LIST-BASED SOFT DEMODULATION OF MIMO QPSK VIA SEMIDEFINITE RELAXATION

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ABSTRACT

We propose a list-based soft demodulator for MIMO systems that generates the soft information by solving a single semidefinite program (SDP). The computational cost of solving this SDP is guaranteed to be a (low-order) polynomial in the problem size, whereas existing list-based demodulators that employ sphere decoding principles do not have such a guarantee. The proposed demodulator also has a computational advantage over an existing semidefinite-relaxation-based soft demodulator that requires the solution of several SDPs. Our simulation results suggest that these computational advantages are achieved without incurring a significant degradation in performance.

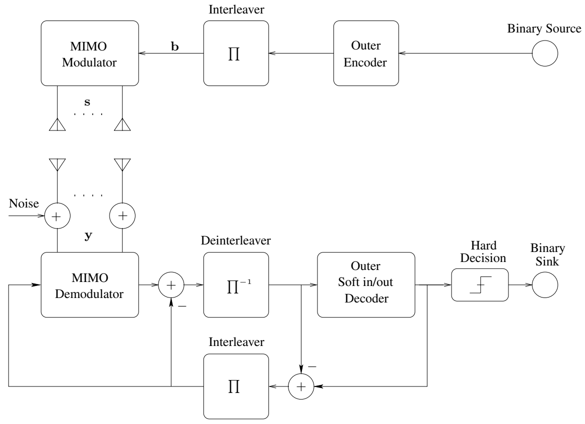

Fig. 1. MIMO BICM-IDD transceiver.

1. INTRODUCTION

The provision of multiple antennas at both the transmitter and receiver of a wireless communication system offers the potential for reliable communication at data rates substantially higher than those of single antenna systems [1]. One of the core challenges in the design of such multiple-input multiple-output (MIMO) systems is to obtain good performance at high data rates without incurring unreasonable computational cost. A standard architecture for doing so consists of an outer binary code and a MIMO modulator at the transmitter, and an iterative "soft" demodulation and decoding (IDD) scheme at the receiver; e.g., [2], see also Fig. 1. The IDD scheme iteratively passes estimates of the likelihood of each bit in the message between a "soft" MIMO demodulator and a soft-input soft-output decoder for the outer binary code. Although the IDD scheme has many desirable features, as the number of bits transmitted per channel use increases the complexity of soft demodulation increases exponentially, and hence there has been considerable interest in the development of reduced-complexity soft MIMO demodulation schemes; e.g., [2–9].

There are two basic approaches to reduced-complexity (joint) soft demodulation of the vector of transmitted bits. The first involves the use of the so-called max-log approximation to approximate the log-likelihood ratio of each bit by the difference between the optimal values of a pair of "hard" demodulation problems. Existing "hard" demodulation algorithms, such as sphere decoding [10] and semidefinite relaxation [11], can then be applied; c.f. [7, 8] and [9], respectively. The second approach to reduced-complexity soft demodulation is based on the idea of list decoding, in which the likelihood of each bit is approximated by partial marginalization over a list of dominant bit-vectors, rather than complete marginalization over the list of all possible vectors. Most of the existing schemes for constructing the list of dominant bit-vectors are based on the application of the principles of "hard" sphere decoding [2, 5], and related tree search algorithms [3, 6].

For moderate problem sizes and at moderate-to-high SNRs, the average computational cost of the sphere decoding and tree search algorithms used in list demodulation is quite reasonable [12]. However, both the average and worst-case complexities remain exponential in the problem size [13], and the "tail" of the computational complexity distribution can be quite significant at low SNRs or for large problem sizes; e.g., [14]. For hard demodulation problems, an alternative to sphere decoding and other tree search methods is to employ semidefinite relaxation (SDR); e.g., [11]. SDR generates a hard decision that is guaranteed to be "close" to the optimal hard decision in an appropriate sense, yet its worst-case computational cost is only a (low-order) polynomial of the problem size, namely $O(n^{3.5})$. SDR techniques have been successfully applied to the first approach to soft demodulation [9], but to the best of our knowledge they have not been applied to list-based techniques. In this paper we will exploit the randomization procedure inherent in SDR to construct an effective list for the list-based technique. Since only one semidefinite program needs to be solved, the (polynomial) worst-case complexity of our approach is an order lower than that of the SDR-based hard demodulation approach in [9], and our simulation results show that the performance of our approach is quite close to that of [9].

For simplicity, in this paper we will focus attention on schemes based on V-BLAST transmission [15] of QPSK symbols, but extensions to general linear dispersion coded schemes follow directly from [16], and extensions to 16-QAM and higher-order constellations follow directly from recent extensions [17, 18] of the SDR approach to hard demodulation. Although we will focus on MIMO systems, iterative multiuser detection and decoding schemes [19] for narrowband (synchronous) CDMA systems can also take advantage of the proposed demodulator.

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2. SYSTEM MODEL

We consider a narrow-band MIMO transmission scheme with N_t transmit antennas and $N_r \geq N_t$ receive antennas. Using a V-BLAST transmission scheme [15] with QPSK symbols, the modulator maps blocks $\mathbf{b}$ of $2N_t$ bits from the encoded and interleaved bit stream to a symbol vector $\mathbf{s}$, each element of which is transmitted from a different antenna. The received signal $\mathbf{y}$ can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v} = \mathbf{H}\mathcal{M}(\mathbf{b}) + \mathbf{v}, \tag{1}$$

where $\mathbf{H}$ is the $N_r \times N_t$ matrix of channel gains and is assumed to be known at the receiver, $\mathbf{v}$ is a vector of additive white circular complex Gaussian noise samples with variance σ^2 per real dimension, and the $\mathcal{M}(\cdot)$ denotes the mapping of $\mathbf{b}$ to $\mathbf{s}$.

A pragmatic choice for a coded MIMO transmission scheme is the MIMO bit interleaved coded modulation scheme (BICM, e.g., [20]) with a "soft" iterative demodulation and decoding (IDD) shown in Fig. 1, and we will develop our soft demodulator within this framework.[1] Since the outer code is binary, the soft output from the demodulator can be in the form of the log-likelihood ratio

$$\begin{aligned} L_i \triangleq L(b_i|\mathbf{y},\mathbf{H}) &= \log \frac{p(b_i = +1|\mathbf{y},\mathbf{H})}{p(b_i = -1|\mathbf{y},\mathbf{H})} \\ &= \log \frac{\Sigma_{\mathcal{L}_{i,+1}} p(\mathbf{y}|\mathbf{b},\mathbf{H})p(\mathbf{b})}{\Sigma_{\mathcal{L}_{i,-1}} p(\mathbf{y}|\mathbf{b},\mathbf{H})p(\mathbf{b})}, \quad i = 1,\ldots,2N_t, \end{aligned} \tag{2}$$

where $\mathcal{L} = \{\mathbf{b} \in \{-1,+1\}^{2N_t}\}$ denotes the (complete) list of bit-vectors, $\mathcal{L}_{i,+1} = \{\mathbf{b} \in \mathcal{L} | b_i = +1\}$ and $\mathcal{L}_{i,-1}$ is defined analogously. (In this paper we will use the antipodal representation of the bits, b_i.) By capturing the *a priori* soft information on the elements of $\mathbf{b}$ in a vector $\boldsymbol{\lambda}$ whose elements are $\lambda_i = \log \frac{p(b_i=+1)}{p(b_i=-1)}$, the log-likelihood ratio can be rewritten as [2]

$$L_i = \log \frac{\Sigma_{\mathcal{L}_{i,+1}} \exp(-D(\mathbf{b})/(2\sigma^2))}{\Sigma_{\mathcal{L}_{i,-1}} \exp(-D(\mathbf{b})/(2\sigma^2))}, \tag{3}$$

where

$$D(\mathbf{b}) \triangleq \|\mathbf{y} - \mathbf{H}\mathcal{M}(\mathbf{b})\|_2^2 - \sigma^2 \boldsymbol{\lambda}^T \mathbf{b}. \tag{4}$$

It can be seen from (2) that as the number of antennas increases, the size of lists $\mathcal{L}_{i,\pm 1}$ increases exponentially, and hence the computational complexity of the demodulator increases exponentially. A common step in the development of reduced complexity soft demodulation is to keep only the dominant summand in each summation in (3). That is:

$$\begin{aligned} L_i &\simeq \log \frac{\max_{\mathbf{b}\in\mathcal{L}_{i,+1}} \exp(-D(\mathbf{b})/(2\sigma^2))}{\max_{\mathbf{b}\in\mathcal{L}_{i,-1}} \exp(-D(\mathbf{b})/(2\sigma^2))} \\ &= \frac{1}{2\sigma^2}\left(\min_{\mathbf{b}\in\mathcal{L}_{i,-1}} D(\mathbf{b}) - \min_{\mathbf{b}\in\mathcal{L}_{i,+1}} D(\mathbf{b})\right). \end{aligned} \tag{5}$$

This is often referred to as the max-log approximation of (3), [2]. Despite this approximation, the cost of computing (5) still grows exponentially in N_t because it requires the solution of two binary quadratic optimization problems. However, this approximation reveals two general classes of approximate soft demodulators:

1. Direct application of "hard" demodulation techniques: The right hand side of (5) consists of two "hard" demodulation problems of size $(2N_t - 1)$. While such problems remain difficult to solve in the general case, the class of sphere decoding algorithms has desirable average complexity properties under a variety of practical conditions [12], and several authors have proposed the adoption of such schemes; e.g., [7, 8]. Unfortunately, the average computational cost of such approaches remains exponential in the problem size [13]. An alternative approach [9] is to approximate the solution to the optimization problems in (5) using the semidefinite relaxation technique [21, 22]; see also [11, 23]. While that approach is approximate, the quality of the approximation is guaranteed to be good and the (worst-case) computational cost is a (low-order) polynomial of the number of bits to be detected.
2. List decoding techniques: These techniques are based on the (efficient) identification of a sub-list, $\hat{\mathcal{L}}$, of $\mathcal{L}$ that contains bit-vectors with small values of $D(\mathbf{b})$. The expression in (5) is then approximated by replacing $\mathcal{L}_{i,\pm 1}$ by $\hat{\mathcal{L}}_{i,\pm 1}$ and solving the resulting optimization problems by enumeration.[2] The computational cost of this approach is dependent on the efficiency of the list generation, and the cardinality of $\hat{\mathcal{L}}$.

Most of the existing list generation techniques for the second class of methods are based on tree-search ideas reminiscent of sphere decoding; e.g., [3]. However, the computational costs of these methods grow rapidly with problem size, especially at low SNR. The purpose of this paper is to develop a list-based demodulator that has a (worst-case) computational cost that is a (low-order) polynomial of the number of bits to be detected. The key step in our development is to exploit the randomization step in the semidefinite relaxation approach for "hard" demodulation. Before we describe the proposed method, we will provide an overview of the SDR technique.

3. SDR FOR HARD DEMODULATION

For ease of exposition, we will use the following real-valued equivalent model for (1),

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\mathbf{b} + \tilde{\mathbf{v}}, \tag{6}$$

where $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{v}}$ are concatenations of the real and imaginary parts of $\mathbf{y}$ and $\mathbf{v}$ in (1), respectively. Given $\boldsymbol{\lambda}$, the maximum *a posteriori* probability bit-vector $\mathbf{b}$ for this channel can be found by solving the following optimization problem:

$$\min_{\mathbf{b}\in\{+1,-1\}^{2N_t}} D(\mathbf{b}) = \min_{\mathbf{b}\in\{+1,-1\}^{2N_t}} \|\tilde{\mathbf{y}} - \tilde{\mathbf{H}}\mathbf{b}\|_2^2 - \sigma^2 \boldsymbol{\lambda}^T \mathbf{b}. \tag{7}$$

Using the following definitions [9, 11]:

$$\tilde{\mathbf{b}} \triangleq \begin{bmatrix} \breve{\mathbf{b}} \\ c \end{bmatrix}, \ \breve{\mathbf{b}} \triangleq c\mathbf{b}, \ \mathbf{Q} = \begin{bmatrix} \tilde{\mathbf{H}}\tilde{\mathbf{H}}^T & \mathbf{a} \\ \mathbf{a}^T & 0 \end{bmatrix}, \ \mathbf{a} \triangleq -\tilde{\mathbf{H}}^T\tilde{\mathbf{y}} - 0.5\sigma^2\boldsymbol{\lambda}, \tag{8}$$

where $c \in \{+1,-1\}$, the problem in (7) can be stated as the following (NP-hard) binary quadratic programming (BQP) problem:

$$\min_{\mathbf{b}\in\{+1,-1\}^{2N_t}} D(\mathbf{b}) = \min_{\tilde{\mathbf{b}}\in\{+1,-1\}^{2N_t+1}} \tilde{\mathbf{b}}^T \mathbf{Q} \tilde{\mathbf{b}}. \tag{9}$$

The semidefinite relaxation approach to approximate the solution to this problem [21, 22] involves the solution of the following semidef-

[1] In CDMA systems, each user has a separate outer code and the receiver may implement joint decoding or separate decoding of each user.

[2] List approximations can also be applied directly to (3) by marginalizing over $\hat{\mathcal{L}}_{i,+1}$ and $\hat{\mathcal{L}}_{i,-1}$, respectively; e.g., [5].

inite program (SDP), which is a (matrix) relaxation of (9),

$$\min \quad \mathrm{Trace}(\mathbf{XQ}) \tag{10a}$$

$$\text{s.t.} \quad \mathbf{X} \succeq 0 \tag{10b}$$

$$\mathbf{X}_{ii} = 1, \quad i = 1, \ldots, 2N_t + 1. \tag{10c}$$

This problem is convex and can be solved in polynomial time using interior point methods [24]. (See [25] for some recent developments.) However, the solution of (10) is a matrix, from which one must extract an approximation of the solution to the original problem (9). If the solution of (10) is rank 1, then this extraction is straightforward and the optimum solution of (10) generates the optimum solution of (9). When this does not happen, an approximation to the solution of (9) can be generated by a randomization procedure [21,22]; see also [11]. This randomization procedure will form the core component of the proposed demodulator, so it is stated here explicitly:

1. Let $\mathbf{X}_o = \mathbf{V}^T\mathbf{V}$ denote a factorization of the solution to (10). Initialize $\gamma = \infty$, $\hat{\mathbf{x}}$ empty, and $m = 0$. Set the maximum number of randomizations, M, and define $\tilde{D}(\tilde{\mathbf{x}}) = \tilde{\mathbf{x}}^T\mathbf{Q}\tilde{\mathbf{x}}$.
2. Choose a random vector $\mathbf{u}$ from the uniform distribution on the unit sphere.
3. Construct $\tilde{\mathbf{x}} = \sigma(\mathbf{V}^T\mathbf{u})$, where $\sigma(\cdot)$ is the component-wise sign operator. Calculate $\tilde{D}(\tilde{\mathbf{x}})$ and increment m.
4. If $\tilde{D}(\tilde{\mathbf{x}}) \leq \gamma$, set $\gamma = \tilde{D}(\tilde{\mathbf{x}})$ and $\hat{\mathbf{x}} = \tilde{\mathbf{x}}$.
5. If $m < M$, return to 2.
6. An approximate solution to (7) is $\hat{\mathbf{b}} = \hat{x}_{2N_t+1}[\hat{x}_1, \ldots, \hat{x}_{2N_t}]^T$.

In addition to its (low-order) polynomial complexity, a key feature of the SDR approach is that even for the worst-case channel, the expected value of $\tilde{D}(\tilde{\mathbf{x}})$ over the randomizations is guaranteed to be within a (reasonably small) constant factor of the optimal value of (7), independent of the number of bits to be detected [22]. Furthermore, since each choice of $\mathbf{u}$ in step 2 is made independently, the probability that $D(\hat{\mathbf{b}})$ is higher than the expected value of $\tilde{D}(\tilde{\mathbf{x}})$ decreases exponentially with M.

When randomization is applied to the hard demodulation problem considered in this section, only the "best" of the randomizations is retained. One of the motivations for the soft demodulator proposed in the next section, was to exploit the fact that multiple randomizations are generated.

4. LIST GENERATION USING A SINGLE SDP

One of the properties of the SDR approach to hard demodulation is that, on average, the randomly generated bit-vectors in step 3 of the randomization procedure yield small values for the objective in (7). That suggests that, on average, these bit-vectors are good candidates for membership of the list in a list-based approach to soft demodulation, and this is the essence of the proposed approach. We will construct a preliminary list $\hat{\mathcal{L}}'$ by simply storing each (unique) bit-vector generated by the randomization procedure. Since it is possible that there may be bit positions for which $\hat{\mathcal{L}}'_{i,+1}$ or $\hat{\mathcal{L}}'_{i,-1}$ is empty, once the randomizations have been completed we will construct an enriched list $\hat{\mathcal{L}}$ consisting of $\hat{\mathcal{L}}'$ plus all those bit-vectors with Hamming distance of 1 of the bit-vectors in $\hat{\mathcal{L}}'$. (This enrichment is based on ideas in [4] and can be implemented by "flipping" individual bits of each element of $\hat{\mathcal{L}}'$.) Once this enriched list has been constructed, we adopt the standard list-based approach and approximate the optimization problems in (5) by enumeration over $\hat{\mathcal{L}}_{i,+1}$ and $\hat{\mathcal{L}}_{i,-1}$, respectively. Since the computational cost of this enumeration grows linearly with the cardinality of $\hat{\mathcal{L}}$, one may wish to bound the cardinality of $\hat{\mathcal{L}}'$, and to use this bound to enable early termination of the randomization procedure should $\hat{\mathcal{L}}'$ be sufficiently rich.[3] The resulting SDR-based list construction procedure takes the following form:

1. Let $\mathbf{X}_o = \mathbf{V}^T\mathbf{V}$ denote a factorization of the solution to (10). Initialize $\hat{\mathcal{L}}'$ empty, $m = 0$, and $k = 0$. Set the maximum number of randomizations, M, and the maximum list size for $\hat{\mathcal{L}}'$, K.
2. Choose a random vector $\mathbf{u}$ from the uniform distribution on the unit sphere.
3. Construct $\tilde{\mathbf{x}} = \sigma(\mathbf{V}^T\mathbf{u})$ and increment m.
4. If $\tilde{\mathbf{b}} = \tilde{x}_{2N_t+1}[\tilde{x}_1, \ldots, \tilde{x}_{2N_t}]^T$ is not in $\hat{\mathcal{L}}'$, add it to $\hat{\mathcal{L}}'$ and increment k.
5. If $k < K$ and $m < M$, return to 2.
6. Construct $\hat{\mathcal{L}}$ as the union of $\hat{\mathcal{L}}'$ and all the single bit-flippings of the bit-vectors in $\hat{\mathcal{L}}'$.

Once $\hat{\mathcal{L}}$ has been constructed in this way, the maximization of problems in (5) can be approximated by enumeration over $\hat{\mathcal{L}}_{i,\pm1}$. However, since $\hat{\mathcal{L}}_{i,\pm1}$ does not necessarily contain the optimal solutions to each of these problems, the soft information generated in this way may be under- or over-estimated. We will take a standard approach to mitigating this effect [6], and will clip the estimated log-likelihood ratios to the interval $[-5, +5]$.

In order to determine the computational complexity of the proposed approach, we note that the worst-case complexity of solving the SDP in (10) using the interior point method in [24] is polynomial and of order $O((2N_t+1)^{3.5}\log\epsilon^{-1})$, where ϵ is a measure of the accuracy of the solution. The complexity of generating each random bit-vector is $O((2N_t+1)^2)$, and the complexity of computing the metric $D(\mathbf{b})$ is $O((2N_t)^2)$. The SDR method of [9] solves one SDP of size $2N_t+1$ and $2N_t$ SDPs of size $2N_t$, so its overall computational complexity is:

$$\begin{aligned} &O((2N_t+1)^{3.5}\log\epsilon^{-1}) + M \times O((2N_t+1)^2) \\ &\quad + 2N_t \times [O((2N_t)^{3.5}\log\epsilon^{-1}) + M \times O((2N_t)^2)] \\ &\qquad \sim O(N_t^{4.5}\log\epsilon^{-1}) + O(MN_t^3). \end{aligned} \tag{11}$$

The proposed list-based approach, requires the solution of only one SDP of size $2N_t+1$. If we let $K \leq M$ denote the maximum cardinality of $\hat{\mathcal{L}}'$ then the cardinality of $\hat{\mathcal{L}}$ is at most $(2N_t+1)K$, and hence the enumeration approach to optimizing the terms on the right hand side of (5) requires at most $(2N_t+1)K$ evaluations of $D(\mathbf{b})$. Therefore, the worst-case complexity of the proposed approach is

$$\begin{aligned} &O((2N_t+1)^{3.5}\log\epsilon^{-1}) + M \times O((2N_t+1)^2) \\ &\quad + (2N_t+1)K \times O((2N_t)^2) \\ &\qquad \sim O(N_t^{3.5}\log\epsilon^{-1}) + O(MN_t^2) + O(KN_t^3), \end{aligned} \tag{12}$$

where these terms correspond to the cost of solving the SDP, the cost of constructing $\hat{\mathcal{L}}$, and the cost of the enumerations in (5) over $\hat{\mathcal{L}}_{i,\pm1}$,

[3] This treatment of the cardinality of $\hat{\mathcal{L}}'$ is rather simplistic, but as we will show in Section 5 its performance is quite good.

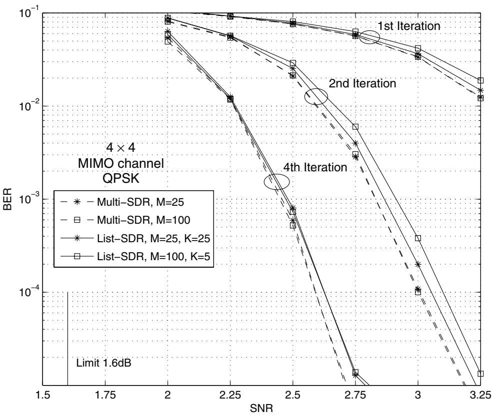

Fig. 2. BER performance comparison, 4×4 MIMO system, maximum number of iterations M and maximum cardinality of $\hat{\mathcal{L}}'$, K.

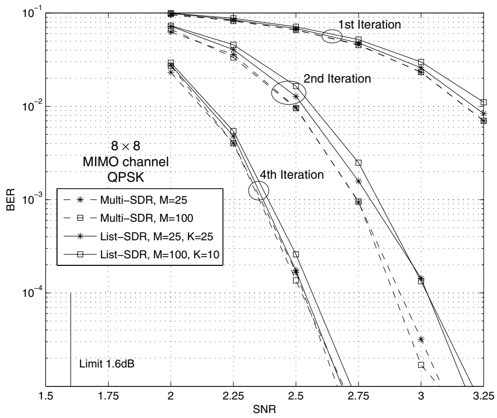

Fig. 3. BER performance comparison, 8×8 MIMO system, maximum number of iterations M and maximum cardinality of $\hat{\mathcal{L}}'$, K.

respectively. This expression shows that the computational cost of the SDP component of the proposed list-based SDR scheme is one order less than that of the SDR scheme in [9] that is based on the hard decision strategy.

5. SIMULATION RESULTS

We now compare the performance of our approach, which we will call List-SDR, to that of the method in [9], which we will denote by Multi-SDR. We consider $N_r \times N_t$ MIMO systems with an i.i.d. Rayleigh block-fading MIMO channel model, and with the transceiver parameters chosen from those used in [2, 5]. In particular, the simple V-BLAST transmission scheme is implemented and the outer code is a rate $1/2$ punctured parallel concatenated turbo code with the $(5, 7)$ recursive systematic convolutional code as the component codes and an (input) block length of 8192. The conventional BCJR algorithm is used to decode the constituent convolutional codes of the turbo code, and 8 turbo decoding iterations are performed before we pass the soft information back to the demodulator. Up to 4 demodulation-decoding iterations are performed. Following [14], the SDP problems are solved to an accuracy of at least 10^{-2} for both the List-SDR and Multi-SDR methods.

Figs. 2, 3 and 4 compare the bit error rate (BER) performance of the List-SDR and Multi-SDR methods after 1, 2 and 4 demodulation-decoding iterations for 4×4, 8×8 and 16×16 MIMO systems, respectively. For each demodulation-decoding iteration we plot four curves. The dashed curves are those for the method in [9] and the solid curves for the proposed method. For the method in [9] we plot curves for two representative values for the number of randomizations M, the lower of which is the value that was selected in [9]. For the proposed method we consider a system with the smaller number of randomizations and $K = M$ so that the list size constraint was inactive, and a system with a larger number of randomizations and a small list size constraint (K) that was almost always active; see also Tab. 1. Each of these curves demonstrates that our proposed approach provides BER performance close to that of the method in [9]. The comparisons in [9] between the Multi-SDR method and other hard-decision-based methods that involve exhaustive search or sphere decoding principles (such as the method in [2]) suggest that the performance of the proposed method is also quite similar to those

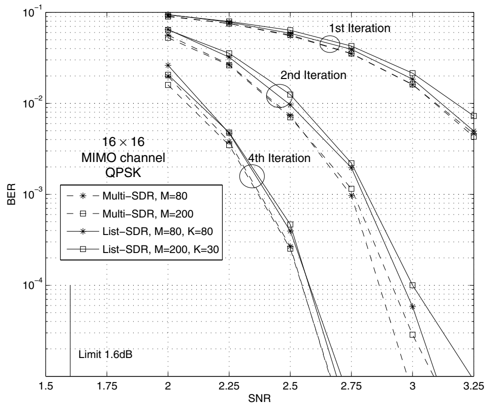

Fig. 4. BER performance comparison, 16×16 MIMO system, maximum number of iterations M and maximum cardinality of $\hat{\mathcal{L}}'$, K.

methods. For reference, in Figs 2–4 we have indicated the SNR at which the mutual information for QPSK symbols is equal to the chosen number of bits per channel use. This indicates that the systems considered provide good performance even when the gap to the input constrained capacity is as small as 1dB.

Although the performance of each of the systems in Figs 2–4 is quite similar, it is instructive to explore the performance-complexity trade-offs therein. First of all, it appears that the number of randomizations chosen in [9] for the Multi-SDR is an appropriate choice, because the larger number of randomizations does not appear to provide significant improvement. When this (smaller) number of randomizations is selected for the proposed List-SDR method (with $K = M$), the performance degrades slightly, but by less than 0.05dB. In order to assess the average computational cost of the enumeration step for the List-SDR algorithm, we have provided in Tab. 1 the average size of the preliminary and enriched lists at each iteration. These numbers suggest that the computational advantage of the proposed list-based method over that in [9] that arises from the fact that only one SDP is solved is not substantially eroded by the enumeration step.

Table 1. Average size of the preliminary list $\hat{\mathcal{L}}'$ and enriched list $\hat{\mathcal{L}}$ in the List-SDR method for different MIMO systems at SNR=2.5dB. For reference, the sizes of the full list $\mathcal{L}$ for each MIMO system are $2^8 = 256$, $2^{16} \approx 6.6 \times 10^4$ and $2^{32} \approx 4.3 \times 10^9$, respectively.

System / Iteration	1st		2nd		4th	
	$\lvert\hat{\mathcal{L}}'\rvert$	$\lvert\hat{\mathcal{L}}\rvert$	$\lvert\hat{\mathcal{L}}'\rvert$	$\lvert\hat{\mathcal{L}}\rvert$	$\lvert\hat{\mathcal{L}}'\rvert$	$\lvert\hat{\mathcal{L}}\rvert$
4×4 (M=25)	7.4	49.4	5.1	36.4	4.1	31.2
8×8 (M=25)	13.5	201.5	7.5	115.7	4.6	74.1
16×16 (M=80)	54.1	1676.9	26.5	819.1	15.6	500.1

In order to explore the effects of limiting the size of the preliminary list generated by the List-SDR method, we chose values for K, the maximum size of the preliminary list, that are smaller than the average sizes reported in Tab. 1, and we repeated the experiments of Figs 2–4, with a large value for M so that the list size constraint was almost always active. Bounding the list size in this way reduces the worst-case complexity of the enumeration step, yet only results in a slight degradation in performance (up to 0.05dB). It is apparent from Tab. 1 that this degradation is due, in large part, to the reduction in the quality of the soft information in the first demodulation-decoding iteration.

6. CONCLUSION

We have proposed a soft MIMO demodulator based on an adaptation of the semidefinite relaxation (SDR) method for hard demodulation to list-based soft demodulation. In contrast to list demodulators based on the principles of sphere decoding, the (worst-case) computational cost of the proposed approach is bounded by a (low-order) polynomial of the number of bits to be demodulated, and in contrast to the SDR-based approach in [9] (that is not based on the list demodulation principle), the proposed approach requires the solution of only one semidefinite program. Our simulation results suggest that these computational advantages are obtained without incurring a significant degradation in performance. While we have focused on BICM-based MIMO systems with V-BLAST transmission of QPSK symbols, there are natural extensions to systems with more general space-time transmission schemes, systems with 16-QAM modulation and narrowband synchronous CDMA, and these extensions are under investigation.

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