FRACTIONAL SPATIAL REUSE PRECODING FOR MIMO DOWNLINK NETWORKS

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ABSTRACT

A linear precoding scheme is developed for unbounded MIMO downlink networks with quasi-static channels that have a hexagonal cell architecture. In the scheme developed herein, the equivalent channel model is structured to be decomposable, and the linear precoders at the base stations are designed to be decomposable as well. The proposed scheme is based the principles of fractional spatial reuse precoding. Spatial reuse precoding (SRP) is a precoding scheme that exploits the fact that interfering sources that employ the same structured precoder arrive in the same subspace, regardless of the particular channel matrices between the interfering sources and the receiver. The proposed scheme is fractional in the sense that each cell is partitioned and different precoders, with different power levels, are assigned in each partition. The proposed fractional SRP scheme enables the elimination of the dominant sources of interference without requiring cooperation between base stations.

Index Terms— interference alignment, spatial reuse, tier, cellular network, Kronecker product, decomposable, MIMO IBC.

1. INTRODUCTION

One of the fundamental aspects of wireless communication networks is interference management. The simplest approach to interference management is to avoid interference by transmitting signals orthogonally in time (TDMA) or frequency (FDMA). More sophisticated approaches that actively manage the interference offer the potential for higher data rates [1, 2]. One such approach is interference alignment (IA) [3–5], which has been shown to achieve more degrees of freedom (DoF) than those that can be achieved using interference avoidance. One typical assumption in the design of IA schemes is the presence of global and perfect channel state information (CSI). In small networks, the gains from IA may outweigh the cost of providing this CSI, but as the network size increases, the amount of CSI to be communicated increases rapidly, which can result in diminishing returns in terms of achievable rates. Another related issue is the need for a central processing unit for the linear precoder design, and the corresponding back-haul requirement. Thus, it would be desirable if each base station could design its own linear precoder without cooperation with other base stations in the network and using only local feedback.

Another typical assumption in the design of IA schemes is that of a fully connected network. In networks with a small number of cells, the receivers are often presumed to be close enough to all the transmitters for the interference to be deemed to be significant. In that scenario, examining the DoF of the network generates considerable insight. However, for larger networks, at moderate SNRs we can often neglect the power of interference from distant transmitters, and practical precoding schemes (and CSI feedback schemes) for such SNRs ought to take advantage of this partial connectivity; see also [6, 7].

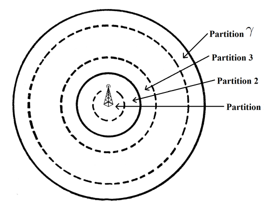

Fig. 1. Partitioning of a cell

With those perspectives in mind, the goal of this paper is to develop a linear precoding scheme for large unbounded networks that provides improved performance over existing schemes. We seek to design this scheme without requiring base stations cooperation and using only local feedback. To achieve these goals, we develop a class of linear precoders that we call fractional spatial reuse precoders. Spatial Reuse Precoding (SRP) [8] describes a network precoding scheme that can be designed so that the signals from the dominant interfering sources at each receiver align in a reduced dimensional subspace. This enables the receiver to eliminate the interference using a simple projection operation. This IA is designed to be achieved regardless of the exact values of the channel matrices between the interfering sources and the user experiencing that interference.

The proposed scheme is fractional in the sense that each cell is partitioned as illustrated in Figure 1, and different precoders are used in each partition. This is analogous to the notion of fractional frequency reuse [9, 10]. The partitioning of the cells enables each base station to assign higher power levels to the cell edge users, and also enables dense reuse patterns of the precoders used in the central partitions of each cell. The proposed fractional SRP scheme is designed to eliminate the dominant sources of interference so that rates higher than those of some existing schemes can be achieved, and is designed to do so using only local feedback.

2. SYSTEM MODEL

The G-cell MIMO interference broadcast channel (IBC) consists of G transmitters or base stations (BSs), each of which has M transmit antennas and communicates to K users, where each user has N receive antennas. The kth user in the ith cell, user (i, k), receives one data stream, and the received signal at that user can be modelled as $\tilde{\mathbf{y}}^{i,k} = \sum_{j=1}^{G} \tilde{\mathbf{H}}_j^{i,k} \tilde{\mathbf{x}}_j + \tilde{\mathbf{n}}^{i,k}$, where $\tilde{\mathbf{y}}^{i,k} \in \mathbb{C}^N$, $\tilde{\mathbf{H}}_j^{i,k} \in \mathbb{C}^{N \times M}$ is the channel matrix between BS j and user (i, k), $\tilde{\mathbf{x}}_j$ is the transmitted signal from BS j and is subject to the average power constraint $\mathrm{E}\big[\|\tilde{\mathbf{x}}_j\|^2\big] \leq P$, and $\tilde{\mathbf{n}}^{i,k}$ represents the additive noise. The channel matrices $\tilde{\mathbf{H}}_j^{i,k}$ are assumed to be full rank, but are otherwise unre-

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stricted. The signaling schemes that we consider are based on blocks of T_c channel uses, over which the channels are assumed to be constant. Defining $\mathbf{y}^{i,k} = \left[\tilde{\mathbf{y}}^{i,k}[1]^T, \tilde{\mathbf{y}}^{i,k}[2]^T, \ldots, \tilde{\mathbf{y}}^{i,k}[T_c]^T\right]^T$ as the concatenation of signals received over a block, and defining $\mathbf{x}_j$ and $\mathbf{n}^{i,k}$ analogously, the received signal over a block can be written as

$$\mathbf{y}^{i,k} = \sum_j \mathbf{H}_j^{i,k}\mathbf{x}_j + \mathbf{n}^{i,k}, \tag{1}$$

where $\mathbf{H}_j^{i,k}$ is a block diagonal matrix, with diagonal blocks $\tilde{\mathbf{H}}_j^{i,k}$, which can be expressed as:

$$\mathbf{H}_j^{i,k} = \mathbf{I}_{T_c} \otimes \tilde{\mathbf{H}}_j^{i,k}, \tag{2}$$

where $\otimes$ denotes the Kronecker product and $\mathbf{I}_{T_c}$ is the identity matrix of size T_c.

In this paper, we consider linear precoding schemes in which the signal transmitted from BS j takes the form

$$\mathbf{x}_j = \mathbf{F}_j\mathbf{s}_j = \sum_k \mathbf{f}_j^k s_j^k, \tag{3}$$

where $\mathbf{f}_j^k$ is the transmit beamformer for user (j,k), and s_j^k is the data symbol for that user. Further, we assume that each cell is divided into a number of partitions γ as shown in Figure 1. We will take a factorized approach to the design of the beamformers $\mathbf{f}_j^k s_j^k$ that depends on the cell index j, the user index k, and the index of the partition in cell j in which user k lies, which we will denote by $\ell(k)$. The beamformers take the form

$$\mathbf{f}_j^k = \sqrt{p_{\ell(k)}}\,\mathbf{\Phi}_j^{\ell(k)}\mathbf{v}_{j,\ell(k)}^k, \tag{4}$$

where $\mathbf{v}_{j,\ell(k)}^k$ is scaled so that $\mathbf{\Phi}_j^{\ell(k)}\mathbf{v}_{j,\ell(k)}^k$ has unit norm. In this factorized form, $p_{\ell(k)}$ denotes the power allocated to the users in partition $\ell(k)$, $\mathbf{\Phi}_j^{\ell(k)}$ is a matrix designed so that receivers in partition $\ell(k)$ can eliminate the dominant sources of interference due to transmissions to users in other cells and due to transmission to users in the same cell, but in other partitions, and $\mathbf{v}_{j,\ell(k)}^k$ is designed to eliminate the intra-partition interference within partition $\ell(k)$. To model the input to the decoder, we let $\mathbf{w}^{i,k} \in \mathbb{C}^N$ is the unit norm receive beamformer used at user (i,k), we let $\mathcal{P}_{i,\ell}$ denote the set of indices, k, of users in partition ℓ of cell i. That signal can be written as

$$\begin{aligned}\hat{\mathbf{y}}^{i,k} &= \mathbf{w}^{i,k\dagger}\mathbf{H}_i^{i,k}\mathbf{\Phi}_i^{\ell(k)}\mathbf{v}_{i,\ell(k)}^k s_i^k \\ &+ \mathbf{w}^{i,k\dagger}\mathbf{H}_i^{i,k}\mathbf{\Phi}_i^{\ell(k)}\textstyle\sum_{\mu\in\mathcal{P}_{i,\ell(k)},\mu\neq k}\mathbf{v}_{i,\ell(k)}^\mu s_i^\mu \\ &+ \mathbf{w}^{i,k\dagger}\mathbf{H}_i^{i,k}\textstyle\sum_{m\neq\ell(k)}\mathbf{\Phi}_i^m\sum_{\mu\in\mathcal{P}_{i,m}}\mathbf{v}_{i,m}^\mu s_i^\mu \\ &+ \mathbf{w}^{i,k\dagger}\textstyle\sum_{j\neq i}\mathbf{H}_j^{i,k}\sum_\mu \mathbf{f}_j^\mu s_j^\mu + \tilde{\mathbf{n}}^{i,k},\end{aligned} \tag{5}$$

Here, the first term represents the desired signal, the second term represents the interference term due to transmissions to users in the same partition, the third term represents the interference term due to transmissions to users in the same cell, but different partitions, the fourth term represents the interference term due to transmissions to users in other cells, and $\tilde{\mathbf{n}}^{i,k} = \mathbf{w}^{i,k\dagger}\mathbf{n}^{i,k}$ is the effective noise.

One of the goals of out approach to the design of $\mathbf{f}_{j,\ell(k)}^k$ is to enable the receivers to eliminate the inter-cell (and inter-partition) interference, without the need for inter-cell CSI at the BSs. To enable that, the design is performed sequentially. First, each BS constructs the matrices $\mathbf{\Phi}_j^\ell$ so that the dominant sources of inter-partition interference and inter-cell can be eliminated by the receivers. The

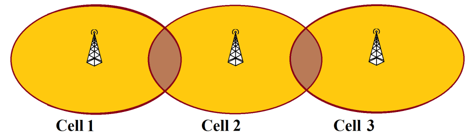

Fig. 2. Cell-arrangement in a linear model

construction of these matrices without CSI is the key contribution of this paper. Next, each user designs its linear receive beamformer $\mathbf{w}^{i,k}$ to eliminate the inter-partition interference and inter-cell interference. (Variations on the choice of $\mathbf{w}^{i,k}$ are discussed in Section 3.4.) With $\mathbf{\Phi}_j^\ell$ and $\mathbf{w}^{i,k}$ designed in this way, the operation of each partition resembles that of an isolated single-cell downlink with effective channels

$$\mathbf{H}_{\text{eff}}^{i,k} = \mathbf{w}^{i,k\dagger}\mathbf{H}_i^{i,k}\mathbf{\Phi}_i^{\ell(k)}. \tag{6}$$

Therefore, each receiver feeds back $\mathbf{H}_{\text{eff}}^{i,k}$ to its serving BS and that BS designs the transmit beamforming vectors $\mathbf{v}_{i,\ell(k)}^k$ as if it were serving a single-cell downlink; e.g., zero-forcing beamforming [11], a quality of service design [12, 13] or one of many other choices.

To develop the proposed SRP scheme, we will first consider an isolated 3-cell network with linear arrangement of cells. We will consider a linear IA scheme for this network that does not require BS cooperation, and involves only a modest number of channel extensions. Then we present the application of the underlying concept of fractional spatial reuse precoding in an unbounded network with hexagonal arrangement of cells.

3. SPATIAL REUSE PRECODING

In the proposed signalling schemes, the matrices $\mathbf{\Phi}_j^\ell$ are chosen to be the Kronecker product of two matrices. That is,

$$\mathbf{\Phi}_j^\ell = \mathbf{T}_j^{\ell,1} \otimes \mathbf{T}_j^{\ell,2}, \tag{7}$$

where the matrices $\mathbf{T}_j^{\ell,1} \in \mathbb{C}^{T_c\times\beta}$ and $\mathbf{T}_j^{\ell,2} \in \mathbb{C}^{M\times M}$ are randomly and independently generated from a continuous distribution and $T_c > \beta$. By construction, the generic rank of $\mathbf{\Phi}_j^\ell$ is βM. In the conceptual development in this section we will use abstract partially-connected models for the networks. In those models weak connections are modelled as being absent.

3.1. A Motivating Example

We begin with a simple example based on the 3-cell linear downlink network illustrated in Figure 2. For the ease of exposition, let us consider a case in which each cell consists only of one partition, i.e., $\gamma = 1$. For that case, we can consider beamformers of the form $\mathbf{f}_j^k = \sqrt{p}\mathbf{\Phi}_j\mathbf{v}_j^k$. If the power transmitted by each BS is controlled so that users that are close to their BSs do not suffer from significant interference, then the abstract partially-connected network model is rather sparse, with only cell-edge users suffering significant interference from one interfering source (users in the brown area in Figure 2). One way to improve the performance of these users is to structure the transmissions in such a way that each user can cancel one source of interference at its side. As one example, the precoders designed for the isolated 2-cell case in [14, 15] provide this structure.

Now, let us consider the case in which each BS increases its power (in order to better service its assigned users) to the point that each user receives non-negligible interference from all its neighboring cells. In this case, users in cell 1 and cell 3 will suffer significant interference from BS 2, and those in cell 2 will suffer significant interference from both BS 1 and BS 3. As such, it appears that although applying signaling techniques from the isolated 2-cell model may be effective strategies for users in cell 1 and 3, they might not be effective for those in cell 2. Indeed, for users in cell 2 we observe that if each BS uses beamformers of the form $\mathbf{f}_j^k = \sqrt{p}\mathbf{\Phi}_j\mathbf{v}_j^k$ with $\mathbf{\Phi}_j$ constructed according to (7) with $T_\text{c} = \beta + 1$, and the vectors $\{\mathbf{v}_j^k\}_k$ being linearly independent, then the subspace spanned by the interference at user $(2, k)$ is the column span of

$$\mathbf{Z}^{2,k} = \begin{bmatrix}\mathbf{H}_1^{2,k}\mathbf{\Phi}_1 & \mathbf{H}_3^{2,k}\mathbf{\Phi}_3\end{bmatrix} \tag{8a}$$

$$= \begin{bmatrix}\mathbf{T}_1^1 \otimes \tilde{\mathbf{H}}_1^{2,k}\mathbf{T}_1^2 & \mathbf{T}_3^1 \otimes \tilde{\mathbf{H}}_3^{2,k}\mathbf{T}_3^2\end{bmatrix}. \tag{8b}$$

Unfortunately, the interference matrix $\mathbf{Z}^{2,k}$ is generically full rank and the receiver cannot design a receive beamformer $\mathbf{w}^{2,k}$ that can eliminate the interfering signals. However, if BSs 1 and 3 use the same precoder, $\mathbf{\Phi}_\text{odd}$, and if $\mathbf{\Phi}_\text{odd} = \mathbf{T}_\text{o}^1 \otimes \mathbf{T}_\text{o}^2$ is designed according to (7), then $\mathbf{Z}^{2,k}$ takes the form

$$\mathbf{Z}^{2,k} = \begin{bmatrix}\mathbf{T}_\text{o}^1 \otimes \hat{\mathbf{H}}_1^{2,k} & \mathbf{T}_\text{o}^1 \otimes \hat{\mathbf{H}}_3^{2,k}\end{bmatrix}, \tag{9}$$

where $\hat{\mathbf{H}}_1^{2,k} = \tilde{\mathbf{H}}_1^{2,k}\mathbf{T}_\text{o}^2$ and $\hat{\mathbf{H}}_3^{2,k} = \tilde{\mathbf{H}}_3^{2,k}\mathbf{T}_\text{o}^2$. Using the same precoder $\mathbf{\Phi}_\text{odd}$ at cell 1 and 3 aligns the subspaces spanned by the signals arriving from BSs 1 and 3, regardless of the particular channel matrices $\tilde{\mathbf{H}}_1^{2,k}$ and $\tilde{\mathbf{H}}_3^{2,k}$. To show that this results in $\mathbf{Z}^{i,k}$ being rank deficient, we begin with the fact that $\mathbf{T}_\text{o}^1 \in \mathbb{C}^{T_\text{c}\times\beta}$. If $T_\text{c} = \beta + 1$, then $\mathbf{T}_\text{o}^1$ is rank deficient with null space of dimension 1, and the receiver can design a vector $\mathbf{q}$ to lie in its null space; i.e., $\mathbf{q}^\dagger\mathbf{T}_\text{o}^1 = \mathbf{0}$. Now, the receiver designs its receive beamformer $\mathbf{w}^{2,k}$ as $\mathbf{w}^{2,k} = \mathbf{q} \otimes \mathbf{u}$, where $\mathbf{u} \in \mathbb{C}^M$ is a degree of design freedom at the receiver, and

$$\begin{aligned}\mathbf{w}^{2,k\dagger}\mathbf{Z}^{2,k} &= \mathbf{w}^{2,k\dagger}\begin{bmatrix}\mathbf{T}_\text{o}^1 \otimes \hat{\mathbf{H}}_1^{2,k} & \mathbf{T}_\text{o}^1 \otimes \hat{\mathbf{H}}_3^{2,k}\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{q}^\dagger\mathbf{T}_\text{o}^1 \otimes \mathbf{u}^\dagger\hat{\mathbf{H}}_1^{2,k} & \mathbf{q}^\dagger\mathbf{T}_\text{o}^1 \otimes \mathbf{u}^\dagger\hat{\mathbf{H}}_3^{2,k}\end{bmatrix} = \mathbf{0}. \end{aligned} \tag{10}$$

Thus, $\mathbf{Z}^{2,k}$ is rank deficient, with null space of dimension at least one. Accordingly any user in cell 2 can eliminate the inter-cell interference at its side, without any cooperation between the BSs. Users in cell 1 and 3 can do the same. The last step in the linear precoder design is the design of the transmit beamformers $\mathbf{v}_{i,\ell(k)}^k$. Each receiver feeds back $\mathbf{H}_\text{eff}^{i,k} = \mathbf{w}^{i,k\dagger}\mathbf{H}_i^{i,k}\mathbf{\Phi}_i$ to its serving BS and that BS designs the matrices $\mathbf{v}_{i,\ell(k)}^k$ to eliminate the intra-cell interference. This is the only component of the design that uses feedback, and that feedback is only local.

3.2. Definitons

The above example conjures a notion of "Spatial Reuse Precoding" (SRP), a precoding scheme designed to exploit the fact that the signals transmitted by interfering sources that use the same precoder align together at the unintended receivers [8]. The system is designed so that that alignment is in a reduced dimensional subspace, and hence inter-cell interference can be removed without requiring the knowledge of the inter-cell channels at the interfering BSs. In a complementary way, we can define the notion of "Spatial Reuse Factor" (SRF) as the rate at which the same precoder is used in the network. We will show below that the structured precoding scheme presented in (7) can be modified in such a way to allow fractional SRP in unbounded downlink networks that are hexagonal in structure.

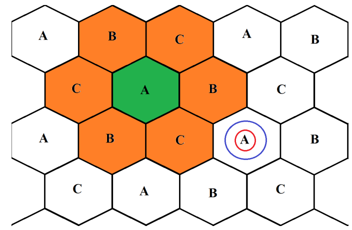

Fig. 3. A spatial reuse precoding scheme for a hexagonal cell arrangement

3.3. Unbounded hexagonal network

This section proposes the main contribution of the paper: fractional SRP for the hexagonal arrangement of cells illustrated in Figure 3, in which, each cell is divided into 3 partitions. As is implicit in our earlier discussion, the key aspect of the proposed approach is the construction of the projection matrices $\mathbf{\Phi}_j^{\ell(k)}$. For the hexagonal network in Figure 3 we construct nine such matrices, one for each partition in three cells denoted **A**, **B**, and **C**. These precoders are then re-used in the network according to the pattern in Figure 3, which has an SRF of 3. We begin with the construction of the precoders for the outer two partitions,

$$\mathbf{\Phi}_j^3 = \mathbf{T}_j^{3,1} \otimes \mathbf{T}_j^{3,2}, \tag{11}$$

$$\mathbf{\Phi}_\mathbf{A}^2 = \mathbf{T}_\text{mid} \otimes \mathbf{T}_\text{A}^2,\ \mathbf{\Phi}_\mathbf{B}^2 = \mathbf{T}_\text{mid} \otimes \mathbf{T}_\text{B}^2,\ \mathbf{\Phi}_\mathbf{C}^2 = \mathbf{T}_\text{mid} \otimes \mathbf{T}_\text{C}^2, \tag{12}$$

where $\mathbf{T}_j^{3,m} \in \mathbb{C}^{T_\text{c}\times\beta}$, $\mathbf{T}_\text{mid} \in \mathbb{C}^{T_\text{c}\times\beta}$ and $\mathbf{T}_\mathbf{A,B,C}^2 \in \mathbb{C}^{M\times M}$ are randomly and independently generated matrices from a continuous distribution and $T_\text{c} = 3\beta + 1$. Finally, we construct the precoders for the first partition as follows

$$\mathbf{\Phi}_\mathbf{A}^1 = \mathbf{T}_\text{A}^1 \otimes \mathbf{T}_\text{center}^2, \mathbf{\Phi}_\mathbf{B}^1 = \mathbf{T}_\text{B}^1 \otimes \mathbf{T}_\text{center}^2, \mathbf{\Phi}_\mathbf{C}^1 = \mathbf{T}_\text{C}^1 \otimes \mathbf{T}_\text{center}^2, \tag{13}$$

where $\mathbf{T}_\text{center}^2 \in \mathbb{C}^{M\times M}$ is a randomly generated matrix, while $\mathbf{T}_\mathbf{A,B,C}^1 \in \mathbb{C}^{T_\text{c}\times\beta}$ are designed according to

$$\mathbf{T}_\mathbf{A}^1 = \mathbf{T}_\mathbf{B}^{3,1} + \mathbf{T}_\mathbf{C}^{3,1}, \mathbf{T}_\mathbf{B}^1 = \mathbf{T}_\mathbf{A}^{3,1} + \mathbf{T}_\mathbf{C}^{3,1}, \mathbf{T}_\mathbf{C}^1 = \mathbf{T}_\mathbf{A}^{3,1} + \mathbf{T}_\mathbf{B}^{3,1}. \tag{14}$$

Here we claim that the above 3-partition Kronecker-structured SRP scheme enables each user to eliminate the dominant sources of interference. Due to space limitations, we will focus on the interference experienced by a user at the cell edge in cell **A**, i.e., at the edge of the third partition in cell **A**. The proposed precoding scheme helps that user eliminate the interfering signals due to transmission to the other partitions in cell **A** and due to transmission to the second and third partitions in any cell indexed as **B** and **C**. To verify that claim, we examine the interference matrix for that user,

$$\begin{aligned}\mathbf{Z}^{\mathbf{A},k} &= \begin{bmatrix}\mathbf{H}_\mathbf{A}^{\mathbf{A},k}\mathbf{\Phi}_\mathbf{A}^1 & \mathbf{H}_\mathbf{A}^{\mathbf{A},k}\mathbf{\Phi}_\mathbf{A}^2 & \mathbf{H}_\mathbf{B}^{\mathbf{A},k}\mathbf{\Phi}_\mathbf{B}^2 & \mathbf{H}_\mathbf{B}^{\mathbf{A},k}\mathbf{\Phi}_\mathbf{B}^3 & \mathbf{H}_\mathbf{C}^{\mathbf{A},k}\mathbf{\Phi}_\mathbf{C}^2 & \mathbf{H}_\mathbf{C}^{\mathbf{A},k}\mathbf{\Phi}_\mathbf{C}^3\end{bmatrix}\\ &= \big[\mathbf{T}_\mathbf{A}^1 \otimes \tilde{\mathbf{H}}_\mathbf{A}^{\mathbf{A},k}\mathbf{T}_\text{center}^2 \ \ \mathbf{T}_\text{mid} \otimes \tilde{\mathbf{H}}_\mathbf{A}^{\mathbf{A},k}\mathbf{T}_\mathbf{A}^2 \ \ \mathbf{T}_\text{mid} \otimes \tilde{\mathbf{H}}_\mathbf{B}^{\mathbf{A},k}\mathbf{T}_\mathbf{B}^2\\ &\qquad \mathbf{T}_\mathbf{B}^{3,1} \otimes \tilde{\mathbf{H}}_\mathbf{B}^{\mathbf{A},k}\mathbf{T}_\mathbf{B}^{3,2} \ \ \mathbf{T}_\text{mid} \otimes \tilde{\mathbf{H}}_\mathbf{C}^{\mathbf{A},k}\mathbf{T}_\mathbf{C}^2 \ \ \mathbf{T}_\mathbf{C}^{3,1} \otimes \tilde{\mathbf{H}}_\mathbf{C}^{\mathbf{A},k}\mathbf{T}_\mathbf{C}^{3,2}\big].\end{aligned} \tag{15}$$

Carefully examining the structure of $\mathbf{Z}^{\mathbf{A},k}$, we find that we can design a vector $\mathbf{q}$ in the null space of $\mathbf{Q} = \begin{bmatrix}\mathbf{T}_\text{mid} & \mathbf{T}_\mathbf{B}^{3,1} & \mathbf{T}_\mathbf{C}^{3,1}\end{bmatrix} \in \mathbb{C}^{(3\beta+1)\times 3\beta}$, i.e., $\mathbf{q}^\dagger\mathbf{Q} = \mathbf{0}$. By construction, $\mathbf{q}^\dagger\mathbf{T}_\mathbf{A}^1 = \mathbf{0}$, as well. Therefore, the receiver can design a receive beamformer $\mathbf{w}^{\mathbf{A},k} = \mathbf{q} \otimes \mathbf{u}$ that lies in the null space of $\mathbf{Z}^{\mathbf{A},k}$, where the receiver is free to

Table 1. Break points in piece-wise linear path loss model

Distance	α_{loss}
$\leq$ 200m	2
500m	3
2km	4
10km	5

choose $\mathbf{u} \in \mathbb{C}^M$. Although the interfering signals due to transmissions to users in the first partitions in cells **B** and **C**, and the interference due to transmissions to third partitions in any other cell indexed **A**, cannot be eliminated, the power assigned to the first partition is lower than the power assigned for the other partitions, because users in the first partition are close to the base station and do not suffer significant interference except at high SNRs. Furthermore, the architecture of the SRP scheme assigns the same precoder to cells that are one cell apart from each other. This reduces the interfering signal power experienced by a cell edge user in cell **A**, due to transmissions to the users in the third partition of other cells indexed by **A**.

3.4. Variations on the theme

The insight that drove the development of the proposed scheme was based on constructing the projection matrices $\mathbf{\Phi}_j^{\ell(k)}$ so that the receivers can eliminate inter-cell/partition interference. However, the receiver is not compelled to completely eliminate that interference, and may choose to employ an alternate interference mitigation technique, such as the maximum SINR receive beamformer, e.g., [16],

$$\mathbf{w}^{i,k} \propto (\mathbf{Q}^{i,k})^{-1}\mathbf{H}_i^{i,k}\mathbf{\Phi}_i^{\ell(k)}\mathbf{v}_{i,\ell(k)}^k. \quad (16)$$

In the proposed system, the intra-partition beamforming vectors $\mathbf{v}_j^\ell$ are designed after the effective channels are fed back to the assigned BSs and hence are not available when $\mathbf{w}^{i,k}$ is designed. Although an iterative design scheme along the lines of [16] can be envisioned, our simulations suggest that a substantial performance gains can be obtained by determining $\mathbf{w}^{i,k}$ in (16) as if all the intra-partition beamforming matrices in the network are identity matrices.

4. SIMULATION RESULTS

We evaluate the performance of the proposed scheme in the case of a network with the hexagonal arrangement of cells shown in Figure 3 and a cell radius of 500m. The effect of the distance between any transmitter and any receiver is captured by a piece-wise linear path loss model [17, 18], where the path loss exponent α_{loss} varies with distance according to linear interpolation between points in Table 1. The BSs and terminals each have four antennas.

We will compare the performance of the proposed scheme (prop), against schemes based on designs for isolated single cell [2], 2-cell [14] and 3-cell [19] networks. In all of the considered networks, there is no cooperation between BSs and only local feedback is employed. In each cell, $K = 12$ users are served, and the inter-partition interference is eliminated by having the receivers feed back their effective channels, and then choosing $\mathbf{v}_{i,\ell(k)}^k$ to be the appropriate column of the zero-forcing beamforming matrix [11]. Furthermore, and for all schemes, we assign different power levels for each partition in the cell. In particular, the powers for the first and second partitions are 20dB and 13dB below that assigned for the third partition, respectively.

In the proposed fractional SRP scheme, the matrices $\mathbf{\Phi}_j^\ell$ are designed with $\beta = 1$ and the SRP pattern in Figure 3. This results in a block length of $T_{\text{c}} = 4$. Each receiver employs the Max-SINR receive beamformer in (16).

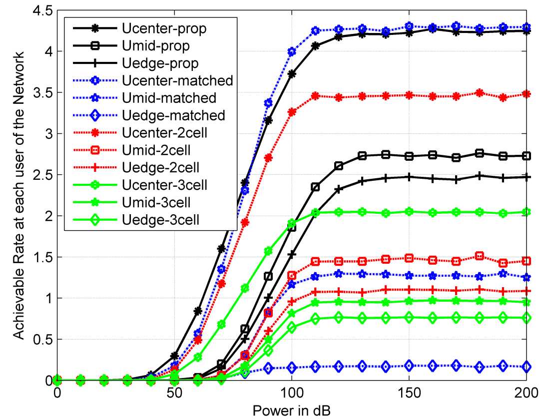

Fig. 4. Achievable rates of various users in a hexagonal cellular network under four different signalling schemes.

The scheme based on insight from the isolated single-cell case (1-cell) ignores (inter-cell) interference. The matrices $\mathbf{\Phi}_j^\ell$ are random matrices of size $MT_{\text{c,1-cell}} \times K$, and hence a block length $T_{\text{c,1-cell}} = 3$ is chosen to enable $K = 12$ users to be served in each cell. Here, there is no spatial reuse; each BS (randomly) chooses its $\mathbf{\Phi}$ matrix individually. Since the 1-cell design ignores the inter-cell interference, the receive beamformer $\mathbf{w}^{i,k}$ is chosen to be the matched filter, i.e., $\mathbf{w}^{i,k}$ is aligned with the left singular vector of $\mathbf{H}_i^{i,k}\mathbf{\Phi}_i^{\ell(k)}$ that corresponds to the largest singular value.

In the schemes based on the isolated 2-cell case (2-cell) and 3-cell case (3-cell), the matrices $\mathbf{\Phi}_j^\ell$ at each BS are chosen using the subspace IA technique in [14] and [19] respectively. For $K = 12$ users per cell, this results in block lengths $T_{\text{c,2-cell}} = 4$ and $T_{\text{c,3-cell}} = 7$, respectively. In an isolated 2-cell network, this choice enables each receiver to project out the interference that it receives from the other BS. To extend that notion to the case of an unbounded network, each receiver chooses its receive beamformer $\mathbf{w}^{i,k}$ to project out the dominant interference source, i.e., $\mathbf{w}^{i,k} \in \mathcal{N}(\mathbf{H}_{j^*}^{i,k}\mathbf{\Phi}_{j^*}^{\ell^*})$, where j^* and ℓ^* index of the dominant interfering signal, and $\mathcal{N}(\cdot)$ denotes the null-space. Similar concepts apply to the 3-cell case, where each receiver projects out the two dominant sources of interference.

In Figure 4, we compare the achievable rates of three users in the network under the four schemes described above. The first user (Ucenter) is located close to its serving BS, the second user (Umid) is located half way to the cell edge, and the third user (Uedge) is located at the cell edge (at the center of a face of a hexagon). Figure 4 shows that at high SNRs the proposed fractional SRP scheme has a significant impact on the rates that can be achieved by users, especially, the cell edge users. In particular, for a cell edge user, it provides 122% and 220% increases over the achievable rates of the schemes based on the 2-cell and 3-cell schemes, respectively, and more than a 10 fold increase over the 1-cell scheme (which does not manage interference). Moreover, the performance of the cell-edge user in the proposed scheme is better than the performance of a user in the middle of the cell using the other schemes.

5. REFERENCES

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