Vector Quantization of Subband Images Using Entropy-Weighted Mean Square Error

Jun Liu and T. N. Davidson

Communications Research Laboratory,
McMaster University,
Hamilton, Ontario, L8S 4K1, Canada.
liuj@sunspot.crl.mcmaster.ca

ABSTRACT

A new distortion measure, entropy-weighted mean square error (EWMSE), is introduced to enhance the perceptual quality of reconstructed images from subband vector quantization schemes. The measure is based on the observation that the subbands containing more information ought to be more accurately represented than those which contain less. A compatible feature extractor for a non-linear interpolative vector quantization (NLIVQ) scheme is proposed in order to extend the method to higher dimensional vector spaces without incurring an excessive computational burden. The experimental results confirm the predictions of improved perceptual quality.

Keywords: Subband coding, vector quantization, entropy, distortion measure, feature extraction, non-linear interpolation.

1 INTRODUCTION

Compression schemes based on vector quantization (VQ) of subband images have been shown to be powerful techniques for image compression. By using interband and/or intraband relationships efficiently, the quality of the reconstruced image can be preserved while the amount of information used to represent it is reduced.[1,2] In many simple VQ schemes, the mean square error (MSE) is chosen as the distortion measure. Although this measure is simple to calculate, low MSE reconstructed images may contain perceptually striking errors.

In order to further enhance the reconstruced image quality, one trend is to find a distortion criterion instead of MSE by optimizing the whole coding system,[3] or by exploiting part of the Human Visual System (HVS).[4–6] In this paper, we propose an alternative distortion criterion, called the entropy-weighted mean square error (EWMSE). The criterion is based on the observation that the subbands containing more information ought to be more accurately represented than those which contain less. Since entropy is a convenient measure of the information in an image, and the mean square error (MSE) is a simple measure of the accuracy of an image representation, a simple distortion criterion which incorporates this observation is the EWMSE. Our experimental results confirm the enhanced perceptual quality of the EWMSE scheme, especially at low bit rates (see Section 3).

The compression ratio of a subband VQ scheme can be raised by increasing the number of subband divisions.

Indeed, Shannon theory states that VQ can perform arbitrarily close to the theoretical optimal performance for a given compression ratio, given vectors of large enough dimension. However, such a scheme involves increasing computational complexity. One approach which may offer similar performance at reduced complexity is nonlinear interpolative VQ (NLIVQ) in which VQ is performed on reduced dimension feature vectors extracted from high dimension signal vectors, and an interpolation process restores full dimensionality at the decoder.[7] However, the construction of an appropriate feature extractor for a given VQ scheme remains an open issue. Here we argue that the feature extractor should work in a way which is compatible with the chosen VQ distortion measure. Since we intend to use NLIVQ with EWMSE on the reduced-dimension vectors, we construct a feature extractor based on the entropy of each subimage. Our preliminary experimental results indicate that such a scheme can provide perceptually enhanced images at low bit rates.

The paper is organized as follows. In the next section, we briefly discuss the uniform subband decomposition. In Section 3 we introduce the EWMSE distortion measure and confirm its desireable performance in the case of a uniform 16 subband decomposition. In Section 4, we extend our scheme to a uniform 64 subband decomposition by applying an entropy-based feature extractor to extract 16-D vectors, which are passed to a NLIVQ scheme based on EWMSE, and provide some experimental results. Some concluding remarks are collected in Section 5.

2 UNIFORM SUBBAND DECOMPOSITION

Subband coding was first applied in the context of image coding by Woods and O'neil.[9] A subband decomposition is produced by an analysis filter bank followed by downsampling. Any or all of the resulting subbands can be further input to an analysis filter bank and downsampling operation for as many stages as desired. Figure 1(a) shows the analysis section of a 2-D seperable filter bank, where first the image rows are passed through the two-channel filter bank, and then the columns are processed, thus a 4-band decomposition is obtained. The righthand side of Figure 1(a) shows the synthesis section to reconstruct the image from the subband images. If all four of the subimages are subjected to another stage of filtering and downsampling, this leads to a uniform decomposition. If only the lowest band is further decomposed, this is refered to as an octave-band decomposition. A uniform 16-subband coding scheme is shown in Figure 1(b). In the next section we will focus on such a subband VQ scheme, and in Section 4 it will be extended to 64 subband decomposition.

Figure 2(a) shows the original Lena image (256×256, 8bpp) and its uniform 16-subband decomposition is shown in Figure 2(b). We have used a Daubechies 4-tap filter bank to perform the subband decomposition due to its simple computation. The entropy of each subimage of Lena is shown in Figure 2(c). Obviously, the subimage with a larger entropy contains more information (see Figure 2(b)).

3 SUBBAND VQ CODING SCHEME WITH EWMSE

For two stages of uniform subband decomposition, the output of the decomposition process consists of 16 signals all of which are sampled at the same rate. Therefore it is natural to consider corresponding samples as vectors in a 16-dimensional space and then to encode this vector source by using vector quantizer.[10] This scheme is shown in Figure 3. This system is simple because no bit allocation procedure needed, while noise shaping can still be achieved by choosing a suitable distortion measure for the VQ. As shown by Westerink et al.,[10] choosing the MSE distortion measure, leads to a simple scheme which outperforms scalar quatization of each subimage with an optimal bit allocation, both in theory and practice. However, the performance of this approach is inferior to that of spatial VQ at low bit rates, as implied by Westerink et al..[10] In this paper, we propose a suitable distortion measure to shape the noise and enhance the performance of this scheme, especially at low bit rates.

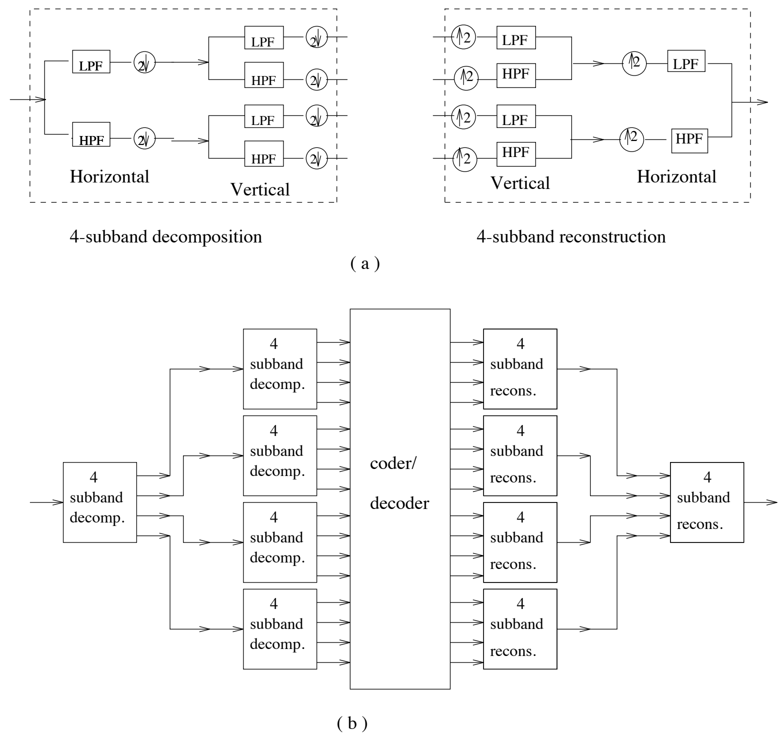


Figure 1. Uniform 16-subband decomposition, coding, and reconstruction scheme.

Figure 1: Uniform 16-subband decomposition, coding, and reconstruction scheme

The mean square error (MSE) distortion measure is defined as

$$D = ||\mathbf{x} - \hat{\mathbf{x}}||_2^2 \ ,$$

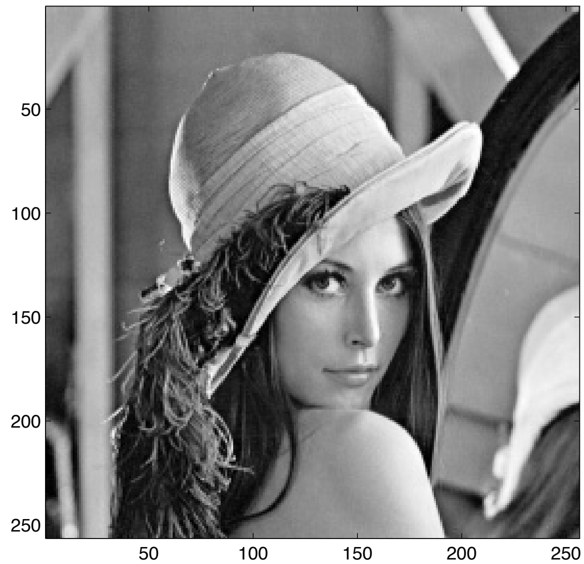
where $\hat{\mathbf{x}}$ is the code vector chosen to represent $\mathbf{x}$ and $||\cdot||_2$ represents the Euclidian norm. It is obvious that if the MSE is used as the error criterion, then each subimage is assumed to have the same importance. However, the subimage with a larger entropy contains more information according to information theory. Therefore, the coding performance may be better if an error criterion is chosen which accords different weights to different components, since the coefficients of the vector are not necessarily of the same importance.

For example, consider a simple 2-D case, for which the table in Figure 4 provides some data from Figure 2. Suppose that $\binom{0}{5}$ and $\binom{-1}{6}$ belong to the same Voronoi cell c_1 and that $\binom{7}{0}$ and $\binom{8}{1}$ are in another Voronoi cell c_2. To which cell should the vector $\binom{4}{4}$ be allocated? In the standard MSE case, it will fall in c_1. Hence, more information is lost from the subimage S_1, which should be more important. To overcome this defect, we introduce an entropy-weighted MSE (EWMSE), in which each component of the error vector between an signal vector and a code vector is weighted by the entropy of the corresponding subimage. The EWMSE distortion measure is

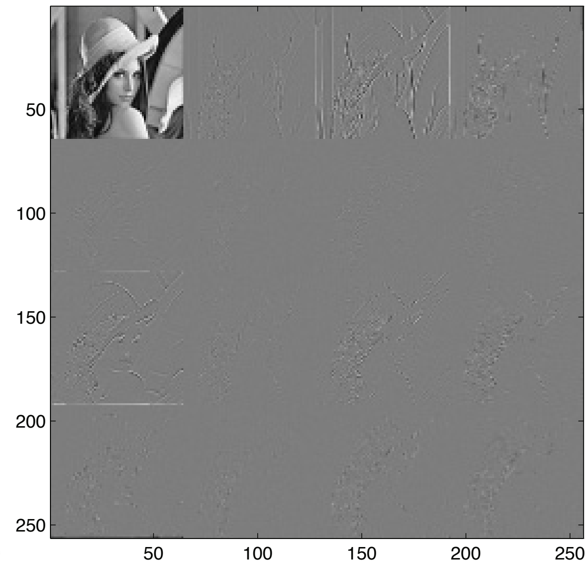
$$D^* = ||\mathbf{H}(\mathbf{x} - \hat{\mathbf{x}})||_2^2 \ ,$$

where $\mathbf{H}$ is a diagonal matrix, $\mathbf{H} = \mathrm{diag}\{H_i\}$, and H_i is the entropy of the i-th subimage. The weight matrix $\mathbf{H}$ ensures that the subimages with larger entropy have higher priority. In this way, the perceptual quality of the reconstructed image may be enhanced, as demonstrated in Example 1. Figure 4 illustrates the difference between MSE and EWMSE for the simple 2-D case above.

Example 1. For the Lena image and its subimages in Figure 2, Figure 5 contains two reconstructed images from SBC+VQ with MSE at 0.5 bpp and 0.375 bpp, respectively; and Figure 6 contains two reconstructed images



(a) Lena image (256X256, 8bpp)



(b) 16 uniform subimages

S1 7.74	S2 1.95	S3 2.70	S4 2.02
S5 1.41	S6 1.02	S7 1.26	S8 1.13
S9 2.21	S10 1.42	S11 1.95	S12 1.60
S13 1.48	S14 1.20	S15 1.45	S16 1.32

(c) entropy of each subimage

Figure 2: Subband decomposition for Example 1.

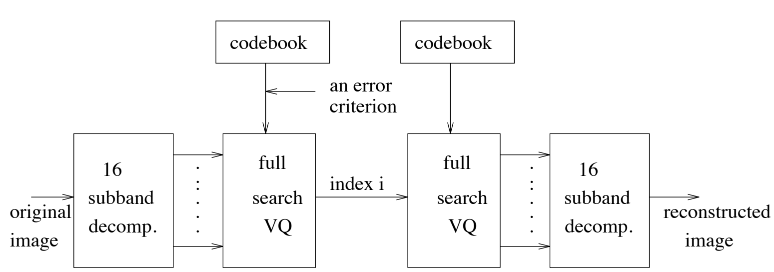


Figure 3: Uniform subband VQ coding scheme (SBC+VQ).

entropy	subimage	pixel values
7.74	S1	0 -1 4 7 8
2.70	S3	5 6 4 0 1

$$\overset{c1}{\begin{pmatrix}0 & -1\\ 5 & 6\end{pmatrix}} \xleftarrow[\text{MSE}]{\text{standard}} \begin{pmatrix}4\\ 4\end{pmatrix} \xrightarrow[\text{MSE}]{\text{entropy-weighted}} \overset{c2}{\begin{pmatrix}7 & 8\\ 0 & -1\end{pmatrix}}$$

Figure 4: A contrast between MSE and EWMSE.

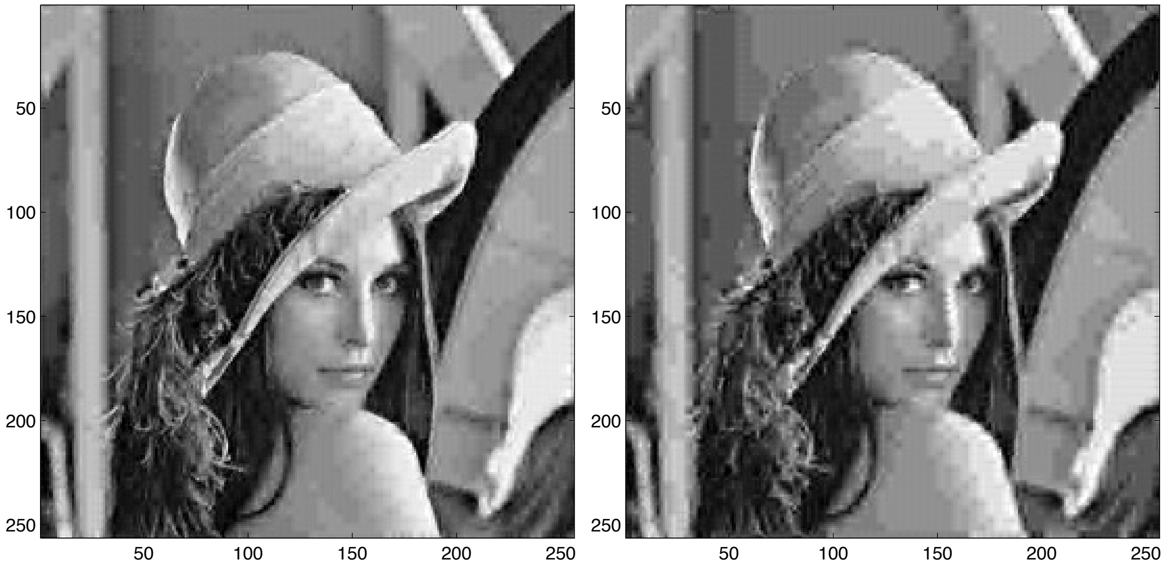
from SBC+VQ with EWMSE at 0.5 bpp and 0.375 bpp, respectively. Here the designed codebook contains only codevectors which represent the statistical properties of the Lena subimages. At the 0.5 bpp rate, Figure 5(a) and Figure 6(a) are of almost the same quality. However, at the lower bit rate 0.375 bpp, the shoulder, hat and background in Figure 6(b) are smoother and have reduced blocking effects in comparison to Figure 5(b); i.e., the EWMSE distortion measure has produced a more pleasing reconstructed image. We note that when an algorithm attempts to minimize a perceptual distortion measure rather than MSE, the PSNR results are of little use, so they are omitted. Also note that we employed the unsophisticated zero padding technique at the image boundaries. The visible "boundary effects" can be reduced by using more sophisticated techniques.

We conclude that the EWMSE can indeed enhance the subjective quality of the reconstructed image, especially at low bit rates, since some general intraband information (subimage entropy) is incorparated in the interband VQ scheme.

4 EXTENSION TO HIGH-DIMENSIONALITY VQ

Shannon theory has stated that VQ can perform arbitrarily close to the theoretical optimal performance for a given bit rate if the vectors have sufficiently large dimension. Unfortunately, for a given bit rate, the computational complexity of VQ rapidly becomes prohibitive as the vector dimension grows. Therefore, when we extend the scheme described in Section 3 to a uniform 64-subband decomposition, we should keep in mind how to enhance the tradeoff between performance and complexity. Recently, a nonlinear interpolative VQ (NLIVQ) has been proposed.[7] The scheme uses reduced-dimensionality feature vectors to perform VQ, instead of the high-dimensionality signal vectors, resulting in reduced computational complexity. The NLIVQ departs from a traditional VQ because the encoder and decoder codebooks have different dimensions. For a given feature vector, an optimal decoder codebook of high dimension can be obtained from the encoder codebook of low dimension by using Gersho's method.[7] Here we argue that such a feature extractor ought to be compatible with the distortion measure chosen for the VQ stage. Therefore, we construct a feature extractor which generates reduced-dimensionality feature vectors by retaining only those coefficients from the subimages with the largest entropies. The feature vectors are then passed to a NLIVQ scheme with the EWMSE distortion measure. The entire coding scheme is shown in Figure 7. Our prelimary work indicates that such a system provides performance close to that of full dimension VQ (with EWMSE) with significantly lower complexity, as shown in the following example.

Example 2. To perform NLIVQ, 16-D feature vectors are first extracted from the 64-D interband vectors by retaining only the components from the subimages with the 16 largest entropies. Vector quantization with



(a) 0.5 bpp (256 codevectors) (b) 0.375 bpp (64 codevectors)

Figure 5: Reconstructed images (256×256) by SBC+VQ with MSE

(a) 0.5 bpp (256 codevectors) (b) 0.375 bpp (64 codevectors)

Figure 6: Reconstructed images (256×256) by SBC+VQ with EWMSE

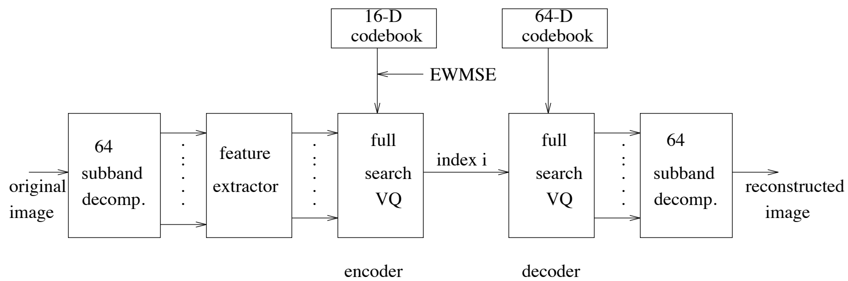
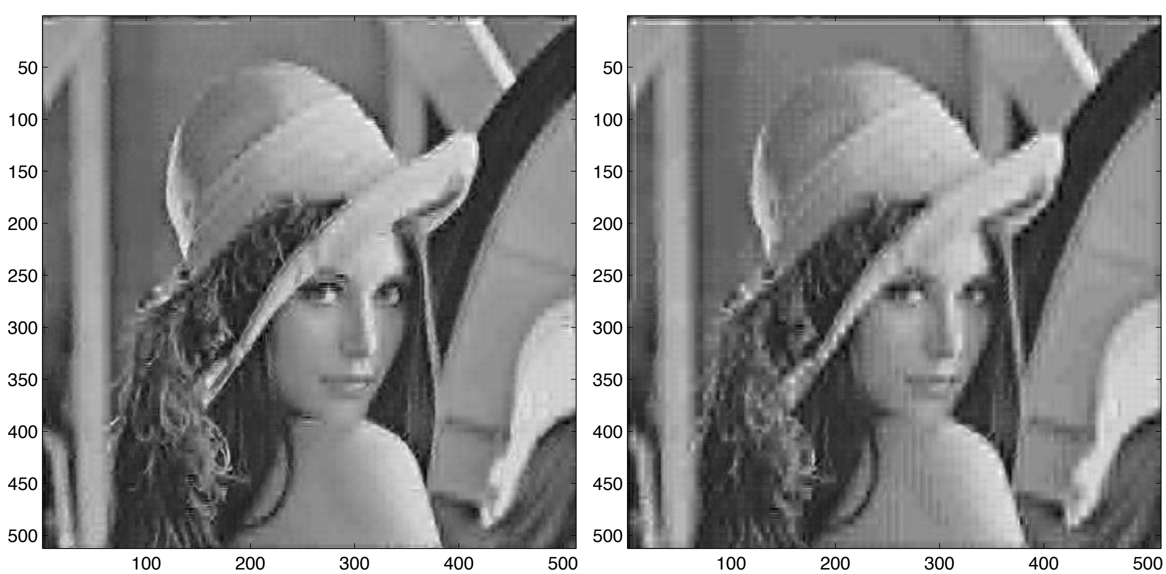


Figure 7: The SBC+NLIVQ scheme with EWMSE.



(a) 0.125 bpp (256 codevectors) (b) 0.0938 bpp (64 codevectors)

Figure 8: Reconstructed images (512×512) by SBC+NLIVQ with EWMSE

EWMSE is then performed in this 16-D vector space. At the decoder, the input 64-D vectors are reconstructed by using a 64-D codebook, which was designed using Gersho's nonlinear interpolative technique.[7,8] Here the original image is Lena (512×512, 8bpp). Two reconstructed images by the above scheme are shown in Figure 8 at bit rates 0.125 bpp and 0.0938 bpp, respectively. Compared with Figures 5 and 6, the image quality in Figure 8 is close to that in Figure 6, with much lower bit rates.

5 CONCLUSION

An entropy-weighted MSE (EWMSE) distortion measure has been introduced in vector quantization with uniform subband images. The coding performance of SBC+VQ is enhanced, especially at low bit rates, since EWMSE allocates different weights to different subband components according to the information content of the subimages.

In order to reduce the computational complexity of the scheme in high dimensional vector spaces, a new feature extractor based on entropy is introduced, and nonlinear interpolative VQ (NLIVQ) with EWMSE is applied. Experimental results show that SBC+NLIVQ with EWMSE in high-dimensionality vector space can provide perceptual pleasing performance even at a compression ratio of 64:1.

6 REFERENCES

[1] J.W. Woods, editor, "Subband image coding", Boston: Kluwer Academic Publishers, 1991.

[2] P.C. Cosman, R.M. Gray, and M. Vetterli, "Vector quantization of image subbands: A survey", IEEE Trans. Image Processing, Vol.5, No.2, PP.202-225, 1996.

[3] Y.H. Kim and J.W. Modestino,"Adaptive entropy coded subband coding of images", IEEE Trans. Image Processing, Vol.1, No.1, PP.31-48, 1992.

[4] L. Vandendorpe, "Human visual weighted quantization", Ann. Télécommun. Vol.47, No.7-8. PP.282-292. 1992.

[5] R. Baseri and V.J. Mathews, "Vector quantization of images using visual masking functions", Proc. ICASSP, Vol.3, PP.365-368, 1992.

[6] D. Taubman and A. Zakhor, "Oritentation adaptive subband coding of image", IEEE Trans. Image Processing, Vol.3, No.4, PP.421-437, 1994.

[7] A. Gersho, "Optimal nonlinear interpolative vector quantization", IEEE Trans. Commun. Vol.38, No.9, PP.1285-1287, 1990.

[8] X. Wang, E. Chan, M.K. Mandal, and S. Panchanathan, "Wavelet-based image coding using nonlinear interpolative vector quantization", IEEE Trans. Image Processing, Vol.5, No.3, PP.518-522, 1996.

[9] J.W. Woods and S.D. O'Neil, "Subband coding of images", IEEE Trans. ASSP, Vol.34, No.5, PP1278-1288, 1986.

[10] P.H. Westerink, D.E. Boekee, J. Biemond, and J.W. Woods, "Subband coding of images using vector quantization", IEEE Trans. Commun., Vol.36, PP713-719, 1988.