On Improving the BER Performance of Rate-Adaptive Block Transceivers, with Applications to DMT

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Abstract— We propose two strategies for improving the (uncoded) bit error rate (BER) performance of practical rate-adaptive block-by-block communication schemes, such as discrete multitone modulation (DMT). Our strategies are inspired by some recent work which showed that for uniformly bit-loaded schemes, the transmission strategy which minimizes the BER for a linear receiver involves allocating power to the subchannels that are implicit in the block-by-block framework in a minimum mean square error (MMSE) fashion and linearly combining these subchannels using a normalized discrete Fourier transform (DFT) matrix. This combining equalizes the decision point signal-to-noise ratios (SNRs) of the subchannels. Given a non-uniformly bit-loaded scheme, our first design strategy simply performs a DFT-based linear combination within the groups of subchannels which share the same constellation. Our second strategy provides further reduction in the BER by re-allocating power within these groups in a MMSE fashion prior to DFT combining. Our examples indicate that our design strategies can provide significant reductions in the BER, and give rise to substantial SNR gains (of the order of several decibels).

I. INTRODUCTION

Block-by-block transmission schemes, such as orthogonal frequency division multiplexing (OFDM) and discrete multitone modulation (DMT), are popular schemes for efficiently and effectively transmitting data over diverse classes of channels; e.g., digital subscriber lines and wireless local area network channels. For applications in which a feedback channel from the receiver to the transmitter is available, and for channels which vary slowly, there are several mature design methods for (linear) block-by-block transceivers; see [1] for an insightful overview. A taxonomy of these methods reveals three dominant classes:

  I. Rate-adaptive designs based on the maximization of the achievable information rate, subject to a bound on the transmitted power; e.g., water-filling based DMT [2–4].

 II. Performance orientated designs based on maximizing a measure of the fidelity of (a linearly processed version of) the received block, subject to a bound on the transmitted power; e.g., minimum mean square error (MMSE) [5–8]; maximum signal-to-noise ratio (SNR) [7]; and minimum (uncoded) bit error rate (BER) [1], [9–11].

III. Constraint satisfaction designs based on minimizing the transmitted power required to achieve a given fidelity at a given rate; e.g., margin adaptive designs [3], [12].

A unifying feature of these three design classes is that the structure of the optimal (linear) transmitter-receiver pair diagonalizes the appropriate channel matrix, and hence exposes a set of parallel subchannels for communication. The classes are differentiated by the way in which the transmission power and the bits to be transmitted are allocated to these subchannels, and the way in which the parallel subchannels are (linearly) combined by the transmitter. The standard transmitter design techniques obtain optimal performance (for the given criterion) without combining the subchannels. However, it has recently been shown [1], [10], [11] that achieving the minimum BER of a uniformly bit-loaded scheme with linear equalization and symbol-by-symbol detection requires linear combination of the subchannels with a special class of unitary matrices; see Section III.

The rate-adaptive designs of Class I are usually derived from the fact that the capacity of a block-by-block transmission system can be achieved if the appropriate channel matrix is diagonalized and power is allocated to the subchannels according to the water-filling distribution [2]. Unfortunately, achieving this capacity involves the implementation of ideal Gaussian codes—a rather awkward task in practice. Standard practical rate-adaptive designs achieve (reasonably) reliable transmission at rates which are a significant fraction of the capacity by employing members of standard constellation and code families on each subchannel. The goal of this paper is to apply some of the principles of the minimum BER designs from Class II to improve the BER of standard rate-adaptive designs from Class I. The key observation is that once the constellation for each subchannel has been determined, the work in [1], [10], [11] indicates that the BER can be improved by re-allocating power among those subchannels which share the same constellation and by linearly combining these subchannels. We will demonstrate in our examples that the resulting reduction in the BER can be quite significant, and can give rise to substantial SNR gains (of the order of several decibels). Our development is based on uncoded error rates for systems with linear equalization and symbol-by-symbol (i.e., parallel) detection. However, we will observe that our design approach equalizes (and maximizes) the decision point SNRs of each set of subchannels which have the same constellation. This fact has the potential to significantly
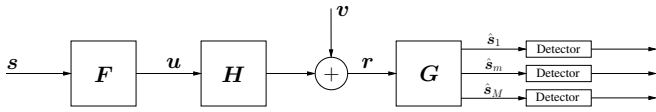
Fig. 1. Generic block-by-block transmission system with symbol-by-symbol (parallel) detection.

simplify the design of appropriate coding schemes, especially when 'hard-decision' decoding is employed. Before we present our results, we point out that while this paper was being prepared we discovered that the principles of some methods from Class III had been independently applied [12] to reduce the power required to achieve a given quality of service at the rates determined by standard rate-adaptive designs. While the technical details of the work in [12] and that herein are quite similar, the design objectives are complementary but somewhat different.

II. LINEAR BLOCK-BY-BLOCK TRANSMISSION

In this paper we consider linear block-by-block transmission schemes of the form in Fig 1. The transmitter (precoder) linearly transforms a block of M data symbols s into a block of $P \geq M$ channel symbols $u = Fs$. For each transmitted block, the receiver collects a block of $N \geq M$ samples r, from which the estimates $\hat{s} = Gr$ of s are linearly obtained. These estimates can be written as

$$\hat{s} = GHFs + Gv, \qquad (1)$$

where H is the $N \times P$ (equivalent) channel matrix, and v denotes the zero-mean Gaussian receiver noise, which has a covariance matrix R_{vv}. The model in Fig 1 and Eq. (1) is quite flexible. In particular, it covers both zero-padded and cyclic-prefixed transmission over both single and multiple antenna frequency-selective channels. For brevity, we will focus on systems with zero-forcing equalization in this paper, and will exploit the results in [10] for minimum BER precoding for zero-forcing equalization. However, the principles of our designs can be directly extended to systems with minimum MSE equalization by exploiting results in [1], [11]; see [12] for a related approach. If zero-forcing equalization is employed, $G = (HF)^{\dagger}$, where $(\cdot)^{\dagger}$ denotes the pseudo-inverse. Assuming that HF has full column rank, Eq. (1) becomes

$$\hat{s} = s + Gv. \qquad (2)$$

A standard simple way to design a practical rate-adaptive transmission scheme is to constrain each element of the signal vector s to come from a member of the family of I–ary QAM constellations. The order of the QAM constellation used on each subchannel is typically chosen according to the decision point SNR. If the elements of s are uncorrelated[1], then a simple way to choose the number of bits to be assigned to

[1]If s is correlated, then the data vectors can be pre-whitened, so long as the covariance matrix of s has full rank.

the mth subchannel is

$$b_m = \left\lfloor \log_2 \left(1 + \frac{\text{SNR}_m}{\Gamma} \right) \right\rfloor, \qquad (3)$$

where $\lfloor \cdot \rfloor$ denotes the greatest integer $\leq x$, Γ is the so-called SNR gap [3] which reflects the desired uncoded error rate performance, and SNR_m is the decision-point signal-to-noise ratio of the mth subchannel. (If $\Gamma > 2$, the 'floor' operation in (3) is often replaced by a rounding operation.) If each element of s is scaled so that $R_{ss} = E\{ss^H\} = E_s I$ (the elements of s have already been assumed to be uncorrelated), then for the zero-forcing equalized scheme in (2)

$$\text{SNR}_m = \frac{E_s}{[GR_{vv}G^H]_{mm}} = \frac{E_s}{[(HF)^{\dagger}R_{vv}((HF)^{\dagger})^H]_{mm}}, \qquad (4)$$

where $[\cdot]_{ij}$ denotes the (i,j)th element of a matrix. Since the subchannel SNR in (4) depends on the precoder, jointly optimal bit-loading/precoder pairs can be rather awkward to obtain directly. However, effective designs can be obtained in a straightforward manner by designing the precoder via water-filling and then assigning bits to the subchannels according to (3). For simplicity, we will focus on such a power load-ing algorithm in this paper. However, the principles of our approach are essentially independent of the manner in which b_m is determined, and hence our approach is also applicable to systems designed via standard bit loading algorithms [3].

If the mth element of s is selected from a Gray-coded square QAM constellation of order 2^{b_m}, where b_m is even, then the average probability of error for the scheme in (2) is closely approximated by [13][2]

$$P_e \approx \frac{2}{M} \sum_{m=1}^{M} \left(\frac{\sqrt{2^{b_m}} - 1}{b_m \sqrt{2^{b_m}}} \text{erfc} \left(\sqrt{\frac{3\text{SNR}_m}{2(2^{b_m} - 1)}} \right) \right.$$
$$\left. + \frac{\sqrt{2^{b_m}} - 2}{b_m \sqrt{2^{b_m}}} \text{erfc} \left(3\sqrt{\frac{3\text{SNR}_m}{2(2^{b_m} - 1)}} \right) \right), \quad (5)$$

where $\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt$. In what follows it will be convenient for us to permute the elements of s so that the symbols from the same constellation are grouped together. To describe that grouping, let K denote the number of distinct constellations used, let $\tilde{b}_i$, $i = 1, 2, \ldots, K$, denote the number of bits in the ith distinct constellation, and let $\mathcal{G}_i$ denote the set of subchannels in the ith group; i.e., $\mathcal{G}_i = \{m|b_m = \tilde{b}_i\}$. Using that notation, Eq. (5) can be rewritten as

$$P_e \approx \frac{2}{M} \sum_{i=1}^{K} \sum_{m \in \mathcal{G}_i} \left(\alpha_i \text{erfc} \left(\sqrt{\frac{\beta_i}{[GR_{vv}G^H]_{mm}}} \right) \right.$$
$$\left. + \zeta_i \text{erfc} \left(3\sqrt{\frac{\beta_i}{[GR_{vv}G^H]_{mm}}} \right) \right), \quad (6)$$

[2]A related formula applies for rectangular QAM constellations [13], but for brevity we only include the square case. Furthermore, although we will use (5), the principles of our approach remain valid when we use the simpler (and more common) approximation of P_e in which the second component of the summand in (5) is deemed negligible.

where $\alpha_i = \frac{\sqrt{2^{\tilde{b}_i}}-1}{\tilde{b}_i\sqrt{2^{\tilde{b}_i}}}$, $\beta_i = \frac{3E_s}{2(2^{\tilde{b}_i}-1)}$, and $\zeta_i = \frac{\sqrt{2^{\tilde{b}_i}}-2}{\tilde{b}_i\sqrt{2^{\tilde{b}_i}}}$.

A key component in our design approach is the observation that the function $\text{erfc}\left(\sqrt{\frac{a}{x}}\right)$ with $a > 0$ is convex in $0 < x < \frac{2a}{3}$. Hence, the summand of $m \in \mathcal{G}_i$ in (6) is a convex function of $[\boldsymbol{GR}_{vv}\boldsymbol{G}^H]_{mm}$ so long as $[\boldsymbol{GR}_{vv}\boldsymbol{G}^H]_{mm} < \frac{2\beta_i}{3}$. If the bit loading is performed via (3), then for all $m \in \mathcal{G}_i$, $[\boldsymbol{GR}_{vv}\boldsymbol{G}^H]_{mm} \leq \frac{E_s}{\Gamma(2^{\tilde{b}_i}-1)} = \frac{2\beta_i}{3\Gamma}$, and hence the summand of $m \in \mathcal{G}_i$ in (6) is convex for all m and i. (If $\Gamma > 1/(\sqrt{2}-1)$ then the summand of $m \in \mathcal{G}_i$ in (6) remains convex when the rounding function replaces the floor function in (3).) Therefore, we can apply Jensen's inequality [2] to (6) to obtain the following lower bound on the average bit error rate,

$$P_e \geq \frac{2}{M}\sum_{i=1}^{K} M_i \left(\alpha_i \, \text{erfc}\left(\sqrt{\frac{\beta_i M_i}{\sum_{m\in\mathcal{G}_i}[\boldsymbol{GR}_{vv}\boldsymbol{G}^H]_{mm}}}\right)\right.$$
$$\left. + \zeta_i \, \text{erfc}\left(3\sqrt{\frac{\beta_i M_i}{\sum_{m\in\mathcal{G}_i}[\boldsymbol{GR}_{vv}\boldsymbol{G}^H]_{mm}}}\right)\right)$$
$$= \frac{2}{M}\sum_{i=1}^{K} \bar{P}_{e,i}$$
$$= \bar{P}_e, \qquad (7)$$

where M_i is the number of subchannels in the ith group (i.e., the cardinality of $\mathcal{G}_i$), and the definition of $\bar{P}_{e,i}$ is implicit.

In the following section we will exploit results in [10] to show how once the bit loading has been established, the lower bound in (7) can be achieved by linearly combining the subchannels within each group with a unitary matrix whose elements have equal magnitude; e.g., a normalized DFT matrix (see Design A). Furthermore, we will show how the power allocation among the subchannels in each group can be modified to reduce the lower bound $\bar{P}_e$. This reduced lower bound can also be achieved by linearly combining the subchannels in the group with a normalized DFT matrix (see Design B). In both Designs A and B the linear combining of the subchannels has the desirable effect of making the decision point SNRs of each subchannel in the same group equal.

III. NEW PRECODER DESIGNS

As we mentioned in Section II, our design approach is essentially independent of the manner in which the constellation for each subchannel is chosen. However, the standard water-filling based rate-adaptive schemes have many desirable properties. Here we will use the simple scheme in (3).

For the block transmission system in (1), the precoder $\boldsymbol{F}$ which maximizes the achievable information rate subject to a transmission power budget $\text{tr}(\boldsymbol{FR}_{ss}\boldsymbol{F}^H) = E_s\,\text{tr}(\boldsymbol{FF}^H) \leq E_s p_0$ is (e.g., [2])

$$\boldsymbol{F}_{\text{WF}} = \boldsymbol{W}\boldsymbol{\Delta}_{\text{WF}}, \qquad (8)$$

where $\boldsymbol{W}\boldsymbol{\Lambda}\boldsymbol{W}^H$ is an eigenvalue decomposition of $\boldsymbol{H}^H\boldsymbol{R}_{vv}^{-1}\boldsymbol{H}$, and hence the matrix $\boldsymbol{W}$ 'exposes' the (eigen) subchannels of the system. The diagonal matrix $\boldsymbol{\Delta}_{\text{WF}}$

is the water-filling power loading matrix. Its mth diagonal element is $\delta_{\text{WF},m} \geq 0$, where

$$\delta_{\text{WF},m}^2 = \max\{\gamma - \Gamma/(E_s\lambda_m), 0\}. \qquad (9)$$

Here, λ_m is the mth diagonal element of $\boldsymbol{\Lambda}$, and γ is chosen so that $\sum_m \delta_{\text{WF},m}^2 = p_0$. The decision point SNR for the mth subchannel for this precoder is

$$\text{SNR}_{\text{WF},m} = E_s\delta_{\text{WF},m}^2\lambda_m. \qquad (10)$$

In the design of our modified precoders below, we consider precoders of the form

$$\boldsymbol{F} = \boldsymbol{W}\boldsymbol{\Delta}\boldsymbol{Q}, \qquad (11)$$

where $\boldsymbol{W}$ was defined after (8), $\boldsymbol{\Delta}$ is a diagonal power loading matrix, and $\boldsymbol{Q}$ is a structured unitary matrix which can be chosen to linearly combine the subchannels which employ the same constellation. To describe the structure of $\boldsymbol{Q}$, let $\boldsymbol{P}$ denote a permutation matrix that groups together the elements of $\boldsymbol{s}$ that employ the same constellation; i.e., for $q \in (\sum_{j=1}^{i-1} M_j, \sum_{j=1}^{i} M_j]$ the elements $[\tilde{\boldsymbol{s}}]_q$ of $\tilde{\boldsymbol{s}} = \boldsymbol{Ps}$ all come from the same $2^{\tilde{b}_i}$–ary QAM constellation. The matrix $\boldsymbol{Q}$ takes the form $\boldsymbol{Q} = \boldsymbol{P}^T\boldsymbol{UP}$, where $\boldsymbol{U}$ is a block diagonal matrix whose ith block is a square unitary matrix of dimension M_i, $1 \leq i \leq K$; i.e., $\boldsymbol{U} = \text{blkdiag}\{\boldsymbol{U}_1, \boldsymbol{U}_2, \ldots, \boldsymbol{U}_K\}$. Observe that $\boldsymbol{F} = \boldsymbol{W}\boldsymbol{\Delta}\boldsymbol{P}^T\boldsymbol{UP} = \boldsymbol{WP}^T\tilde{\boldsymbol{\Delta}}\boldsymbol{UP}$, where $\tilde{\boldsymbol{\Delta}} = \boldsymbol{P}\boldsymbol{\Delta}\boldsymbol{P}^T$ is the power loading matrix when the subchannels are grouped according to their assigned constellations. To simplify the notation, we let $\tilde{\boldsymbol{\Delta}}_i$ denote the ith diagonal block of $\tilde{\boldsymbol{\Delta}} = \boldsymbol{P}\boldsymbol{\Delta}\boldsymbol{P}^T$, where the blocks are of size M_i, and let $\tilde{\boldsymbol{\Lambda}}_i$ denote the ith diagonal block of $\tilde{\boldsymbol{\Lambda}} = \boldsymbol{P}\boldsymbol{\Lambda}\boldsymbol{P}^T$. We now proceed with our designs.

A. Design A: Subchannel combining

In this design we choose $\boldsymbol{\Delta} = \boldsymbol{\Delta}_{\text{WF}}$, and hence our only degree of freedom is the block diagonal unitary matrix $\boldsymbol{U}$; i.e., $\boldsymbol{F}_A = \boldsymbol{W}\boldsymbol{\Delta}_{\text{WF}}\boldsymbol{P}^T\boldsymbol{UP}$. If we let $\boldsymbol{G}_A = (\boldsymbol{HF}_A)^\dagger$, then for each symbol in each group which is allocated bits (i.e., for which $\tilde{b}_i > 0$), the decision point SNR of the mth subchannel is

$$\text{SNR}_{A,m} = \frac{E_s}{[\boldsymbol{G}_A\boldsymbol{R}_{vv}\boldsymbol{G}_A^H]_{mm}} = \frac{E_s}{[\boldsymbol{U}_i^H\boldsymbol{\Theta}_i\boldsymbol{U}_i]_{\ell\ell}} \qquad (12)$$

where $m \in \mathcal{G}_i$, and ℓ, $1 \leq \ell \leq M_i$, is the position of the mth symbol within that group (as determined by $\boldsymbol{P}$). Here, $\boldsymbol{\Theta}_i = \tilde{\boldsymbol{\Delta}}_i^{-1}\tilde{\boldsymbol{\Lambda}}_i^{-1}\tilde{\boldsymbol{\Delta}}_i^{-1}$, where $\tilde{\boldsymbol{\Lambda}}_i$ and $\tilde{\boldsymbol{\Delta}}_i$ are guaranteed to be invertible if $\tilde{b}_i > 0$. (Note that if $\tilde{\boldsymbol{\Lambda}}_i$ is singular, then $\tilde{b}_i = 0$, according to (3).) Since each $\boldsymbol{U}_i$ is unitary, for any subchannel in the ith group, $m \in \mathcal{G}_i$,

$$\min_{m\in\mathcal{G}_i} \text{SNR}_{\text{WF},m} \leq \text{SNR}_{A,m} \leq \max_{m\in\mathcal{G}_i} \text{SNR}_{\text{WF},m}, \qquad (13)$$

where $\text{SNR}_{\text{WF},m}$ is the decision point SNR on the mth subchannel of the water filling precoder. Equation (13) shows that for any $\boldsymbol{U}_i$, the decision point SNR of subchannel $m \in \mathcal{G}_i$ for Design A lies between the largest and smallest SNRs achieved by the subcarriers from the same group in the water-filling

design. Hence, the bit loading determined for the water-filling precoder via (3) remains valid for Design A.

Now, since U_i is unitary, the lower bound, $\bar{P}_{e,i}$, on the BER of the ith group for the water-filling design remains valid for Design A. What remains to be shown is that this lower bound can be achieved by specific choices of U_i. Using Jensen's inequality [2], we observe that $\bar{P}_{e,i}$ is achieved if all the decision point SNRs in the ith group are equal. Using an argument similar to that in [10], these SNRs can be made equal by choosing U_i to be a normalized $M_i \times M_i$ discrete Fourier transform (DFT) matrix, D_{M_i}. Therefore, the precoder

$$F_A = W \Delta_{\text{WF}} P^T \text{blkdiag}\{D_{M_1}, D_{M_2}, \ldots, D_{M_K}\} P \quad (14)$$

is guaranteed to have a lower (uncoded) BER than the water-filling precoder.

B. Design B: Power re-allocation and subchannel combining

Design A achieves the lower bound in (7) on the BER of the water-filling based design. However, this lower bound can be reduced by re-allocating power among the subchannels in each group. Furthermore, this reduced lower bound is also achievable. Deriving such a precoder involves the design of both Δ and U in (11). The role of Δ is to reduce $\bar{P}_e$ while ensuring that power allocated to each group of subchannels remains the same as it was for the water-filling design. This constraint ensures that the original bit loading continues to satisfy (3) with the original SNR gap. Using the work in [10], the lower bound on the group BER, $\bar{P}_{e,i}$, is minimized if the power loading over the ith group is the minimum mean square error (MMSE) power loading for that group (and zero-forcing equalization) subject to the total power allocated to that group remaining the same as the water-filling design. So long as $\tilde{b}_i > 0$, the appropriate MMSE power loading for group i is $\tilde{\Delta}_{B,i} = \phi_i \tilde{\Lambda}_i^{-1/4}$, where $\phi_i > 0$ satisfies $\phi_i^2 = \text{trace}(\tilde{\Delta}_{\text{WF},i}^2)/\text{trace}(\tilde{\Lambda}_i^{-1/2})$. The scalar ϕ_i ensures that the total power allocated to the ith group is the same as it was for the water-filling design. As shown in the Appendix, for any subchannel in the ith group (i.e., $m \in \mathcal{G}_i$), a relationship analogous to (13) holds:

$$\min_{m \in \mathcal{G}_i} \text{SNR}_{\text{WF},m} \leq \text{SNR}_{B,m} \leq \max_{m \in \mathcal{G}_i} \text{SNR}_{\text{WF},m}. \quad (15)$$

Hence, the original bit loading continues to satisfy (3) under the modified power loading scheme.

Since the modified group power loading, $\tilde{\Delta}_{B,i}$, minimizes $\bar{P}_{e,i}$, the lower bound $\bar{P}_e$ on the total BER is lower for Design B than it was for Design A and the water-filling design. Moreover, using an analogous argument to that for Design A, this reduced lower bound on the BER is achieved by choosing $U_i = D_{M_i}$. That is, the reduced lower bound on the BER is achieved by the following precoder:

$$F_B = W \Phi \Lambda^{-1/4} P^T \text{blkdiag}\{D_{M_1}, D_{M_2}, \ldots, D_{M_K}\} P, \quad (16)$$

where $\Phi = P^T \tilde{\Phi} P$ and

$$\tilde{\Phi} = \text{blkdiag}\{\phi_1 I_{M_1}, \phi_2 I_{M_2}, \ldots, \phi_K I_{M_K}\}.$$

Here, for brevity, we have assumed that $\tilde{b}_i > 0$ for all i.

IV. Performance analysis

In this section, we compare the BER performance of our Designs A and B to that of the underlying water-filling based rate-adaptive design in a simple single-antenna discrete multitone system. In this case, W is a normalized inverse DFT matrix. The noise on each subcarrier is assumed to be Gaussian, zero mean, and independent from that on the other subcarriers (as is often assumed in standard DMT designs). Therefore, the mth diagonal element of Λ is $|H(e^{j2\pi(m-1)/M})|^2/\sigma_m^2$, where $H(e^{j\omega})$ is the frequency response of the channel, and σ_m^2 is the noise variance on the mth subcarrier. The particular DMT scheme that we will consider has $M = 32$ subcarriers, and a cyclic prefix of length 4. Hence, $P = 36$ is the transmitted block size. The noise is modelled as being white; i.e., $\sigma_m^2 = \sigma^2$. To determine the bit loading, we applied (3) to the water-filling power loaded precoder with $\Gamma = 8$ dB and with a rounding operation replacing the floor operation in (3). For simplicity of implementation, we restricted our attention to square QAM constellations, so b_m was always even. In Fig. 2, we provide the BER performance of our precoders and the water-filling precoder averaged over 1000 realizations of a frequency-selective Rayleigh fading channel of length 5. (The channel taps were generated independently from a circular complex Gaussian distribution of variance 0.5 per dimension.) The block SNR is defined as $E_s p_0/(M\sigma^2)$, which is equal to the average symbol SNR. Since we are performing a rate-adaptive design, as the block SNR increases the number of bits transmitted per block also increases. For each realization of the channel, all three precoders transmit the same number of bits for each subcarrier and hence the same number of bits per block. The number of bits per transmitted block at each SNR, averaged over the randomly generated channels is shown in Fig. 3.

As predicted by our derivations, Fig 2 illustrates the fact that Design A provides a lower BER than the underlying water-filling design, and that Design B provides a further improvement in the BER. At various points on these BER curves the SNR gain of Design A over the water-filling design is around 12 to 15 dB. Design B provides an additional SNR gain of around 2 to 3 dB.

V. Conclusion

We have provided two strategies for improving the (un-coded) BER of rate-adaptive block-by-block communication schemes. These strategies were based on extending recent results on minimum BER transmission for uniformly bit loaded schemes [1], [10], [11] to the case of non-uniform bit loading. The resulting precoders involve a DFT-based linear combination of the subchannels which share the same constellation. In the case of the second strategy, a re-allocation of power across these subchannels in a minimum mean square error sense is also performed. Our precoders have the rather appealing property that they equalize the decision point SNRs of those subchannels which share the same constellation. The examples indicated that our precoders can provide significant reductions in the BER and can give rise to substantial SNR
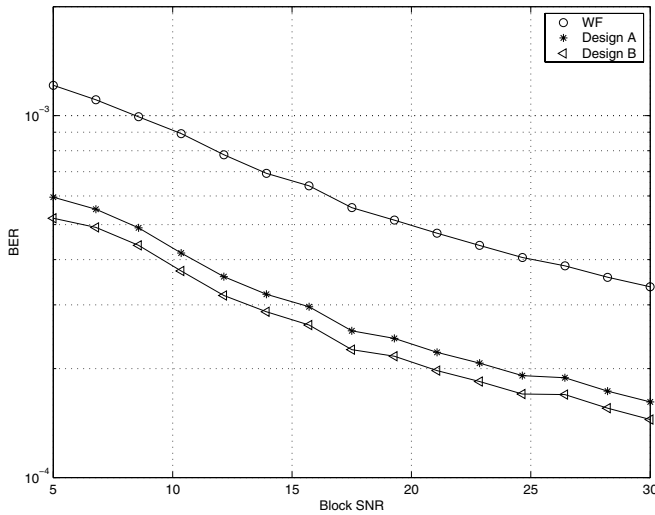
Fig. 2.   Average BER performance of the three precoders for the system described in Section IV.
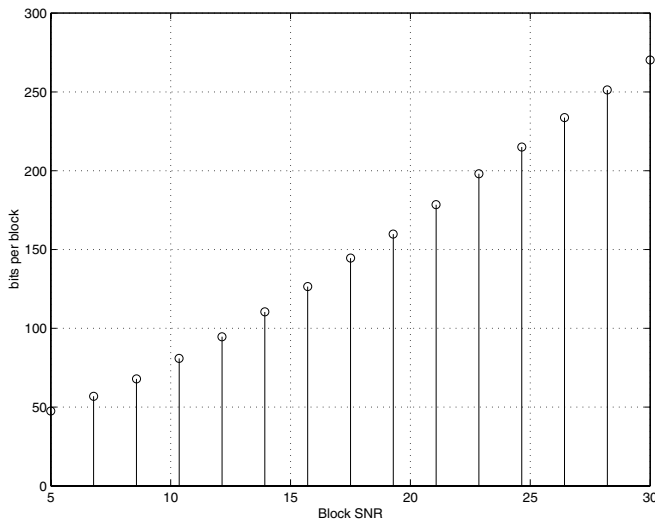

Fig. 3.   Number of bits transmitted per block in Fig. 2, averaged over the 1000 realizations of the channel. Recall that each precoder transmits the same number of bits.

gains (of the order of several decibels). Since our precoders have a simple structured closed form expression (viz. Eqs (14) and (16)), they are also relatively simple to implement in practice.

For simplicity, the focus of this paper was on block-by-block schemes with linear zero-forcing equalization. However, the principles behind our design extend immediately to the case of MMSE equalization [1], [11]. Some independent complementary work in that direction appeared recently in [12].

APPENDIX
DERIVATION OF (15)

If we let $x_m = [\boldsymbol{\Lambda}^{-1}]_{mm}$ and $y_m = [\boldsymbol{\Delta}_{\mathrm{WF}}^2]_{mm}$, then if subchannel m is a member of the ith group (and $\tilde{b}_i > 0$),

$$[\boldsymbol{G}_B \boldsymbol{R}_{vv} \boldsymbol{G}_B^H]_{mm} \geq \frac{\sum_{m \in \mathcal{G}_i} x_m^{1/2}}{\sum_{m \in \mathcal{G}_i} y_m} \left(\min_{m \in \mathcal{G}_i} x_m \right)^{1/2}$$

$$\geq \frac{M_i (\min_{m \in \mathcal{G}_i} x_m)^{1/2}}{M_i \max_{m \in \mathcal{G}_i} y_m} \left(\min_{m \in \mathcal{G}_i} x_m \right)^{1/2}$$

$$= \min_{m \in \mathcal{G}_i} \frac{x_m}{y_m}$$

$$= \min_{m \in \mathcal{G}_i} [\boldsymbol{G}_{\mathrm{WF}} \boldsymbol{R}_{vv} \boldsymbol{G}_{\mathrm{WF}}^H]_{mm},$$

and hence the right inequality in (15). The inequality on the left of (15) can be derived in an analogous fashion.

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