

A Block Alternating Likelihood Maximization Approach to Multiuser Detection

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Abstract—In this paper, we address the maximum-likelihood (ML) multiuser detection problem for asynchronous code-division multiple access (CDMA) systems with multiple receiver antennas in frequency-selective fading environments. Multiuser ML detection (MLD) in this case provides attractive symbol error performance, but it requires the solution of a large-scale combinatorial optimization problem. To deal with the computational complexity of this problem, we propose an efficient approximation method based on a block alternating likelihood maximization (BALM) principle. The idea behind BALM is to decompose the large-scale MLD problem into smaller subproblems. Assuming binary or quaternary phase shift keying (BPSK or QPSK) (which are often employed in CDMA), the combinatorial subproblems are then accurately and efficiently approximated by the semidefinite relaxation (SDR) algorithm—an algorithm that has been recently shown to lead to quasi-ML performance in synchronous CDMA scenarios. Simulation results indicate that this BALM detector provides close-to-optimal bit error rate (BER) performance. The BALM principle is quite flexible, and we demonstrate this flexibility by extending BALM to multicarrier (MC) multiuser systems. By exploiting the special signal correlation structure of MC systems, we develop a variation of BALM in which dynamic programming (DP) is used to solve the subproblems. It is shown using simulations that the BER performance of this DP-based BALM detector is as promising as that of the SDR-based BALM detector.

Index Terms—Coordinate ascent, dynamic programming, maximum likelihood detection, multicarrier systems, multiuser detection, relaxation methods, semidefinite programming.

I. INTRODUCTION

IN WIDEBAND code-division multiple access (CDMA) multiuser communication systems, a practically imperative problem is to suppress the multiuser interference effects caused by various factors such as multipath fading, timing asynchronism between users, and the near-far problem. The problem in multiuser detection is to effectively combat multiuser interference, thereby improving system performance as well as user

capacity. One of the most powerful multiuser detectors is the maximum-likelihood (ML) multiuser detector [1], [2], which, under a few common assumptions, is optimal in the sense that it provides the minimum error probability in jointly detecting all the transmitted symbols. In practice, the ML multiuser detector delivers a significantly better symbol error performance than many other (suboptimal) multiuser detectors. Unfortunately, direct implementation of ML multiuser detection is computationally difficult because the multiuser ML detection (MLD) problem is an integer quadratic programming problem, for which there is no known efficient algorithm in most practical cases [3]. (Some restricted cases in which efficient MLD is possible have been determined [4]–[7].)

Since MLD offers attractive symbol error performance, there has been much interest in implementing the ML detector in an approximate but computationally efficient manner. Such interest has led to a variety of approximate MLD approaches, such as the alternating variable methods [8]–[10], expectation maximization [11], [12], relaxation [13]–[18], and heuristic and local search [19]–[26]. (Some lattice searching techniques [see [27] and references therein] could also be applied to ML multiuser detection.) Recently, it has been shown [15]–[18] that for the standard scenario of antipodally-modulated synchronous CDMA, MLD can be accurately (and efficiently) approximated using a so-called semidefinite relaxation (SDR) algorithm [28]–[30]. This SDR-ML detector is effective in that it performs substantially better than other standard suboptimal detectors and is efficient in that its time complexity per symbol [1] is of the order of $K^{2.5}$, where K is the number of users. In fact, it has been shown [16] that several existing suboptimal multiuser detectors, including the decorrelator, the linear minimum mean squared error (LMMSE) detector, the bound relaxation detector [13], [14], and an interference canceling detector based on linear clipper decisions [11], [13], [14] can be viewed as “degenerate” versions of the SDR detector. Furthermore, a recent investigation [31] has suggested that at high signal-to-noise ratios, there is a high probability that SDR-MLD provides the true ML decision.

The performance advantages of the SDR-ML detector in synchronous CDMA systems have motivated our development of extensions to more general multiuser communication scenarios. In this paper, the SDR-ML detection method will be applied to asynchronous CDMA communications over frequency-selective channels with multiple receiver antennas [12], [32]. As will be explained in our problem formulation in Section II, the MLD problem in this asynchronous CDMA case is not only computationally hard (as in synchronous CDMA) but is very

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large in scale due to the presence of interblock interference. It will be illustrated in Section III that directly applying SDR to this large-scale MLD problem remains computationally infeasible. (Many other suboptimal detection methods are also susceptible to this high dimensionality problem.) To overcome this problem, we will propose an effective block alternating likelihood maximization (BALM) technique in Section IV. The idea behind the BALM technique is to decompose the MLD problem into a multitude of subproblems, each of which has a problem size much smaller than that of the complete MLD. Assuming binary or quaternary phase shift keying (BPSK or QPSK) modulation, the BALM constituent subproblems are then accurately and efficiently approximated by the SDR algorithm. The idea behind BALM is reminiscent of that of coordinate ascent [10], but BALM is more general in that it partitions the MLD problem on an overlapping block-by-block basis, rather than on a symbol-by-symbol basis. We will show by simulations that the resulting BALM-SDR detector (with appropriate configurations) provides close-to-optimal bit error rates (BERs). Our simulation results will also show that the BER performance of the BALM-SDR detector is significantly better than that of several suboptimal detectors, including the coordinate ascent detector.

The concept of BALM multiuser detection can also be extended to multiuser communication systems other than CDMA. In Section V, BALM will be considered for a multicarrier (MC) system. We will show that the BALM subproblems in the MC case are structured and can be exactly solved using dynamic programming (DP). The computational advantages of this BALM-DP detector will be discussed. Simulation results will illustrate that the BALM-DP detector provides BER performance as promising as that of the BALM-SDR.

II. BACKGROUND

In this section, we will formulate a received signal model for the uplink of an asynchronous CDMA system with multiple receiving antennas and will discuss the difficulty of implementing ML multiuser detection in this scenario.

A. Asynchronous CDMA Signal Model

We consider an uplink asynchronous CDMA scenario where K users transmit information to the base station through their respective multipath channels. The transmitted signal of the k th user can be represented by

$$x_k(t) = \sum_{n=1}^N b_k[n]c_k(t - (n-1)T_b - \tau_k) \quad (1)$$

where

- $b_k[n] \in \mathcal{A}$ k th-user symbol sequence;
- \mathcal{A} alphabet set;
- N data frame length;
- T_b symbol interval;
- $\tau_k \in [0, T_b)$ transmission delay due to asynchronism between users;
- $c_k(t)$ k th-user spreading code waveform.

In this work, we focus on the BPSK and QPSK constellations, i.e., $\mathcal{A} = \{-1, 1\}$ for BPSK and $\mathcal{A} \in \{-1-j, -1+j, 1-j, 1+j\}$

for QPSK. At the base station, an antenna array of M_r elements is used to receive the user signals. Let $r_i(t)$ be the signal output of the i th antenna, and let $\mathbf{r}(t) = [r_1(t), \dots, r_{M_r}(t)]^T$. Assuming that the channels are time-invariant within the data frame, the vector received signal can be modeled as

$$\mathbf{r}(t) = \sum_{k=1}^K \int_{\mathcal{T}} \mathbf{h}_k(t-u)x_k(u)du + \mathbf{v}(t). \quad (2)$$

Here, \mathcal{T} is the interval on which the signals are defined, $\mathbf{v}(t)$ is complex circular additive white Gaussian noise with zero mean and spectral density \mathcal{N}_0 , and $\mathbf{h}_k(t)$ is the vector impulse response of the k th-user's channel. In a multipath propagation scenario, the impulse response $\mathbf{h}_k(t)$ will typically take the form

$$\mathbf{h}_k(t) = \sum_{l=1}^{L_p} \mathbf{a}(\theta_{kl})\alpha_{kl}\delta(t - d_{kl}) \quad (3)$$

where θ_{kl} , α_{kl} , and d_{kl} are the direction of arrival (DOA), complex gain, and time delay of the l th path of the k th user, respectively, L_p is the number of paths, and $\mathbf{a}(\theta) \in \mathbb{C}^{M_r}$ is the array response vector for a signal from direction θ . Assuming a linear structure with uniform spacing between antenna elements for the array, $\mathbf{a}(\theta)$ is given by

$$\mathbf{a}(\theta) = [1, e^{-j(2\pi d/\lambda)\sin\theta}, \dots, e^{-(M_r-1)j(2\pi d/\lambda)\sin\theta}]^T \quad (4)$$

where d is the inter-element spacing, and λ is the carrier wavelength.

Let us define $\tilde{\mathbf{s}}_k(t)$ to be the received signal waveform of the k th user

$$\tilde{\mathbf{s}}_k(t) = \int_{\mathcal{T}} \mathbf{h}_k(t-u)c_k(u - \tau_k)du. \quad (5)$$

Let $E_k = \int_{\mathcal{T}} \|\tilde{\mathbf{s}}_k(t)\|^2 dt$ denote the received signal energy and $\mathbf{s}_k(t) = \tilde{\mathbf{s}}_k(t)/\sqrt{E_k}$ denote the normalized version of $\tilde{\mathbf{s}}_k(t)$. The received signal in (2) can now be expressed in a convenient multiuser multiple-output form as follows:

$$\mathbf{r}(t) = \sum_{n=1}^N \sum_{k=1}^K \sqrt{E_k}b_k[n]\mathbf{s}_k(t - (n-1)T_b) + \mathbf{v}(t). \quad (6)$$

B. Maximum-Likelihood Multiuser Detection

In multiuser detection, we are concerned with detecting all symbols $b_k[n]$, given that the received waveforms $\mathbf{s}_k(t)$ and the user energies E_k are known. In this paper our emphasis is placed on the maximum-likelihood (ML) multiuser detector, which, under the assumption of $b_k[n]$ being identically and independently distributed, attains the minimum probability of incorrectly detecting $\{b_k[n]\}_{k,n}$ [1], [2]. Let

$$y_k[n] = \int_{\mathcal{T}} \mathbf{s}_k^H(t - (n-1)T_b)\mathbf{r}(t)dt \quad (7)$$

denote the (multiple receiver antenna) matched filter output. Define $\mathbf{b}[n] = [b_1[n], \dots, b_K[n]]^T$, and $\mathbf{y}[n] =$

$[y_1[n], \dots, y_K[n]]^T$. The ML decision is the solution of the following maximization problem [1], [2], [32]:

$$\max_{\substack{\mathbf{b}[n] \in \mathcal{A}^K \\ n=1, \dots, N}} 2 \sum_{n=1}^N \operatorname{Re}\{\mathbf{b}^H[n] \mathbf{A} \mathbf{y}[n]\} - \sum_{n=1}^N \sum_{q=1}^N \mathbf{b}^H[n] \mathbf{H}[n-q] \mathbf{b}[q]. \quad (8)$$

Here, $\mathbf{A} = \operatorname{diag}(\sqrt{E_1}, \dots, \sqrt{E_K})$, $\mathbf{H}[n] = \mathbf{A} \mathbf{R}[n] \mathbf{A}$, and $\mathbf{R}[n] \in \mathbb{C}^{K \times K}$ is an energy normalized signal correlation sequence with (k, l) th entry

$$R_{kl}[n] = \int_{-T}^T s_k^H(t - nT_b) s_l(t) dt. \quad (9)$$

If we assume that all received waveforms $s_k(t)$ are supported on $[0, (M+1)T_b)$, where the integer M is dependent on the ratio of the multipath delay spread plus the maximum user transmission delay to the symbol interval, then we have that

$$\mathbf{R}[n] = \mathbf{0}, \quad |n| > M. \quad (10)$$

Since the coefficients $\mathbf{R}[m]$ for $m \neq 0$ describe the interference between symbol blocks $\mathbf{b}[n]$ and $\mathbf{b}[n-m]$, M is referred to as the *interblock interference (IBI) order*.

To derive a more convenient expression for the maximum likelihood detection (MLD) problem, we define $\mathbf{b}_N = [\mathbf{b}^T[1], \dots, \mathbf{b}^T[N]]^T$, and $\mathbf{y}_N = [\mathbf{y}^T[1], \dots, \mathbf{y}^T[N]]^T$. Now, (8) can be rewritten as

$$\max_{\mathbf{b}_N \in \mathcal{A}^{KN}} J(\mathbf{b}_N) \quad (11)$$

where

$$J(\mathbf{b}_N) = 2 \operatorname{Re}\{\mathbf{b}_N^H \mathbf{A}_N \mathbf{y}_N\} - \mathbf{b}_N^H \mathbf{H}_N \mathbf{b}_N \quad (12)$$

$$\mathbf{A}_N = \operatorname{diag}(\mathbf{A}, \dots, \mathbf{A}) \in \mathbb{R}^{KN \times KN} \quad (13)$$

and

$$\mathbf{H}_N = \begin{bmatrix} \mathbf{H}[0] & \mathbf{H}[-1] & \dots & \mathbf{H}[-M] & & \mathbf{0} \\ \mathbf{H}[1] & \mathbf{H}[0] & \mathbf{H}[-1] & \dots & \ddots & \\ \vdots & \ddots & \ddots & \ddots & \ddots & \mathbf{H}[-M] \\ \mathbf{H}[M] & & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \ddots & \mathbf{H}[M] & \dots & \mathbf{H}[1] & \mathbf{H}[0] \end{bmatrix} \in \mathbb{C}^{KN \times KN}. \quad (14)$$

Problem (11) is a computationally hard combinatorial optimization problem. (Some further details regarding the inherent computational difficulty of MLD can be found in [3].) In addition, (11) is a large-scale problem because the practical values of the frame length N are often very large. Thus far, the most effective method of exactly solving (11) is the Viterbi algorithm [32]–[34] (see also [1] and [2] for the conventional asynchronous CDMA scenario). Unfortunately, this algorithm yields a time complexity per symbol¹ of $\mathcal{O}((1/K)|\mathcal{A}|^{K(M+1)})$ (with

¹The time complexity per symbol is defined to be the ratio of the total time complexity to the total number of detected symbols [1].

$|\mathcal{A}|$ being the alphabet size), which is prohibitive for practical cases of large number of users.

The computational difficulty mentioned above has motivated the development of various suboptimal detectors with substantially lower complexity. The following two sections will focus on our development of a new computationally efficient quasi-ML detector.

III. SEMIDEFINITE RELAXATION AND ITS APPLICATION TO ML DETECTION

In this section, we review the semidefinite relaxation (SDR) algorithm and its application to efficient approximate ML multiuser detection. (More complete descriptions are available elsewhere [15]–[18], [28]–[30].) We will point out that the direct application of SDR to the ML detection of asynchronous CDMA is likely to be computationally expensive. However, SDR will play a central role in the BALM multiuser detection approach outlined in the next section.

A. Semidefinite Relaxation

Semidefinite relaxation [28]–[30] is an accurate approximation method for certain difficult optimization problems. Here, our emphasis is placed on the following Boolean quadratic programming (BQP) problem:

$$\max_{\mathbf{x} \in \{-1, 1\}^m} \mathbf{x}^T \mathbf{Q} \mathbf{x} \quad (15)$$

where m is the problem size, and $\mathbf{Q} = \mathbf{Q}^T \in \mathbb{R}^{m \times m}$. The BQP problem is nondeterministic polynomial-time (NP)-hard, meaning that it is unlikely to be solved with polynomial time complexity in m . The SDR algorithm can approximate the BQP with polynomial time complexity in m . It consists of three steps.

- i) Relax some of the constraints of the BQP problem to obtain a relaxed problem.
- ii) Solve the relaxed problem.
- iii) Convert the solution of the relaxed problem to an approximate BQP solution. A summary of the SDR algorithm is given in Table I.

To show the most important step of relaxation [i.e., Step i)], we use $\mathbf{x}^T \mathbf{Q} \mathbf{x} = \operatorname{Trace}(\mathbf{x} \mathbf{x}^T \mathbf{Q})$ to reformulate Problem (15) as

$$\begin{aligned} \max \quad & \operatorname{Trace}(\mathbf{X} \mathbf{Q}) \\ \text{s.t.} \quad & \mathbf{X} = \mathbf{x} \mathbf{x}^T, \quad \mathbf{x} \in \mathbb{R}^m \\ & X_{ii} = 1, \quad i = 1, \dots, m. \end{aligned} \quad (16)$$

The constraint $\mathbf{X} = \mathbf{x} \mathbf{x}^T$ implies that \mathbf{X} is symmetric, positive semi-definite (PSD), and of rank 1. Now, if the rank 1 constraint is removed and replaced by the constraint that \mathbf{X} is merely symmetric and PSD (i.e., $\mathbf{X} \succeq \mathbf{0}$), then we obtain a relaxed problem

$$\begin{aligned} \max \quad & \operatorname{Trace}(\mathbf{X} \mathbf{Q}) \\ \text{s.t.} \quad & \mathbf{X} \succeq \mathbf{0} \\ & X_{ii} = 1, \quad i = 1, \dots, m. \end{aligned} \quad (17)$$

Problem (17) is referred to as a semidefinite relaxation (SDR) of the BQP problem. The SDR problem in (17) is a semidefinite program [35]: a class of convex optimization problems for

TABLE I

SUMMARY OF THE SDR ALGORITHM. FUNCTION $\sigma : \mathbb{R}^m \rightarrow \mathbb{R}^m$ DENOTES THE ELEMENT-WISE THRESHOLD DECISION FUNCTION, I.E., THE i TH ELEMENT OF $\sigma(\mathbf{x})$ IS 1 IF $x_i \geq 0$, AND -1 OTHERWISE

Step 1. Given $\mathbf{Q} \in \mathbb{R}^{m \times m}$, and a parameter M_{rand} denoting the number of randomizations.

Step 2. Solve the semidefinite program

$$\hat{\mathbf{X}} = \arg \max_{\substack{\mathbf{X} \succeq 0 \\ x_{ii} = 1 \forall i}} \text{Trace}(\mathbf{X}\mathbf{Q})$$

Step 3. (Randomization) Factorize $\hat{\mathbf{X}} = \hat{\mathbf{V}}^T \hat{\mathbf{V}}$.

for $i=1,2, \dots, M_{\text{rand}}$

Randomly generate a vector $\mathbf{u}_i \in \mathbb{R}^m$ uniformly distributed on an m -dimensional unit sphere. Compute $\tilde{\mathbf{x}}_i = \sigma(\hat{\mathbf{V}}^T \mathbf{u}_i)$.

end;

Choose $\hat{\mathbf{x}} = \tilde{\mathbf{x}}_q$ as the approximate solution of \mathbf{x}^* , where $q = \arg \max_{i=1, \dots, M_{\text{rand}}} \tilde{\mathbf{x}}_i^T \mathbf{Q} \tilde{\mathbf{x}}_i$.

which the globally optimal solution can be efficiently obtained. Helmberg *et. al* [36] have developed an optimization algorithm tailor-made for Problem (17), which yields an attractive computational cost of $\mathcal{O}(m^{3.5})$.

Once we have a solution to (17), we need to extract an approximate solution $\hat{\mathbf{x}}$ to (15). This can be done in several ways, an obvious one of which is to use the principal eigenvector of the solution of (17) [15], [18]. Here, we employ a randomization algorithm [28]–[30], which we have found to be very effective in the MLD application [16]. The randomization process is provided in Step 3 of Table I, (detailed descriptions for this randomization algorithm can be found in [16] and [28]–[30]), where the input parameter M_{rand} denotes the number of randomizations (i.e., the number of trials of the random search process). A few issues regarding the randomization are as follows (see [16] for more detailed explanations):

- 1) Simulation results have shown that the randomization algorithm can achieve accurate approximations with modest M_{rand} .
- 2) The complexity of the randomization process (with modest M_{rand}) is much smaller than that of solving SDR.

B. Application of SDR to MLD

To illustrate how SDR can be applied to ML multiuser detection, we use the conventional synchronous CDMA scenario [1] as an example. In the conventional synchronous CDMA scenario, where $\tau_1 = \dots = \tau_K = 0$ and $\mathbf{h}_1(t) = \dots = \mathbf{h}_K(t) = \delta(t)$, there is no IBI, i.e., $M = 0$. Thus, the MLD problem in (11) can be reduced to N independent problems

$$\hat{\mathbf{b}}_{\text{ML}}[n] = \arg \max_{\mathbf{b}[n] \in \mathcal{A}^K} 2\text{Re}\{\mathbf{b}^H[n]\mathbf{A}\mathbf{y}[n]\} - \mathbf{b}^H[n]\mathbf{H}[0]\mathbf{b}[n] \quad (18)$$

where $\hat{\mathbf{b}}_{\text{ML}}[n]$ denotes the ML decision of $\mathbf{b}[n]$. Note that the problem size of (18) is much smaller than that of the more general MLD problem in (11). To apply SDR to (18), we first focus

on the case of BPSK modulation, in which Problem (18) can be rewritten as

$$\max_{\mathbf{b}[n] \in \{-1,1\}^K} 2\mathbf{b}^T[n]\text{Re}\{\mathbf{A}\mathbf{y}[n]\} - \mathbf{b}^T[n]\text{Re}\{\mathbf{H}[0]\}\mathbf{b}[n]. \quad (19)$$

Problem (19) is equivalent to the following BQP:

$$\max_{\substack{\mathbf{b}[n] \in \{-1,1\}^K \\ c \in \{-1,1\}}} [\mathbf{b}^T[n] \ c] \begin{bmatrix} -\text{Re}\{\mathbf{H}[0]\} & \text{Re}\{\mathbf{A}\mathbf{y}[n]\} \\ (\text{Re}\{\mathbf{A}\mathbf{y}[n]\})^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{b}[n] \\ c \end{bmatrix}. \quad (20)$$

It can be verified that if $(\mathbf{b}^*[n], c^*)$ is the solution of (20), then the solution of (19) is $\hat{\mathbf{b}}_{\text{ML}}[n] = c^* \mathbf{b}^*[n]$. Thus, we can use the previously described SDR algorithm to approximate the equivalent problem in (20) and then use the relation mentioned above to obtain an approximate $\hat{\mathbf{b}}_{\text{ML}}[n]$. It can be shown [15] that for the QPSK case, Problem (18) is equivalent to

$$\max_{\substack{\tilde{\mathbf{b}}[n] \in \{-1,1\}^{2K} \\ c \in \{-1,1\}}} [\tilde{\mathbf{b}}^T[n] \ c] \begin{bmatrix} -\tilde{\mathbf{H}}[0] & \mathbf{g}[n] \\ \mathbf{g}^T[n] & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{b}}[n] \\ c \end{bmatrix} \quad (21)$$

where $\tilde{\mathbf{b}}[n] = [\text{Re}\{\mathbf{b}^T[n]\} \ \text{Im}\{\mathbf{b}^T[n]\}]^T$, $\mathbf{g}[n] = [(\text{Re}\{\mathbf{A}\mathbf{y}[n]\})^T \ (\text{Im}\{\mathbf{A}\mathbf{y}[n]\})^T]^T$, and

$$\tilde{\mathbf{H}}[0] = \begin{bmatrix} \text{Re}\{\mathbf{H}[0]\} & -\text{Im}\{\mathbf{H}[0]\} \\ \text{Im}\{\mathbf{H}[0]\} & \text{Re}\{\mathbf{H}[0]\} \end{bmatrix} \quad (22)$$

and that the SDR algorithm can be applied in the same way as that for BPSK.

The above SDR-ML detection process for synchronous CDMA is efficient in that its time complexity per symbol is $\mathcal{O}(K^{2.5})$. Now, a question of interest is whether or not SDR can be efficiently applied to the more general MLD problem in (11) for which IBI is present. Since Problem (11) is in the same form as that in Problem (18), the SDR algorithm can be directly applied to (11). However, such an SDR application requires a time complexity per symbol of $\mathcal{O}((KN)^{2.5})$, which is unlikely to be affordable for large N . In the next section, we will propose a remedy for this dimensionality problem.

IV. BALM

In this section, we propose a BALM approach, the objective of which is to mitigate the computational difficulty of processing the large-scale MLD problem in (11). It will be illustrated that the previously described SDR algorithm turns out to be a good match for the BALM approach. In particular, we will realize a BALM-based detector by using SDR to accurately and efficiently approximate the optimization problems constituting BALM. (In the next section, an alternative BALM implementation for multicarrier systems will be considered.) The BALM principle and the BALM-SDR multiuser detection method will be presented in Section IV-A. Simulation results showing the potential of the BALM-SDR detector will be given in Section IV-B.

A. BALM Principle and its Combination with SDR

To illustrate the BALM principle, it is instructive to study the coordinate ascent (CA) method from which BALM is generalized. (The CA method was applied to a single-user system

in [10]. A method closely related to CA [called space alternating generalized expectation maximization] was used to perform suboptimal ML multiuser detection in [11].) Coordinate ascent is an iterative technique; at each iteration, the MLD objective is maximized with respect to only one decision variable, whereas the other decision variables are held fixed. By doing so, CA decouples the MLD problem into a multitude of one-dimensional (1-D) subproblems, each of which is much easier to solve than its full counterpart. Let $\hat{\mathbf{b}}_N^{(m)}$ denote the approximate MLD solution generated at the m th cycle of the CA method, and let $\hat{b}_{N,\ell}^{(m)}$ denote the ℓ th element of $\hat{\mathbf{b}}_N^{(m)}$. The CA method consists of the following steps, :

Given a number of cycles C and an initialization $\hat{\mathbf{b}}_N^{(0)}$.
 for $m = 1, \dots, C$
 for $\ell = 1, \dots, KN$

$$\hat{b}_{N,\ell}^{(m)} = \arg \max_{b \in \mathcal{A}} J(\hat{b}_{N,1}^{(m)}, \dots, \hat{b}_{N,\ell-1}^{(m)}, b, \hat{b}_{N,\ell+1}^{(m-1)}, \dots, \hat{b}_{N,KN}^{(m-1)}).$$

end;
 end;

where the MLD objective function $J(\mathbf{b}_N)$ is defined in (12). The CA method has the following advantages: i) The 1-D subproblem in (23) can be very easily solved [10], and ii) the objective values $J(\hat{\mathbf{b}}_N^{(m)})$ are monotonically nondecreasing with m , i.e.,

$$J(\hat{\mathbf{b}}_N^{(0)}) \leq J(\hat{\mathbf{b}}_N^{(1)}) \leq \dots \leq J(\hat{\mathbf{b}}_N^{(C)}). \quad (24)$$

Hence, the approximate MLD solution of any given cycle is expected to yield either improved or equal performance compared to that of its previous cycles.

In our BALM approach, the objective $J(\mathbf{b}_N)$ is maximized with respect to a block of symbols at each iteration. In addition, overlap between the updating blocks is permitted. Define $\{\hat{\mathbf{b}}^{(m)}[n]\}_{n=1}^N$ to be the approximate MLD solution generated at the m th cycle of BALM, and let $J(\mathbf{b}_N) = J(\mathbf{b}[1], \dots, \mathbf{b}[N])$. The BALM approach consists of the following procedures:

Given a number of cycles C , a window length P , a time shift factor $Q, Q \leq P \leq N$, and an initialization $\{\hat{\mathbf{b}}^{(0)}[n]\}_{n=1}^N$.
 for $m = 1, \dots, C$
 for $\ell = 1, Q+1, 2Q+1, \dots, N-P+1$
 Update $\{\hat{\mathbf{b}}^{(m)}[\ell], \hat{\mathbf{b}}^{(m)}[\ell+1], \dots, \hat{\mathbf{b}}^{(m)}[\ell+P-1]\}$
 to be the solution of (25), shown at the bottom of the page.
 end;
 end;

Here, we make a simplifying assumption that $(N-P)/Q$ has no remainder. Fig. 1 depicts the BALM updating process within

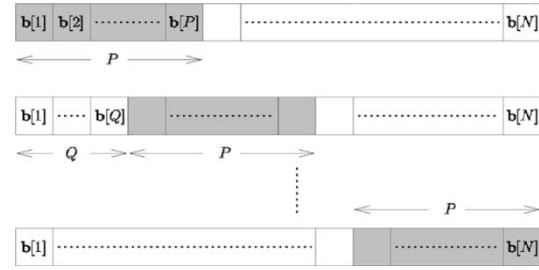


Fig. 1. Block update of BALM within a cycle.

a cycle. It can be shown that the monotonic nondecreasing property of $J(\hat{\mathbf{b}}_N^{(m)})$ in (24) also holds for BALM; see [37] for the details. Consider the following lemma for reformulating the BALM subproblem in (25).

Lemma 1: Given a quadratic function $J(\mathbf{b}) = 2\text{Re}\{\mathbf{b}^H \mathbf{g}\} - \mathbf{b}^H \mathbf{H} \mathbf{b}$, where $\mathbf{H} = \mathbf{H}^H$, partition $\mathbf{b} = [\mathbf{b}_1^T \mathbf{b}_2^T \mathbf{b}_3^T]^T$, $\mathbf{g} = [\mathbf{g}_1^T \mathbf{g}_2^T \mathbf{g}_3^T]^T$, and

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \mathbf{H}_{13} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \mathbf{H}_{23} \\ \mathbf{H}_{31} & \mathbf{H}_{32} & \mathbf{H}_{33} \end{bmatrix} \quad (26)$$

where $\mathbf{b}_i, \mathbf{g}_i \in \mathbb{C}^{n_i}$, and $\mathbf{H}_{i\ell} = \mathbf{H}_{\ell i}^H \in \mathbb{C}^{n_i \times n_\ell}$ for $i, \ell \in \{1, 2, 3\}$. For any given $\mathbf{b}_1 \in \mathbb{C}^{n_1}$, $\mathbf{b}_3 \in \mathbb{C}^{n_3}$, and for any set $\mathcal{S} \subseteq \mathbb{C}^{n_2}$, the partial quadratic maximization problem

$$\max_{\mathbf{b}_2 \in \mathcal{S}} J([\mathbf{b}_1^T \mathbf{b}_2^T \mathbf{b}_3^T]^T) \quad (27a)$$

$$= \max_{\mathbf{b}_2 \in \mathcal{S}} 2 \sum_{i=1}^3 \text{Re}\{\mathbf{b}_i^H \mathbf{g}_i\} - \sum_{i=1}^3 \sum_{\ell=1}^3 \mathbf{b}_i^H \mathbf{H}_{i\ell} \mathbf{b}_\ell \quad (27b)$$

is equivalent to the quadratic maximization problem

$$\max_{\mathbf{b}_2 \in \mathcal{S}} 2\text{Re}\{\mathbf{b}_2^H (\mathbf{g}_2 - \mathbf{H}_{21} \mathbf{b}_1 - \mathbf{H}_{23} \mathbf{b}_3)\} - \mathbf{b}_2^H \mathbf{H}_{22} \mathbf{b}_2. \quad (28)$$

Lemma 1 is easily shown by removing components independent of \mathbf{b}_2 from (27b).

If we define $\mathbf{b}_P[\ell] = [\mathbf{b}^T[\ell], \dots, \mathbf{b}^T[\ell+P-1]]^T$ and $\mathbf{y}_P[\ell] = [\mathbf{y}^T[\ell], \dots, \mathbf{y}^T[\ell+P-1]]^T$, the BALM subproblem in (25) can be rewritten as

$$\max_{\mathbf{b}_P[\ell] \in \mathcal{A}^{KP}} J([\hat{\mathbf{b}}_{\ell-1}^{(m)}]^T \mathbf{b}_P^T[\ell] (\hat{\mathbf{b}}_{N-\ell-P+1}^{(m-1)}[\ell+P])^T]^T). \quad (29)$$

Furthermore, we can partition

$$\mathbf{y}_N = [\mathbf{y}_{\ell-1}^T[1] \mathbf{y}_P^T[\ell] \mathbf{y}_{N-\ell-P+1}^T[\ell+P]]^T$$

and

$$\mathbf{H}_N = \begin{bmatrix} \mathbf{H}_P & \mathbf{L} & & \mathbf{0} \\ \vdots & \vdots & \ddots & \\ & \mathbf{U} & \mathbf{H}_P & \mathbf{L} \\ & & \vdots & \vdots & \ddots \\ \mathbf{0} & & & \mathbf{U} & \mathbf{H}_P \end{bmatrix} \quad (30)$$

$$\max_{\substack{\mathbf{b}[n] \in \mathcal{A}^K, \\ n=\ell, \dots, \ell+P-1}} J(\hat{\mathbf{b}}^{(m)}[1], \dots, \hat{\mathbf{b}}^{(m)}[\ell-1], \mathbf{b}[\ell], \dots, \mathbf{b}[\ell+P-1], \hat{\mathbf{b}}^{(m-1)}[\ell+P], \dots, \hat{\mathbf{b}}^{(m-1)}[N]). \quad (25)$$

TABLE II
 SIMULATION SETTINGS IN EXAMPLE 1

user no. k	spreading codes $\{c_{kp}\}$	transmission delays (T_b) τ_k	multipath parameters							
			DOAs ($^\circ$)		delays (T_b)		gains			
			θ_{k1}	θ_{k2}	d_{k1}	d_{k2}	$ \alpha_{k1} $	$\angle\alpha_{k1}$	$ \alpha_{k2} $	$\angle\alpha_{k2}$
1	1 1 1	0.53	16	65	0.30	0.05	0.65	-2.96	1.05	-1.35
2	1 -1 1	0.28	69	40	0.23	0.15	0.78	2.31	1.30	0.64
3	1 1 -1	0.11	64	42	0.08	0.02	0.48	-0.37	1.39	-2.38

where \mathbf{H}_N is defined in (14):

$$\mathbf{H}_P = \begin{bmatrix} \mathbf{H}[0] & \mathbf{H}[-1] & \dots & \mathbf{H}[-P+1] \\ \mathbf{H}[1] & \mathbf{H}[0] & \dots & \mathbf{H}[-P+2] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}[P-1] & \mathbf{H}[P-2] & \dots & \mathbf{H}[0] \end{bmatrix} \quad (31)$$

$$\mathbf{U} = \begin{bmatrix} \mathbf{H}[M] & \dots & \mathbf{H}[2] & \mathbf{H}[1] \\ \mathbf{H}[M+1] & \dots & \mathbf{H}[3] & \mathbf{H}[2] \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{H}[M+P-1] & \dots & \mathbf{H}[P+1] & \mathbf{H}[P] \end{bmatrix} \quad (32)$$

and

$$\mathbf{L} = \begin{bmatrix} \mathbf{H}[-P] & \mathbf{H}[-P-1] & \dots & \mathbf{H}[-P-M+1] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}[-2] & \mathbf{H}[-3] & \dots & \mathbf{H}[-M-1] \\ \mathbf{H}[-1] & \mathbf{H}[-2] & \dots & \mathbf{H}[-M] \end{bmatrix}. \quad (33)$$

Applying Lemma 1 to (29), the BALM subproblem in (25) can be reformulated as

$$\max_{\mathbf{b}_P[\ell] \in \mathcal{A}^{KP}} 2\text{Re}\{\mathbf{b}_P^H[\ell] \boldsymbol{\gamma}_P^{(m)}[\ell]\} - \mathbf{b}_P^H[\ell] \mathbf{H}_P \mathbf{b}_P[\ell] \quad (34)$$

where

$$\begin{aligned} \boldsymbol{\gamma}_P^{(m)}[\ell] &= \mathbf{A}_P \mathbf{y}_P[\ell] - [\mathbf{0} \ \mathbf{U}] \hat{\mathbf{b}}_{\ell-1}^{(m)}[1] - [\mathbf{L} \ \mathbf{0}] \hat{\mathbf{b}}_{N-\ell-P+1}^{(m-1)}[\ell+P] \\ &= \mathbf{A}_P \mathbf{y}_P[\ell] - \mathbf{U} \hat{\mathbf{b}}_M^{(m)}[\ell-M] - \mathbf{L} \hat{\mathbf{b}}_M^{(m-1)}[\ell+P] \end{aligned} \quad (35)$$

and \mathbf{A}_P is defined in (13). The BALM constituent subproblem in (34) is similar to the synchronous CDMA MLD subproblem in (18) in that a quadratic function is maximized subject to alphabet constraints on the symbols. Hence, we can use the SDR algorithm described in Section III to approximate the solution of (34). We call this combined method of BALM and SDR the *BALM-SDR detector*.

To determine the computational complexity of the BALM-SDR detector, we observe that for each BALM constituent subproblem, a time complexity of $\mathcal{O}(K^{3.5} P^{3.5})$ is required by the SDR algorithm. Since BALM partitions the original MLD problem into $(N-P)/Q + 1$ subproblems, the total time complexity of the BALM-SDR detector is

$$\mathcal{O}\left(C \left(\frac{N-P}{Q} + 1\right) K^{3.5} P^{3.5}\right). \quad (36)$$

In practice, we often choose $Q \leq P \ll N$ for computational economy. Under such circumstances, the total time complexity

of BALM-SDR can be simplified to $\mathcal{O}((CN/Q)K^{3.5}P^{3.5})$. It follows that the time complexity per symbol of BALM-SDR is

$$\mathcal{O}\left(\frac{C}{Q} K^{2.5} P^{3.5}\right). \quad (37)$$

It is clear that for large number of users, the time complexity per symbol of BALM-SDR is much lower than that of the ML Viterbi algorithm [$\mathcal{O}((1/K)|\mathcal{A}|^{K(M+1)})$ operations per symbol]. The computational cost of BALM-SDR is also considerably smaller than that of directly using SDR to approximate MLD [$\mathcal{O}((KN)^{2.5})$ operations per symbol].

The BALM-SDR performance characteristics with respect to P , Q , and C will be examined by simulation results in the next section.

B. Simulation Results

Three simulation examples are used to demonstrate the promising performance of the BALM-SDR detector.

Example 1: This example aims to study the behavior of the BALM-SDR detector under various configurations and to compare the performance of the BALM-SDR detector to that of the true ML detector, which is obtained via the Viterbi algorithm. As the ML Viterbi algorithm is computationally acceptable only for small number of users, we set $K = 3$. A two-element, $\lambda/2$ -spaced uniformly linear array is used at the receiver. The rest of the system and channel parameter settings are given in Table II. We should point out that the system is fully loaded, i.e., the number of users is equal to the number of chips. Data modulation is BPSK. In addition to the ML Viterbi algorithm and our BALM-SDR detector, the following suboptimal detectors were also tested: the (multiple receiver antenna) matched filter detector, the FIR linear minimum mean square error (LMMSE) detector, and the CA detector.

Fig. 2(a) shows the performance of the various detectors (the configuration of the BALM-SDR detector can be found in the legend). As illustrated, the BER performance of the BALM-SDR detector is close to that of the true ML detector and is substantially better than that of the other suboptimal detectors. Moreover, unlike the CA detector, the BALM-SDR detector yields promising performance, even with poor initialization (i.e., the matched filter detector initialization).

Fig. 2(b)–(d) illustrate the performance characteristics of BALM-SDR for various values of the window length P , the time shift factor Q , and the number of cycles C . In Fig. 2(b), we illustrate a situation where Q is fixed, and matched filter detector initialization is used. As seen, increasing P enhances the BER performance substantially. In contrast, increasing C only modestly improves the BER. Similar results are observed in Fig. 2(c), in which matched filter detector initialization is

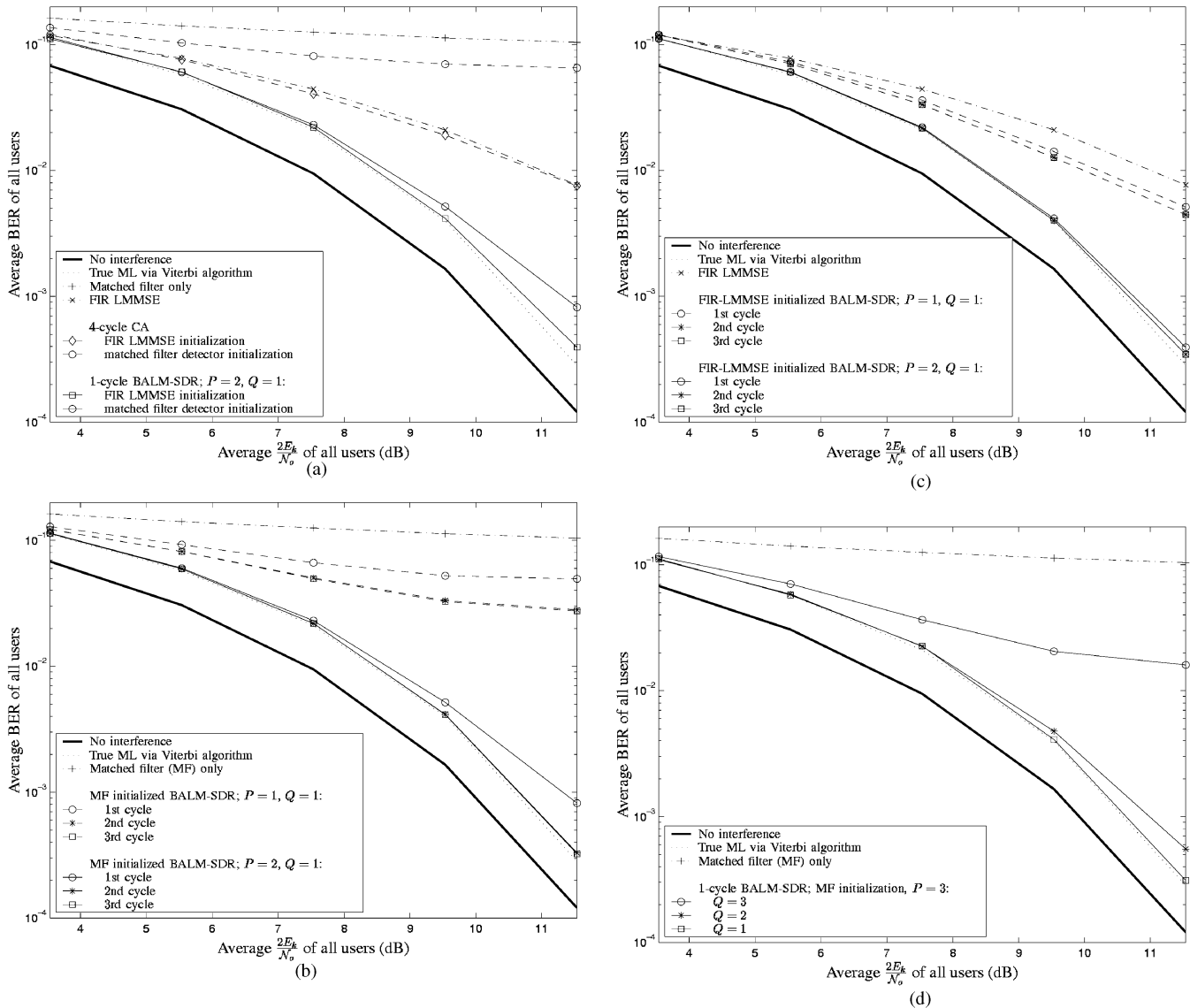


Fig. 2. BALM-SDR detection performance in a three-user, two-antenna, uplink asynchronous CDMA system. (a) Performance comparison with various multiuser detectors. (b)–(d) Performance behaviors for various parameter and initialization settings. The filter order of the FIR LMMSE detector is 3. The number of randomizations for the BALM-SDR detector is 20. The number of cycles of the CA detector, set to be 4, was found to be sufficient for (local) convergence.

replaced by FIR-MMSE detector initialization. In Fig. 2(d), the BER performance for various Q is given. The other parameters are fixed to be $P = 3, C = 1$. Fig. 2(d) indicates that using the smallest Q yields the best BER performance. This result, along with those observed in Fig. 2(b) and (c), indicates that significant performance gain can be achieved by increasing the difference between P and Q , i.e., increasing window overlap. \square

Example 2: We now consider a multiuser communication system employing both CDMA and space division multiple access (SDMA). Taking advantage of spatial diversity, this system allows some users to share the same CDMA signature waveform in order to increase the user capacity. In our simulation scenario, each spreading-code waveform is used by two users. The total number of users is $K = 14$, and the spreading code sequences are the length-7 Gold codes. Data modulation is QPSK. The receiving antenna array is uniform linear, contains three el-

ements, and is $\lambda/2$ -spaced. Each multipath channel consists of three paths. At each trial, the multipath parameters and the user transmission delays [see(3) for definitions] are randomly generated, with θ_{kl} being uniformly distributed on $[-\pi/6, \pi/6]$; τ_k being uniformly distributed on $[0, T_b)$; d_{kl} being uniformly distributed on $[0, (4/7)T_b)$; and α_{kl} being unit-variance complex circular Gaussian distributed. At each trial, the transmitted power of each user is adjusted such that all received user waveforms have equal energy. The simulation results are shown in Fig. 3. Clearly, the BALM-SDR detector yields better average BER performance than the other suboptimal detectors. \square

Example 3: In the conventional asynchronous CDMA scenario (where one receiving antenna is used and multipath fading is negligible), a major factor limiting system performance is the near-far problem, i.e., when the user energies exhibit disparity. In this example, the near-far resistant capability of the BALM-SDR detector is demonstrated in such a scenario.

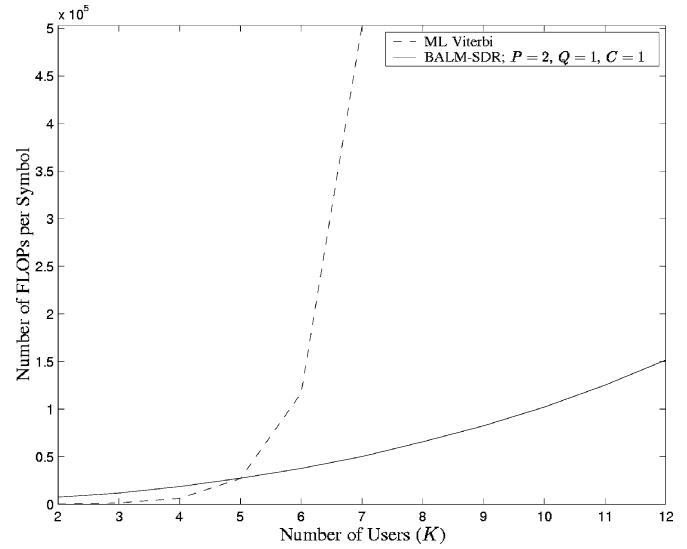
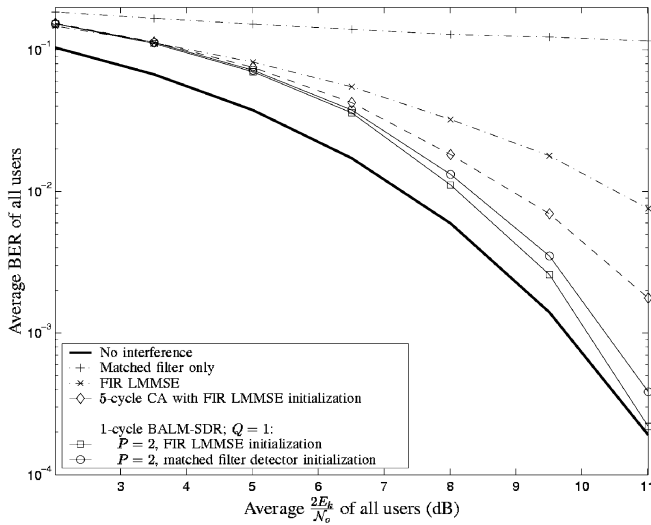


Fig. 3. BER performance of the various detectors in a 14-user asynchronous CDMA/SDMA system. The filter order of the FIR LMMSE detector is 3. The number of randomizations for the BALM-SDR detector is 20.

Fig. 5. Complexity comparison of the exact ML detector via Viterbi algorithm and the BALM-SDR detector.

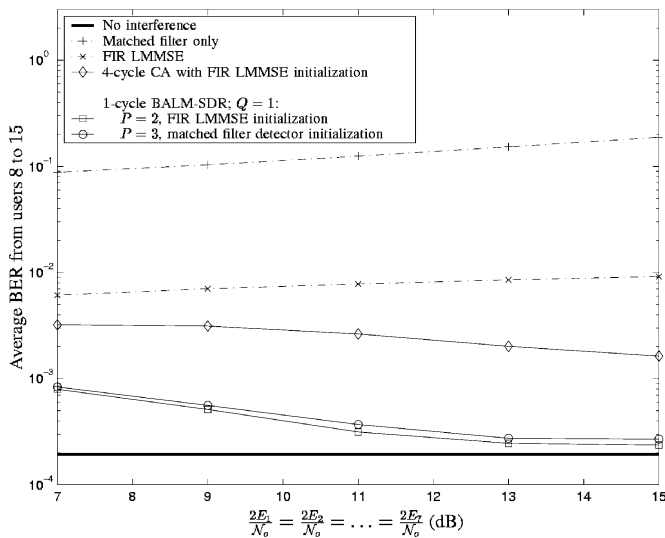


Fig. 4. Near-far performance of the various detectors in a 15-user asynchronous CDMA system. The filter order of the FIR LMMSE detector is 9. The number of randomizations for the BALM-SDR detector is 20.

The number of users is 15, and data modulation is BPSK. The spreading sequences are the length-15 Kasami (large set) codes. Users 1 to 7 are the interferers, and their SNRs are identical. Users 8 to 15 are the desired users, and their SNRs are fixed at $2E_k/\mathcal{N}_o = 11$ dB. At each trial, the delays τ_k are randomly generated following a uniform distribution on $[0, T_b)$. Fig. 4 shows the average BER performance of the desired users versus various interferers' SNRs. Compared with the other suboptimal detectors, the BALM-SDR detector shows substantially better BER performance in various near-far situations. \square

Example 4: As mentioned in the previous subsection, the BALM-SDR detector offers substantially lower computational complexity order than the exact ML detector when the number of users (K) is large. This computational advantage is demonstrated by simulations in this example. The simulation settings are similar to those of the previous example, except

that the SNRs are fixed at $2E_k/\mathcal{N}_o = 11$ dB for all users. The SDR optimization process for the BALM-SDR detector is implemented using the standard interior-point optimization algorithm in [36]. The numbers of floating-point operations (FLOPs) per symbol of the exact ML detector (via Viterbi algorithm) and the BALM-SDR detector are plotted in Fig. 5, which indicates that the computational cost of the BALM-SDR detector is much lower than that of the ML Viterbi algorithm when $K \geq 6$. \square

V. BALM: VARIATIONS ON A THEME

In this section, we demonstrate the flexibility of the BALM technique by applying it to multicarrier (MC)-based multiuser communication systems. In MC-based multiuser schemes, the signal correlation has an interesting structure induced by the frequency localization of the user signature waveforms. (In the CDMA case, the signal correlation is virtually unstructured.) We will show that by exploiting this correlation structure, the BALM constituent subproblems can be exactly solved using dynamic programming (DP). It will be illustrated that with appropriate MC waveforms and detector parameter settings, the complexity of this BALM-DP method can be of low order. In Section V-A, the MC signal correlation will be described. Then, the DP method for BALM will be investigated in Section V-B. Simulation results will be shown in Section V-C.

A. Multicarrier Communications

Consider a multiuser communication scenario reminiscent of that presented in Section II-A, with the CDMA signature waveforms $c_k(t)$ being replaced by the multicarrier (MC) signature waveforms:

$$c_k(t) = e^{j2\pi(k-1)t/T_b} g(t) \tag{38}$$

where $g(t)$ is a baseband pulse. In this MC system, each user occupies a subcarrier (i.e., $j2\pi(k-1)t/T_b$), and the associated

subcarrier-modulated signals are spectrally overlapping for increasing spectral efficiency. Define an integer $D \geq 0$ such that the two-sided bandwidth of $g(t)$ is $(D + 1)/T_b$, i.e.,

$$|G(f)| \simeq 0, \quad |f| \geq \frac{D+1}{2T_b} \quad (39)$$

where $G(f)$ is the Fourier transform of $g(t)$. Since D is proportional to the amount of spectral overlap between contiguous MC signals, we refer to D as the *subcarrier overlap factor*. The following observation is noted.

Observation 1: If the MC signature waveforms are employed, then every correlation matrix $\mathbf{R}[n]$ has its dominant elements lying within a diagonal band from the D th subdiagonal to the D th super-diagonal, i.e.,

$$R_{kl}[n] \simeq 0, \quad |k - l| > D \quad (40)$$

for all n .

Observation 1 is the direct consequence that a MC signature waveform exhibits significant spectral overlap only with $2D$ of its neighboring MC signature waveforms. The proof of Observation 1 is as follows. Define $\check{s}_k(f)$ to be the element-wise Fourier transform of $\mathbf{s}_k(t)$, i.e., the q th element of $\check{s}_k(f)$ is the Fourier transform of the q th element of $\mathbf{s}_k(t)$. By Parseval's relation, the correlation coefficient in (9) can be expressed as

$$R_{kl}[n] = \text{Re} \left\{ \int_{-\infty}^{\infty} e^{j2\pi fnT_b} \check{s}_k^H(f) \check{s}_l(f) df \right\}. \quad (41)$$

From (5) and (38)

$$\check{s}_k(f) = \frac{1}{\sqrt{E_k}} \check{\mathbf{h}}_k(f) G(f - f_k) \quad (42)$$

where $f_k = (k - 1)/T_b$, and $\check{\mathbf{h}}_k(f)$ is the element-wise Fourier transform of $\mathbf{h}(t)$. Substituting (42) into (41) yields

$$R_{kl}[n] = \text{Re} \left\{ \int_{-\infty}^{\infty} \frac{e^{j2\pi fnT_b}}{\sqrt{E_k E_l}} \check{\mathbf{h}}_k^H(f) \check{\mathbf{h}}_l(f) G^*(f - f_k) G(f - f_l) df \right\}. \quad (43)$$

Since the magnitude of $G(f)$ is negligible for $|f| > (D + 1)/2T_b$, $G(f - f_k)$ and $G(f - f_l)$ do not have significant overlap for $|f_k - f_l| \geq (D + 1)/T_b$, and thus, (40) follows.

B. Dynamic Programming for BALM Implementation

Clearly, the BALM multiuser detection principle in Section IV-A can be directly applied to the above described MC system. Here, we consider a variation of BALM, for which the BALM constituent subproblems are exactly solved using dynamic programming and the structure of (40) of the MC system. For notational simplicity, we rewrite the BALM subproblem [in (34)] in the following form:

$$\max_{\mathbf{b}_P \in \mathcal{A}^{KP}} 2\text{Re}\{\mathbf{b}_P^H \boldsymbol{\gamma}_P\} - \mathbf{b}_P^H \mathbf{H}_P \mathbf{b}_P \quad (44a)$$

$$= \max_{\mathbf{b}[n] \in \mathcal{A}^{KP}, n=1, \dots, P} \sum_{n=1}^P 2\text{Re}\{\mathbf{b}^H[n] \boldsymbol{\gamma}[n]\} - \sum_{n=1}^P \sum_{q=1}^P \mathbf{b}^H[n] \mathbf{H}[n - q] \mathbf{b}[q] \quad (44b)$$

where $\mathbf{b}_P = [\mathbf{b}^T[1], \dots, \mathbf{b}^T[P]]^T$, and $\boldsymbol{\gamma}_P = [\boldsymbol{\gamma}^T[1], \dots, \boldsymbol{\gamma}^T[P]]^T$. Define $\tilde{\mathbf{b}}_k = [b_k^T[1], \dots, b_k^T[P]]^T$, and $\tilde{\boldsymbol{\gamma}}_k = [\boldsymbol{\gamma}_k^T[1], \dots, \boldsymbol{\gamma}_k^T[P]]^T$. Problem (44b) can be rewritten as

$$\max_{\tilde{\mathbf{b}}_k \in \mathcal{A}^P, k=1, \dots, K} 2 \sum_{k=1}^K \text{Re}\{\tilde{\mathbf{b}}_k^H \tilde{\boldsymbol{\gamma}}_k\} - \sum_{k=1}^K \sum_{l=1}^K \tilde{\mathbf{b}}_k^H \tilde{\mathbf{H}}_{kl} \tilde{\mathbf{b}}_l \quad (45)$$

where

$$\tilde{\mathbf{H}}_{kl} = \begin{bmatrix} H_{kl}[0] & H_{kl}[-1] & \dots & H_{kl}[-P] \\ H_{kl}[1] & H_{kl}[0] & \dots & H_{kl}[-P+1] \\ \vdots & \vdots & \ddots & \vdots \\ H_{kl}[P] & H_{kl}[P-1] & \dots & H_{kl}[0] \end{bmatrix}. \quad (46)$$

Notice that $\tilde{\mathbf{H}}_{kl} = \tilde{\mathbf{H}}_{lk}^H$. In addition, Observation 1 leads to a property that

$$\tilde{\mathbf{H}}_{kl} = \mathbf{0}, \quad |k - l| > D. \quad (47)$$

Using (47), the BALM subproblem in (45) can be turned to a DP problem. To see this, we use (47) to rewrite the quadratic term in (45) as

$$\sum_{k=1}^K \sum_{l=1}^K \tilde{\mathbf{b}}_k^H \tilde{\mathbf{H}}_{kl} \tilde{\mathbf{b}}_l = \sum_{k=1}^K \left(\tilde{\mathbf{b}}_k^H \tilde{\mathbf{H}}_{kk} \tilde{\mathbf{b}}_k + 2 \sum_{l=k-D}^{k-1} \text{Re}\{\tilde{\mathbf{b}}_k^H \tilde{\mathbf{H}}_{kl} \tilde{\mathbf{b}}_l\} \right) \quad (48)$$

where, without loss of generality, the matrices $\tilde{\mathbf{b}}_k$ and $\tilde{\mathbf{H}}_{kl}$ are assumed to be zero for either $k \leq 0$ or $l \leq 0$. Using (48), Problem (45) becomes

$$\max_{\tilde{\mathbf{b}}_k \in \mathcal{A}^P, k=1, \dots, K} \sum_{k=1}^K \lambda_k(\tilde{\mathbf{b}}_k, \tilde{\mathbf{b}}_{k-1}, \dots, \tilde{\mathbf{b}}_{k-D}) \quad (49)$$

where

$$\lambda_k(\tilde{\mathbf{b}}_k, \tilde{\mathbf{b}}_{k-1}, \dots, \tilde{\mathbf{b}}_{k-D}) = \text{Re} \left\{ \tilde{\mathbf{b}}_k^H \left(2\tilde{\boldsymbol{\gamma}}_k - \tilde{\mathbf{H}}_{kk} \tilde{\mathbf{b}}_k - 2 \sum_{l=k-D}^{k-1} \tilde{\mathbf{H}}_{kl} \tilde{\mathbf{b}}_l \right) \right\}. \quad (50)$$

Define a state vector $\boldsymbol{\alpha}_k = [\tilde{\mathbf{b}}_{k-1}^T, \tilde{\mathbf{b}}_{k-2}^T, \dots, \tilde{\mathbf{b}}_{k-D}^T]^T$, and denote $\lambda_k(\tilde{\mathbf{b}}_k; \boldsymbol{\alpha}_k) = \lambda_k(\tilde{\mathbf{b}}_k, \tilde{\mathbf{b}}_{k-1}, \dots, \tilde{\mathbf{b}}_{k-D})$. Equation (49) can be reformulated as

$$\begin{aligned} & \max_{\tilde{\mathbf{b}}_k \in \mathcal{A}^P, k=1, \dots, K} \sum_{k=1}^K \lambda_k(\tilde{\mathbf{b}}_k; \boldsymbol{\alpha}_k) \\ & \text{s.t. } \boldsymbol{\alpha}_1 = \mathbf{0} \\ & \boldsymbol{\alpha}_{k+1} = \begin{bmatrix} \mathbf{0}_{P, (D-1)P} & \mathbf{0}_{P, P} \\ \mathbf{I}_{(D-1)P} & \mathbf{0}_{(D-1)P, P} \end{bmatrix} \boldsymbol{\alpha}_k + \begin{bmatrix} \mathbf{I}_P \\ \mathbf{0}_{(D-1)P, P} \end{bmatrix} \tilde{\mathbf{b}}_k \end{aligned} \quad (51)$$

where \mathbf{I}_n and $\mathbf{O}_{m,n}$ are an $n \times n$ identity matrix and an $m \times n$ zero matrix, respectively. Problem (51) is a *dynamic programming* (DP) problem and can be solved using a standard DP algorithm [38] reminiscent of the Viterbi algorithm. The complexity of solving DP is dependent on the cardinality of the state vector set. For Problem (51), $\mathcal{O}(K|\mathcal{A}|^{(D+1)P})$ operations are required to find the optimal solution.

We call the combined BALM and DP method *the BALM-DP detector*. Following the complexity development in Section IV-A, it can be verified that the time complexity per symbol of the BALM-DP detector is

$$\mathcal{O}\left(\frac{C}{Q}|\mathcal{A}|^{(D+1)P}\right) \quad (52)$$

again for the practical settings of $P \ll N$. If both D and P are small, then the time complexity per symbol of BALM-DP is of low order (even for very large K) and is substantially lower than that of the ML Viterbi algorithm [$\mathcal{O}((1/K)|\mathcal{A}|^{K(M+1)})$] for large number of users. As illustrated in the asynchronous CDMA simulations in Section IV-B, the values of P , which are sufficient for achieving good BER performance, are often quite small. Moreover, the subcarrier overlap factor D can be made small by employing a spectrally efficient pulse shape.

C. Simulation Results

The following MC communication simulation example illustrates the performance advantages of the BALM-DP detector.

Example 5: We consider a QPSK-modulated asynchronous MC multiple access system with 20 users. The baseband pulse shape is a time-overlapping half sine wave:

$$g(t) = \begin{cases} \sin\left(\frac{\pi t}{(1+\alpha)T_b}\right), & 0 \leq t \leq (1+\alpha)T_b \\ 0, & \text{otherwise} \end{cases} \quad (53)$$

where the factor α controls the percentage of time overlap of the transmitted pulses. In this example, we set $\alpha = 1/4$. Using the 99% energy bandwidth criterion, the spectral overlap factor was numerically found to be $D = 1$ [7]. The other simulation settings follow those of the random multipath channel simulation in Example 2. The BER performance of the BALM-DP detector and the other suboptimal detectors was plotted against the SNR in Fig. 6. Similar to the simulation results for the BALM-SDR detector in the last section, the BALM-DP detector yields BER performance substantially better than that of the other suboptimal detectors. \square

VI. CONCLUSION AND DISCUSSION

Practically realizable approximate ML detectors based on a block alternating likelihood maximization (BALM) technique have been developed for the asynchronous CDMA and multicarrier systems with frequency-selective multipath fading. The BALM technique proceeds by decomposing the ML detection problem into overlapping subproblems and then solving these subproblems in a cyclic fashion. The BALM approach is effective in reducing the computational difficulty of processing the large-scale MLD problem. For the CDMA case with either BPSK or QPSK constellations, we have used an accurate

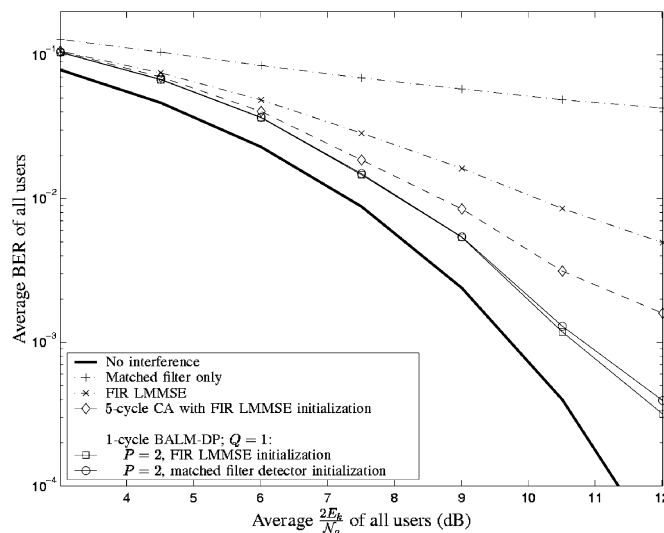


Fig. 6. BER performance of the various detectors in a 20-user asynchronous MC multiple access system. The filter order of the FIR LMMSE detector is 3.

semidefinite relaxation (SDR) approximation algorithm to efficiently solve the subproblems (in a suboptimal fashion). In the multicarrier case where the BALM constituent subproblems are structured, we have developed a dynamic programming (DP) method for solving them. Simulation studies have indicated that the BER performance of these BALM detectors closely approaches the optimal, when the window overlap of BALM (i.e., the difference between the parameters P and Q) is large enough.

Since BALM can achieve quasi-ML performance with a substantially reduced computational cost compared with complete MLD, an interesting future direction would be to rigorously analyze its performance and to investigate the conditions under which the performance of the BALM-SDR and BALM-DP detectors approaches that of the true ML detector. In addition, since the BALM technique is quite flexible, it can be implemented using other efficient suboptimal detection methods, such as the other relaxation methods [13], [14] or various search methods [21]–[27] to solve the constituent subproblems. Likewise, the standard SDR optimization algorithm employed here [36] could be replaced by computationally faster SDR algorithms [18], [39], [40], the solution accuracy of which might be reduced, compared with that of the standard algorithm. The tradeoffs between computational complexity and symbol error performance are different for such combinations, and they remain interesting avenues for future work.

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