DESIGN OF BRANCH-HOPPED WAVELET PACKET DIVISION MULTIPLEXING SCHEMES

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ABSTRACT

Wavelet Packet Division Multiplexing (WPDM) is a high-capacity, flexible and robust orthogonal multiplexing technique in which wavelet packet basis functions are chosen as the coding waveforms. Branch-Hopped WPDM (BH-WPDM) is an extension of WPDM which incorporates hopping strategies analogous to those of frequency-hopped schemes. It is based on an efficient modular switched transmultiplexer structure which provides the advantages of hopping whilst retaining many of the desirable features of WPDM. In previous work we have identified classes of slow and fast BH-WPDM schemes and have evaluated a number of switching strategies. In the present paper we provide a method for re-designing the filters within the transmultiplexer modules to provide further robustness to narrow frequency-selective fading channels under a given switching strategy.

Key Words: multiplexing, wavelet packets, hopping, frequency-selective fading.

1. INTRODUCTION

Wavelet Packet Division Multiplexing (WPDM) [1] is an emerging orthogonal multiplexing technique in which wavelet packet basis functions [2] are chosen as the coding waveforms. In contrast to the conventional time division (TDM), frequency division (FDM) and code division (CDM) multiplexing schemes, the waveforms used to represent the data symbols of each user overlap in *both* time and frequency. The fact that the waveforms are of finite duration and overlap in time and frequency provides a substantial increase in capacity over TDM and FDM [1] and robustness to certain adverse channel environments [1,3], whilst their close relationships with multi-rate filter banks (transmultiplexers) provide particularly simple transmitter and receiver structures [1].

Orthogonal multiplexing schemes are sensitive to the effects of frequency-selective channels. Whilst the frequency overlapping of the WPDM waveforms provides some robustness to these effects, the commonly used wavelet packet basis functions are still 'localized' in frequency, and hence are susceptible to the perturbation of frequency-selective channels. In frequency-hopped communication schemes, the susceptibility of a narrow-band communication scheme to an unknown frequency-selective channel is reduced by 'hopping' the carrier frequency between several frequencies in a pattern which is known by the receiver. The *Branch-Hopped WPDM* (BH-WPDM) scheme employs an efficient modular switched transmultiplexer structure to achieve an analogous hopping effect for WPDM. In previous work [4,5] we evaluated a number of different switching strategies for BH-WPDM and identified hopping schemes with performance advantages which are analogous to those of slow and (coherently combined) fast frequency hopping. In this paper we provide a method for re-designing the filters within the transmultiplexer modules to provide further robustness to narrow frequency-selective fading channels under a given switching strategy.

2. WAVELET PACKET DIVISION MULTIPLEXING

We begin with a brief review of the WPDM scheme [1]. (See [6] and references therein for some related work.) To define the wavelet packet basis functions, let g_0 be a unit-energy real causal FIR filter of length L which is orthogonal to its even translates; i.e., $\sum_n g_0[n]g_0[n-2m] = \delta[m]$, where $\delta[m]$ is the Kronecker delta, and let g_1 be the (conjugate) quadrature mirror filter, $g_1[n] = (-1)^n g_0[L-1-n]$. If g_0 satisfies some mild technical conditions [2], we can use an iterative algorithm to find the function $\phi_{01}(t) = \sqrt{2}\sum_n g_0[n]\phi_{01}(2t-nT_0)$ for an arbitrary interval T_0.

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Subsequently, we can define the family of functions $\phi_{\ell m}$, $\ell \geq 0$, $1 \leq m \leq 2^{\ell}$ in the following (binary) tree-structured manner:

$$\phi_{\ell+1,2m-1}(t) = \sum_n g_0[n]\phi_{\ell m}(t - nT_{\ell}), \qquad (1a)$$

$$\phi_{\ell+1,2m}(t) = \sum_n g_1[n]\phi_{\ell m}(t - nT_{\ell}), \qquad (1b)$$

where $T_{\ell} = 2^{\ell}T_0$. For any given tree structure, the functions at the *terminals* of the tree form a *wavelet packet* [2]. They have a finite duration, $(L-1)T_{\ell}$, and are self- and mutually-orthogonal at integer multiples of dyadic intervals, and hence they are a natural choice for multiplexing applications. More precisely, if $\mathtt{T}$ denotes the set of terminal index pairs, then for $(\ell,m),(\lambda,\mu) \in \mathtt{T}$

$$\langle \phi_{\ell m}(t - nT_{\ell}), \phi_{\lambda\mu}(t - kT_{\lambda}) \rangle = \delta[\ell - \lambda]\,\delta[m - \mu]\,\delta[n - k]. \qquad (2)$$

In the WPDM scheme, the (binary) message data at the (ℓ,m)th terminal, $d_{\ell m}[n]$, are waveform coded by pulse amplitude modulation (PAM) of the function at that terminal. Hence the WPDM composite signal is

$$s(t) = \sum_{(\ell,m)\in\mathtt{T}} \sum_n d_{\ell m}[n]\phi_{\ell m}(t - nT_{\ell}).$$

(Note that the terminals on different levels have different symbol rates, $1/T_{\ell}$.) Due to the orthogonality relationship in Eq. (2) the data can be extracted from the transmitted signal without inter-symbol interference or crosstalk using a simple matched filter receiver for each terminal. By exploiting the structure in Eq. (1) we obtain an alternative transmitter structure using a tree-structured multi-rate 'synthesis' filter bank and a single PAM modulator (see Fig. 1)

$$s(t) = \sum_k \sigma_{01}[k]\phi_{01}(t - kT_0), \qquad (3)$$

where

$$\sigma_{01}[k] = \sum_{(\ell,m)\in\mathtt{T}} \sum_n f_{\ell m}[k - 2^{\ell}n]d_{\ell m}[n], \qquad (4)$$

and $f_{\ell m}$ is the *equivalent filter* from the (ℓ,m)th terminal to the root node, which can be found recursively using Eq. (1) and $f_{\ell m}[k] = \langle \phi_{\ell m}(t), \phi_{01}(t - kT_0)\rangle$. The orthogonality property of the equivalent filters, $\langle f_{\ell m}[k - 2^{\ell}n], f_{\lambda\mu}[k - 2^{\lambda}i]\rangle = \delta[\ell-\lambda]\delta[m-\mu]\delta[n-i]$, for $(\ell,m),(\lambda,\mu) \in \mathtt{T}$, confirms that an alternative receiver structure consisting of a single matched filter and a tree-structured multi-rate 'analysis' filter bank is available (see Fig. 1). By substituting Eq. (4) into Eq. (3) we can view the WPDM scheme as a member of a class of generalized orthogonal CDM schemes in which the 'codes' are the equivalent filters $f_{\ell m}$ and the 'chip' waveform is ϕ_{01}. This generalized class includes the conventional orthogonal CDM schemes (i.e., Walsh-Hadamard schemes), but extends those schemes to allow for real-valued orthogonal codes which overlap in time, and orthogonal chip waveforms which have a duration longer than the chip interval.

3. BRANCH-HOPPED WPDM

The BH-WPDM scheme is based on a modular switched transmultiplexer structure in which a two-input two-output memoryless switching unit is attached to the input of each 'merge' module at the transmitter and to the output of each corresponding 'split' module at the receiver, as illustrated in Fig. 1. If we toggle the state of each switch at the transmitter in a pattern which is known at the receiver, we 'hop' the branches of the tree-structured filter banks. 'Long' or 'short' intervals between switch state changes lead to schemes with analogies to the slow and fast frequency-hopped schemes, respectively. Of course, the BH-WPDM hopping schemes require synchronization of the switches, but that is no more arduous than the synchronization of frequency-hopped schemes.

The BH-WPDM composite signal can be written in a form analogous to Eq. (3) as (see Fig. 1)

$$\tilde{s}(t) = \sum_k \tilde{\sigma}_{01}[k]\phi_{01}(t - kT_0), \qquad (5)$$

where $\tilde{\sigma}_{01}$ is obtained from the 'switched' synthesis filter bank,[1]

$$\tilde{\sigma}_{01}[k] = \sum_{(\ell,m)\in\mathtt{T}} \sum_n \tilde{f}_{\ell m}[k,n]d_{\ell m}[n], \qquad (6)$$

and $\tilde{f}_{\ell m}[k,n]$ is the equivalent filter from the (ℓ,m)th terminal to the root node 'seen' by a unit sample at instant n at the (ℓ,m)th terminal. If we define $x_{\lambda\mu}[i]$ to be the state of the switch at the (λ,μ)th node at the ith instant, with zero representing a 'parallel' connection and one representing a 'cross' connection then $\tilde{f}_{\ell m}[k,n]$ can be found recursively as

[1] An alternative matrix-based notation for the filter banks in Eqs (4) and (6) has also been developed [4].

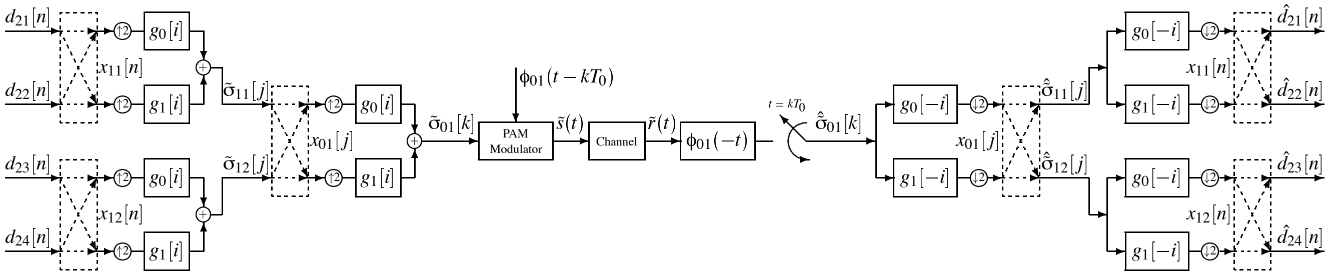


Fig. 1: A four-user BH-WPDM scheme. The dashed boxes represent 'switching' units which provide either a 'parallel' or a 'cross' connection at each instant. If the switches remain in the 'parallel' state we have a WPDM scheme.

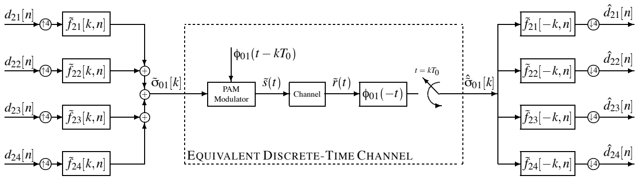


Fig. 2: An equivalent model for Fig. 1.

$$\tilde{f}_{\ell+1,2m-1}[k,n] = \sum_i \big(g_0[i-2n](1-x_{\ell m}[n]) + g_1[i-2n]x_{\ell m}[n]\big)\tilde{f}_{\ell m}[k,i], \quad \text{(7a)}$$

$$\tilde{f}_{\ell+1,2m}[k,n] = \sum_i \big(g_0[i-2n]x_{\ell m}[n] + g_1[i-2n](1-x_{\ell m}[n])\big)\tilde{f}_{\ell m}[k,i], \quad \text{(7b)}$$

with $\tilde{f}_{01}[k,n] = \delta[k-n]$. (If all the switches are in the parallel state then $\tilde{f}_{\ell m}[k,n] = f_{\ell m}[k-2^{\ell}n]$ and we return to the underlying WPDM scheme.) The equivalent filters retain the orthogonality property of those in the WPDM scheme, $\langle \tilde{f}_{\ell m}[k,n], \tilde{f}_{\lambda\mu}[k,i]\rangle = \delta[\ell-\lambda]\delta[m-\mu]\delta[n-i]$, for $(\ell,m),(\lambda,\mu) \in \mathtt{T}$, and hence the receiver consists of a single matched filter and a 'switched' analysis filter bank. The properties of $\tilde{f}_{\ell m}[k,n]$ ensure that (with white binary data) the BH-WPDM composite signal, $\tilde{s}$, has the same cyclic spectra as that of the underlying WPDM scheme. Hence BH-WPDM retains the capacity advantages of WPDM. What the BH-WPDM scheme does change is the way in which the spectra of $\tilde{s}$ are allocated to the user at each terminal. By substituting Eq. (6) into Eq. (5) the BH-WPDM scheme can be viewed as a 'code-hopped' extension to the class of generalized orthogonal CDM schemes discussed at the end of Section 2 in which the orthogonal codes allocated to the data symbols at a given terminal may vary from symbol to symbol, as illustrated in Fig. 2.

4. SWITCHING STRATEGIES

Once the underlying tree structure of a BH-WPDM scheme has been chosen, its performance depends on the switching strategy and the filter g_0. Central to the analysis of the effects of the switching strategies at each node are the following observations [5], obtained by careful inspection of Eq. (7):

1. The equivalent filter $\tilde{f}_{\ell m}[k,n]$ depends on switch states at all the nodes along the path from the (ℓ,m)th terminal to the $(0,1)$ node. Those nodes are (λ,μ), where $0 \le \lambda \le \ell-1$, $\mu = \lceil 2^{\lambda-\ell}m \rceil$, and $\lceil w \rceil$ denotes the least integer $\ge w$.
2. The number of states of the switch at the (λ,μ)th node (along the above mentioned path) which affect $\tilde{f}_{\ell m}[k,n]$ is the length of the equivalent filter from the (ℓ,m)th terminal to the $(\lambda+1, \lceil 2^{\lambda+1-\ell}m \rceil)$th node. The length of that filter is $L_{\ell}^{\lambda+1}$, where $L_{\ell}^{\lambda} \triangleq (2^{\ell-\lambda}-1)(L-1)+1$.

Combining these two observations, we find that $\tilde{f}_{\ell m}[k,n]$ depends on $x_{\lambda\mu}[i]$ for $0 \le \lambda \le \ell-1$, $\mu = \lceil 2^{\lambda-\ell}m \rceil$ and $i \in [2^{\ell-\lambda-1}n, 2^{\ell-\lambda-1}n + L_{\ell}^{\lambda+1}-1]$. We collect these states, in a particular arrangement, in the vector $\boldsymbol{x}_{\ell m}[n]$. There are $K_\ell \triangleq \sum_{\lambda=0}^{\ell-1} L_{\ell}^{\lambda+1} = L_\ell - \ell L - 1$ elements in $\boldsymbol{x}_{\ell m}[n]$, where $L_\ell \triangleq L_{\ell}^{0}$. Therefore, there are 2^{K_ℓ} possible values of $\boldsymbol{x}_{\ell m}[n]$, each generating a *distinct equivalent filter*. Distinct equivalent filters are, in general, *distinct* in the sense that they are not shifted versions of each other. The role of the switching strategy is to carve out a subset of those 2^{K_ℓ} distinct equivalent filters and hop $\tilde{f}_{\ell m}[k,n]$ amongst (shifted versions of) that set.

To capture that notion, let $\boldsymbol{x}_{\ell m}^{(i)}$ denote the ith possible value of $\boldsymbol{x}_{\ell m}[n]$, and let $h_{\ell m}^{(i)}[k]$ denote the (time-invariant) equivalent filter $\tilde{f}_{\ell m}[k,0]$ when $\boldsymbol{x}_{\ell m}[0] = \boldsymbol{x}_{\ell m}^{(i)}$.

With those definitions, if $\boldsymbol{x}_{\ell m}[n] = \boldsymbol{x}_{\ell m}^{(i)}$ then

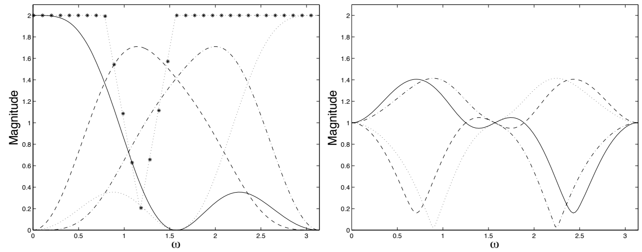
$$\tilde{f}_{\ell m}[k,n] = h_{\ell m}^{(i)}[k - 2^{\ell} n]. \tag{8}$$

For a given switching strategy we assess the relative frequency of the assignment in Eq. (8) by associating a weight, $W_{\ell m}^{(i)}$, with each $\boldsymbol{x}_{\ell m}^{(i)}$ (and hence with each $h_{\ell m}^{(i)}[k]$). For a given interval of interest $\mathtt{N}_{\ell}$ of length N_ℓ symbols, $W_{\ell m}^{(i)} \triangleq N_{\ell m}^{(i)}/N_\ell$, where $N_{\ell m}^{(i)}$ is the number of instants in $\mathtt{N}_{\ell}$ for which the switching strategy makes the assignment $\boldsymbol{x}_{\ell m}[n] = \boldsymbol{x}_{\ell m}^{(i)}$. We collect the filters $h_{\ell m}^{(i)}[k]$ with non-zero weights in the *set of distinctly generated equivalent filters* for the given switching strategy, $\mathtt{F}_{\ell m} = \left\{ h_{\ell m}^{(i)}[k] \middle| W_{\ell m}^{(i)} \neq 0 \right\}$, and let $U_{\ell m} \leq 2^{K_\ell}$ denote the number of elements in that set.

In order to classify BH-WPDM schemes, we observe that if $\boldsymbol{x}_{\ell m}^{(i)}$ contains only one state of each switch, then $h_{\ell m}^{(i)}[k] = f_{\ell m'}[k]$ for some $1 \leq m' \leq 2^{\ell}$; i.e., the filter $h_{\ell m}^{(i)}[k]$ is an equivalent filter from an underlying WPDM scheme. Otherwise, $h_{\ell m}^{(i)}[k]$ depends on both filters in the 'merge' unit at at least one node. If the intervals between switch state changes are 'long',[2] the only elements of $\mathtt{F}_{\ell m}$ with substantial weights are equivalent filters from an underlying WPDM scheme. In that case, the scheme will be said to be a *slow BH-WPDM scheme*. If the intervals between switch state changes are 'short', the scheme will be said to be a *fast BH-WPDM scheme*. In that case, the elements of $\mathtt{F}_{\ell m}$ with substantial weights will not be filters from an underlying scheme, but will be filters which depend on both filters in the 'merge' unit at at least one node. Since g_0 and g_1 tend to be 'low-pass' and 'high-pass', respectively, such filters tend to have a broader frequency response (as a fraction of the bandwidth of the whole multiplexing scheme) than the corresponding WPDM filters.

In previous work [4, 5] we have shown that in a slowly varying frequency-selective channel, slow and fast BH-WPDM schemes provide performance averaging analogous to that of slow and fast frequency hopping schemes, respectively. Slow BH-WPDM schemes tend to provide performance averaging amongst individual users, but they tend not to provide an overall performance improvement. In contrast, the broader spectra of the equivalent filters in the fast schemes provide an overall performance improvement in such channels. This effect is illustrated in Figs 3 and 4. (See [4,5,7] for further details.)

[2]By 'long' we mean that for the switch at the (λ,μ)th node the intervals between switch state changes are long with respect to $L_{\ell_{\max}}^{\lambda+1}$, where $\ell_{\max}$ is the maximum value of ℓ over the terminals, (ℓ,m), affected by the switch at the (λ,μ)th node.



(a) (b)

Fig. 3: Spectra of the equivalent filters for the four-user system illustrated in Fig. 1 based on a standard Daubechies filter [2] of length four: (a) the WPDM scheme (no switching), with a snapshot of the notch channel used to produce Fig. 4 (asterisks); (b) a fast BH-WPDM scheme in which each switch is toggled at each instant.

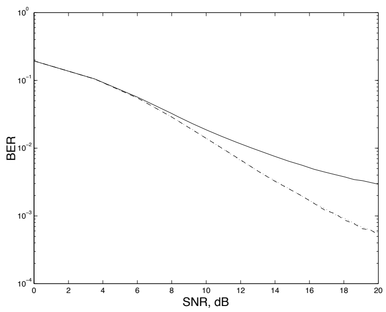


Fig. 4: Simulated overall BER against SNR for the schemes in Fig. 3, with uncoded binary data, in a slowly and uniformly varying notch channel with additive white Gaussian noise. A snapshot of the notch channel is given in Fig. 3(a). Legend: WPDM and any slow BH-WPDM scheme (indistinguishable): solid; the fast BH-WPDM scheme: dot-dashed.

5. DESIGN OF BH-WPDM FILTERS

In the previous section, we made the (heuristic) observation that the spectral broadening of the equivalent filters induced by the switching transients in a fast BH-WPDM scheme leads to improved performance in (unknown) slowly fading frequency-selective channels which are narrow with respect to the bandwidth of the whole multiplexing scheme. (Related observations have been made for spread spectrum communication systems.) In this section we exploit that observation to re-design the filter g_0 in the transmultiplexer modules for a given switching strategy, in order to gain further improved performance in such channels. As a measure of the breadth of the spectrum of the equivalent filter $h_{\ell m}^{(i)}[k]$ (as a fraction of the bandwidth of the multiplexing scheme as a whole) we define the *(RMS) deviation from frequency flatness* of the equivalent filter $h_{\ell m}^{(i)}[k]$ to be

$$D_{\ell m}^{(i)} \triangleq \left(\frac{1}{\pi} \int_0^\pi \left(\left| H_{\ell m}^{(i)}(e^{j\omega}) \right|^2 - 1 \right)^2 d\omega \right)^{1/2}, \quad (9)$$

where $H_{\ell m}^{(i)}(e^{j\omega})$ is the Discrete-Time Fourier Transform of $h_{\ell m}^{(i)}[k]$. The deviation from frequency flatness is simple to compute and small values of $D_{\ell m}^{(i)}$ can be an effective guide towards 'good' switching strategies for narrow frequency-selective channels. For example, for the WPDM scheme in Fig. 3(a) the deviations are 1.10 and 2.22 (two of each), and for the BH-WPDM scheme in Fig. 3(b), the deviations are 0.30 and 0.35 (again two of each). The performance advantage of the BH-WPDM scheme predicted by its smaller deviations is clearly achieved in the scenario of Fig. 4.

The derivation of our design method proceeds by reiterating the observation that for a given switching strategy, a filter g_0 for which all distinct equivalent filters $h_{\ell m}^{(i)}[k]$ in each set $\mathtt{F}_{\ell m}$ have small values of $D_{\ell m}^{(i)}$ ought to lead to 'good' performance in slowly fading narrow frequency-selective channels. In order to simplify the exposition, we will restrict our attention to BH-WPDM schemes in which all the terminals are at the same level, and to switching strategies for which each distinct equivalent filter has the same weight.[3] An initial design problem can be phrased as follows: For the given switching strategy, find a filter g_0 which minimizes a particular $D_{\ell_0 m_0}^{(i_0)}$ subject to $D_{\ell m}^{(i)}$ for all the other equivalent filters being no greater than $(1+\varepsilon_D)D_{\ell_0 m_0}^{(i_0)}$, for a small positive ε_D, and to g_0 being of unit energy and self-orthogonal at even translations. Note that we have implicitly decoupled the design of g_0 from that of $\phi_{01}(t)$ so that no 'regularity' constraints [2] on g_0 are required. This decoupling is known as the 'splitting trick' in the wavelet literature [2].

[3]Many periodic switching strategies satisfy the second criterion [7]. The extension of this work to more general schemes is straightforward but notationally cumbersome.

Careful inspection of Eq. (9) reveals that if $g_0[n] = \delta[n]$ then $D_{\ell m}^{(i)} = 0$ for all $(\ell, m) \in \mathtt{T}$ and $1 \leq i \leq U_{\ell m}$. Therefore, such a solution, which corresponds to a (scrambled) TDM scheme, is a global optimum for this problem. However, TDM schemes tend be sensitive to time-selective effects such as impulsive noise [3], so we impose a constraint on the time localization of the equivalent filters. By analogy with Eq. (9) we define the *(RMS) deviation from ideal time spread* for the equivalent filter $h_{\ell m}^{(i)}[k]$ to be

$$\tau_{\ell m}^{(i)} \triangleq \left(\sum_{k=0}^{L_\ell - 1} \left(\left(h_{\ell m}^{(i)}[k] \right)^2 - 1/L_\ell \right)^2 \right)^{1/2}.$$

We constrain the deviation from ideal time spread of all the equivalent filters $h_{\ell m}^{(i)}[k]$ to be of the same order as that obtained by the BH-WPDM scheme with the given switching strategy and with g_0 being the standard Daubechies filter [2] of the desired length. This will ensure that the robustness of the designed BH-WPDM scheme to time selective effects is of the same order as that of the BH-WPDM scheme with the same switching strategy and the Daubechies filter.

The design problem can now be formally stated: For a given switching strategy and a given filter length L, select a particular $(\ell_0, m_0) \in \mathtt{T}$, and i_0, $1 \leq i_0 \leq U_{\ell_0 m_0}$, and find a filter $g_0[n]$, $0 \leq n \leq L-1$, which achieves the minimum of

$$\min_{g_0[n]} D_{\ell_0 m_0}^{(i_0)}$$

subject to

$$\sum_{n=2k}^{L-1} g_0[n] g_0[n-2k] = \delta[k], \quad 0 \leq k \leq \lceil (L-1)/2 \rceil, \quad (10)$$
$$D_{\ell m}^{(i)} \leq (1+\varepsilon_D) D_{\ell_0 m_0}^{(i_0)}, \qquad (\ell, m) \in \mathtt{T}, \ 1 \leq i \leq U_{\ell m},$$
$$\tau_{\ell m}^{(i)} \leq (1+\varepsilon_\tau)\tau^*, \qquad (\ell, m) \in \mathtt{T}, \ 1 \leq i \leq U_{\ell m},$$

where ε_D and ε_τ are small positive constants, τ^* is the maximal $\tau_{\ell m}^{(i)}$ for the BH-WPDM scheme with the same switching strategy and the standard Daubechies filter [2] of length L. Note that L must be even in order to satisfy Eq. (10) [2]. Based on our heuristic observation, we would expect such an optimized scheme to perform better than a scheme employing the standard Daubechies filter of that length in a slowly fading narrow frequency-selective environment. In the following example we demonstrate that significant performance gains are indeed achievable in this manner.

Example 1 Consider the four-user BH-WPDM scheme illustrated in Fig. 1, and two switching strategies: the WPDM scheme (with no switching) and the fast scheme in which each switch is toggled at each instant. An optimal g_0 of length four was found for each scheme by solving the design problem with $\varepsilon_D = \varepsilon_\tau = 0.1$, using a standard Sequential Quadratic Programming technique. This resulted in $D^{(i_0)}_{\ell_0,m_0} = 0.81$ for the WPDM scheme and $D^{(i_0)}_{\ell_0,m_0} = 0.17$ for the fast scheme. For comparison, the deviations for the schemes based on the standard Daubechies filter of length 4 are 1.10 and 2.22 (two of each) for the WPDM scheme, and 0.30 and 0.35 (two of each) for the fast scheme. The spectra of the equivalent filters for the optimized fast scheme are plotted in Fig. 5, and are clearly flatter in frequency than those for the fast scheme with the Daubechies filter. Using the deviations and the preceding discussion, we predict that each optimized scheme will perform better than the corresponding scheme based on the Daubechies filter in a slowly fading narrow frequency-selective channel. Furthermore, we predict that the optimized fast scheme will perform better than the optimized WPDM scheme. Simulated BER curves for transmission through the slowly and uniformly varying notch channel used to produce Fig. 4, a snapshot of which was provided in Fig. 3, are plotted in Fig. 6. These figures demonstrate that a significant improvement in BER performance is indeed achievable. For example, at 20dB SNR, the BER performance gain of the the optimized fast scheme over the Daubechies filter based fast scheme is around 50%. (Equivalently, the SNR gain of the optimized fast scheme is more than 2dB at an SNR of around 20dB.) Finally, note that the prediction that the optimized fast scheme would perform better than the optimized WPDM scheme is confirmed. This demonstrates an intrinsic advantage of an optimized hopped scheme over an optimized static scheme. Numerical experiments in a number of other scenarios confirm the trends illustrated in this example [7]. □

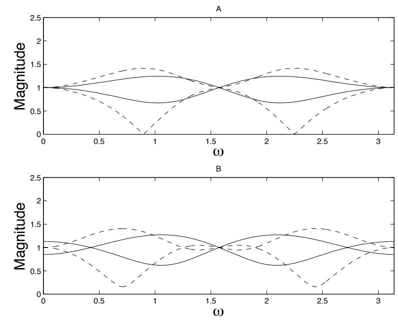
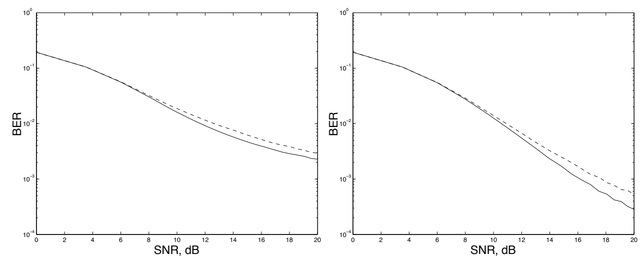


Fig. 5: Spectra of the equivalent filters for the fast schemes in Ex. 1. Optimized scheme: solid; Daubechies filter based scheme: dashed.



(a) (b)

Fig. 6: Simulated overall BER against SNR for the optimized schemes (solid) and the Daubechies filter based schemes (dashed) in Ex. 1. (a) WPDM; (b) fast BH-WPDM.

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