless Commun.*, vol. 10, no. 9, pp. 3053-3063, Sept. 2011.

[20] R. Vaze, "Transmission capacity of spectrum sharing ad hoc networks with multiple antennas," *IEEE Trans. Wireless Commun.*, vol. 10, no. 7, pp. 2334–2340, Jul. 2011.

[21] W. Ren, Q. Zhao, and A. Swami, "Power control in cognitive radio networks: How to cross a multi-lane highway," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1283-1296, Sept. 2009.

[22] R. Cai, W. Zhang, and P. C. Ching, "Spectrum sharing between random networks," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, pp. 3137-3140, Kyoto, Japan, Mar. 2012.

[23] M. Song, C. Xin, Y. Zhao, and X. Cheng, "Dynamic spectrum access: from cognitive radio to network radio," *IEEE Wireless Commun. Mag.*, vol. 19, no. 1, pp. 23-29, Feb. 2012.

[24] N. Golmie, N. Chevrollie, and O. Rebala, "Bluetooth and WLAN coexistence: Challenges and solutions," *IEEE Wireless Commun. Mag.*, vol. 10, no. 6, pp. 22-29, Dec. 2003.

[25] P. C. Pinto, A. Ginoretti, M. Z. Win, and M. Chinai, "A stochastic geometry approach to coexistence in heterogeneous wireless networks," *IEEE. J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1268-1282, Sept. 2009.

[26] K. Huang, V. K. N. Lau, and Y. Chen, "Spectrum sharing between cellular and mobile ad hoc networks: Transmission-capacity trade-off," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1256-1267, Aug. 2009.

[27] M. Haenggi and R. K. Ganti, "Interference in large wireless networks", *Foundations and Trends in Networking*, vol. 3, no. 2, pp. 127-248, 2008.

[28] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "An aloha protocol for multihop mobile wireless networks," *IEEE Trans. Inf. Theory*, vol. 52, pp. 421-436, Feb. 2006.

[29] S. Weber, J. G. Andrews, and N. Jindal, "An overview of the transmission capacity of wireless networks," *IEEE Trans. Commun.*, vol. 58, no. 2, pp. 3593-3604, Dec. 2010.

[30] W. Rudin and J. Cofman, *Principles of Mathematical Analysis*. McGrawHill, 1964.

[31] S. Boyd and L. Vandenberge, *Convex Optimization*. Cambridge Univeristy Press, 2004.