

Outage-based design of robust Tomlinson-Harashima transceivers for the MISO downlink with QoS requirements[☆]

Michael Botros Shenouda^{a,b}, Timothy N. Davidson^a, Lutz Lampe^b

^a*Department of Electrical and Computer Engineering, McMaster University, 1280 Main Street West, Hamilton, Ontario, L8S 4K1, Canada. Tel: +1 905 525 9140, Ext. 27352.*

Fax: +1 905 521 2922.

^b*Department of Electrical and Computer Engineering, University of British Columbia, BC, Canada.*

Abstract

We consider a broadcast channel in which the base station is equipped with multiple antennas and each user has a single antenna, and we study the design of transceivers based on Tomlinson-Harashima precoders with probabilistic Quality of Service (QoS) requirements for each user, in scenarios with uncertain channel state information (CSI) at the transmitter. Each user's QoS requirement is specified as a constraint on the maximum allowed outage probability of the receiver's mean square error (MSE) with respect to a specified target MSE, and we demonstrate that these outage constraints are associated with constraints on the outage of the received signal-to-interference-plus-noise-ratio (SINR). We consider four different stochastic models for the channel uncertainty, and we design the downlink transceiver so as to minimize the total transmitted power subject to the satisfaction of the probabilistic QoS constraints. We present three conservative approaches to solving the resulting chance constrained optimization problems. These approaches are based on efficiently-solvable deterministic convex design formulations that guaran-

[☆]This work was supported in part by the Killam Trust, the Natural Sciences and Engineering Research Council of Canada and the Canada Research Chair program. A preliminary version of a portion of this manuscript appears in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing*, Taipei, April 2009

Email addresses: michael.botros@gmail.com (Michael Botros Shenouda), davidson@mcmaster.ca (Timothy N. Davidson), lampe@ece.ubc.ca (Lutz Lampe)

tee the satisfaction of the probabilistic QoS constraints. We also demonstrate how to apply these approaches in order to obtain computationally-efficient solutions to some related design problems. Our simulations indicate that the proposed methods can significantly expand the range of QoS requirements that can be satisfied in the presence of uncertainty in the CSI.

Keywords: downlink; robust optimization; chance constraints

1. Introduction

We consider the design of transceivers for the downlink of a narrowband cellular system with multiple transmit antennas at the basestation and multiple single antenna receivers. Motivated by the increasing demand for a variety of low-latency interactive services, we adopt a design framework in which we seek to minimize the total transmitted power required to satisfy (physical layer) quality of service (QoS) constraints specified by the users. In static environments, the quality of the channel state information (CSI) that can be provided to the transmitter is sometimes high enough to warrant design methodologies based on the assumption that the available CSI is perfect. This assumption facilitates the design process, and several effective methods for the design of linear precoders for instances of the QoS problem with perfect CSI are available [1–8]. The availability of perfect CSI at the transmitter also offers the possibility of sequential interference pre-subtraction using Tomlinson-Harashima precoding [9–15], and the application of the significantly more complicated dirty-paper coding techniques, which can be used to achieve points on the boundary of the capacity region [16].

In practical downlink systems, however, the CSI that is available at the transmitter is subject to a variety of sources of imperfection, such as the channel estimation errors that arise in systems with uplink-downlink reciprocity, and the quantization errors that arise in systems with quantized feedback [17, 18]. In dynamic environments, the delay between the (imperfect) estimation of the channel and the application of that estimate in transmission can lead to errors due to the estimate becoming “out of date”. While channel prediction can mitigate such errors to some degree, the prediction errors may still be significant. Given the pervasive nature of uncertainties in the CSI, one might consider operating the downlink without CSI, either via isotropic signalling to one user at a time, or by feeding back receiver measurements (rather than channel estimates) in order to retrospectively align

interfering signals [19]. However, insight from an analysis of the sum-rate degrees of freedom suggests that in slowly-varying environments, systems based on feeding back (imperfect) channel state estimates will be preferable [20]. Nevertheless, downlink precoder design methods that assume perfect CSI at the transmitter are sensitive to channel uncertainties because knowledge of the CSI is used to mitigate interference at the receivers, and this sensitivity is heightened in the context of QoS-based designs. Furthermore, the interference pre-subtraction in Tomlinson-Harashima precoding is inherently sensitive to CSI mismatch; e.g., [21].

One approach to addressing the sensitivities of downlink precoder design to uncertain CSI is to incorporate the channel uncertainty into the design process. In the context of the QoS problem that we are addressing, there are several ways that this can be done. One is to consider a bounded model for the error in the transmitter’s estimate of the channels and to constrain the design of the precoder so that the users’ QoS requirements are satisfied for all channels admitted by this model. That “worst-case” robustness problem has been considered for linear precoders [3, 22–25] and Tomlinson-Harashima precoders [24]. In this paper, we consider a different approach in which the uncertainty in the transmitter’s estimate of the users’ channel coefficients is modelled stochastically, and we design downlink transceivers so as to minimize the total transmitted power subject to the QoS requirements being satisfied with a given probability; that is, we minimize the power subject to constraints on the probability of outage. Problems from this class have been considered for linear downlink precoders with a stochastic uncertainty model for the channel [26, 27], or a stochastic uncertainty model for the channel covariance matrix [28]. Similar problems have also been considered in related contexts; e.g., [29]. The main purpose of the present paper is to develop design techniques for Tomlinson-Harashima precoders that satisfy outage-based QoS constraints. The modulo operators inherent in the precoders and receivers of Tomlinson-Harashima-based systems make direct analysis quite intricate, and hence we formulate the QoS requirements as constraints on the maximum allowed outage probability of a mean square error (MSE) measure of each user’s received signal, and we show that these outage constraints are related to corresponding constraints on the outage of the received signal-to-interference-plus-noise-ratio (SINR). (This MSE measure [30] has been widely employed in the design of Tomlinson-Harashima precoding systems; e.g., [12, 21, 24, 31, 32].) Using this MSE framework, we provide a unified approach to the design of Tomlinson-Harashima and linear

transceivers with probabilistic QoS constraints.

In the development of our design techniques, we consider four stochastic models for the uncertainty in the channel coefficients of each user. Taken together, these models cover a wide range of communication scenarios with uncertain CSI. For each model, the design problem is formulated as a chance constrained optimization problem in which each chance constraint involves a randomly perturbed second order cone (SOC) constraint. Chance constraints of this type are generically intractable [33], and (conservative) approaches that guarantee that the chance constraints are satisfied are usually considered. We present three conservative design approaches that are based on efficiently-solvable deterministic convex design formulations that guarantee the satisfaction of the probabilistic QoS constraints. These formulations are then extended to solve some related design problems, including minimizing the largest MSE (over the users) at which a specified outage probability can be attained, and maximizing the variance of the uncertainty that can be tolerated at a specified probabilistic QoS level. Our simulations indicate that the proposed methods can significantly expand the range of QoS requirements that can be satisfied in the presence of uncertainty in the CSI.

In closing this introduction, we emphasize that our focus is on systems with individual QoS requirements and the goal is to minimize the power required to satisfy the constraints. For systems in which the goal is to optimize a performance metric subject to constraints on the transmitted power, a variety of approaches to the design of robust downlink transceivers with Tomlinson-Harashima precoding have been proposed; e.g., [31, 32, 34–37].

The rest of the paper is organized as follows: Section 2 will present the system model and design formulations for transceivers with MSE-based QoS constraints assuming perfect CSI. Section 3 will present different stochastic models for channel uncertainty. Section 4 will present the problem of designing transceivers with probabilistic constraints, while Sections 5, 6, and 7 will present different approaches to obtaining convex design formulations that guarantee the satisfaction of these outage-based QoS constraints. Section 8 will briefly demonstrate how these approaches can be applied to solve other related design problems, and Section 9 will provide the results of some comparative simulations.

Our notation is as follows: We will use boldface capital letters to denote matrices, boldface lower case letters to denote vectors and medium weight lower case letters to denote individual elements; \mathbf{A}^T and \mathbf{A}^H denote the transpose and the conjugate transpose of the matrix \mathbf{A} , respectively. The

notation $\|\mathbf{x}\|$ denotes the Euclidean norm of vector \mathbf{x} , while $\|\mathbf{A}\|$ denotes the spectral norm (maximum singular value) of the matrix \mathbf{A} , and $E\{\cdot\}$ denotes the expectation operator. The term $\text{tr}(\mathbf{A})$ denotes the trace of matrix \mathbf{A} , and for symmetric matrices \mathbf{A} and \mathbf{B} , $\mathbf{A} \geq \mathbf{B}$ denotes the fact that $\mathbf{A} - \mathbf{B}$ is positive semidefinite. Some of our design formulations will be based on a second-order cone program, and others will be based on a semidefinite program [38]. These classes of optimization problems are convex and can be solved efficiently.

2. System Model

We consider a narrowband downlink with N_t antennas at the transmitter and K users, each with one receive antenna. We consider systems of the form in Fig. 1, in which Tomlinson-Harashima precoding (THP) is used at the transmitter for spatial multi-user interference pre-subtraction; e.g., [9, 30]. Linear transceivers are a special case of this model in which the feedback matrix $\mathbf{B} = \mathbf{0}$.

The elements of the vector $\mathbf{s} \in \mathbb{C}^K$ in Fig. 1 are the data symbols destined for each user, and each symbol s_k is chosen independently from a square quadrature amplitude modulation (QAM) constellation \mathcal{S} with cardinality M and is normalized so that $E\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}$. The output symbols of the feedback loop in Fig. 1, v_k , are generated sequentially, and hence \mathbf{B} is a strictly lower triangular matrix. In the absence of the modulo operation, the output symbols could be written as $v_k = [\mathbf{v}]_k = s_k - \sum_{j=1}^{k-1} \mathbf{B}_{k,j} v_j$. The modulo operation in the transmitter controls the transmitted power by ensuring that v_k remains within the boundaries of the Voronoi region of the constellation. For the case of square QAM, this region is a square of side length D . A standard approach to mitigating the difficulties of analyzing the model in Fig. 1 is to adopt the equivalent model in Fig. 2 [30]; see also [12, 21, 24, 31, 32]. In that model,

$$\mathbf{v} = (\mathbf{I} + \mathbf{B})^{-1}\mathbf{u}, \quad (1)$$

where $\mathbf{u} = \mathbf{s} + \mathbf{i}$ is the modified data symbol and the elements of \mathbf{i} , which depend on \mathbf{s} , take the form $i_k = i_k^{re} D + j i_k^{imag} D$, where $i_k^{re}, i_k^{imag} \in \mathbb{Z}$, and $j = \sqrt{-1}$. The elements of \mathbf{v} are almost uncorrelated and approximately uniformly distributed over the Voronoi region [30, Th. 3.1], and hence they have slightly higher average energy than the elements of \mathbf{s} ; an effect that is termed precoding loss [30]. For square M -ary QAM we have $\sigma_v^2 = E\{|v_k|^2\} =$

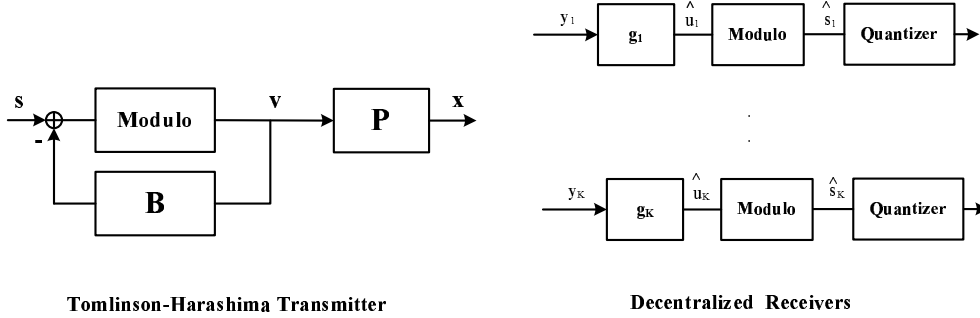


Figure 1: A broadcast channel with a Tomlinson-Harashima transceiver.

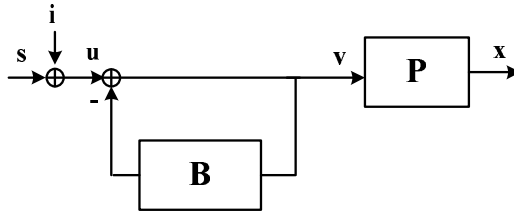


Figure 2: Linear model for the Tomlinson-Harashima transmitter.

$\frac{M}{M-1} \mathbb{E}\{|s_k|^2\}$ for all k except the first one [30]. For moderate to large values of M this precoding loss can be neglected and the approximation $\mathbb{E}\{\mathbf{v}\mathbf{v}^H\} = \mathbf{I}$ is often used; e.g., [9, 12, 21, 24, 30–32]. Hence, the average transmitted power constraint can be approximated by $\mathbb{E}_{\mathbf{v}}\{\mathbf{x}^H \mathbf{x}\} = \text{tr}(\mathbf{P}^H \mathbf{P})$.

The signal received by user k can be written as

$$y_k = \mathbf{h}_k \mathbf{x} + n_k = \mathbf{h}_k \mathbf{P} (\mathbf{I} + \mathbf{B})^{-1} \mathbf{u} + n_k, \quad (2)$$

where the row vector $\mathbf{h}_k \in \mathbb{C}^{1 \times N_t}$ contains the channel gains from the transmitting antennas to the k^{th} receiver, and n_k represents additive circular white noise with zero mean and variance $\sigma_{n_k}^2$. At each receiver, the equalizing gain g_k is used to obtain an estimate $\hat{u}_k = g_k \mathbf{h}_k \mathbf{P} (\mathbf{I} + \mathbf{B})^{-1} \mathbf{u} + g_k n_k$ of the modified data symbol u_k , upon which the modulo operation is performed to obtain \hat{s}_k .

The actions of the modulo operators at the transmitter and receiver have, over time, been somewhat resistant to insightful analysis. Therefore, we will adopt the conventional approach (e.g., [12, 21, 24, 30–32]) and define an error signal in terms of the modified data symbols,

$$\hat{u}_k - u_k = (g_k \mathbf{h}_k \mathbf{P} - \mathbf{m}_k - \mathbf{b}_k) \mathbf{v} + g_k n_k, \quad (3)$$

where \mathbf{m}_k and \mathbf{b}_k are the k^{th} rows of the matrices \mathbf{I} and \mathbf{B} , respectively. When the integer i_k is eliminated by the modulo operation at the receiver, this error signal is equivalent to the error in the receiver's estimate of the transmitted symbol, $\hat{s}_k - s_k$. Using the error signal in (3), the corresponding Mean Square Error (MSE) for the k^{th} user is

$$\begin{aligned} \text{MSE}_k &= \text{E}\{|\hat{u}_k - u_k|^2\} = \|g_k \mathbf{h}_k \mathbf{P} - \mathbf{m}_k - \mathbf{b}_k\|^2 + |g_k|^2 \sigma_{n_k}^2 \\ &= \left\| \begin{bmatrix} g_k \mathbf{h}_k \mathbf{P} - \mathbf{m}_k - \mathbf{b}_k & g_k \sigma_{n_k} \end{bmatrix} \right\|^2. \end{aligned} \quad (4)$$

2.1. Transceiver design with QoS: Perfect CSI

The quality of service constraints that we will consider in this paper take the form of upper bounds on the mean square errors, MSE_k . This approach has several advantages: it enables a unified treatment of linear and Tomlinson-Harashima transceivers with QoS constraints; it overcomes the difficulty of formulating SINR-based QoS constraints for Tomlinson-Harashima transceivers; and, as stated formally below, guaranteeing an upper bound on MSE_k is closely related to guaranteeing a lower bound on the signal-to-interference-plus-noise-ratio, SINR_k . In particular, as shown in [39],

Lemma 1. *For any given set of user's channels $\{\mathbf{h}_k\}_{k=1}^K$, if there exists a transceiver design $\mathbf{P}, \mathbf{B}, g_k$ that guarantees that $\text{MSE}_k \leq \zeta_k$, then that design also guarantees that when the modulo operator at the receiver removes i_k , $\text{SINR}_k \geq (1/\zeta_k) - 1$.*

In the case in which accurate CSI is available at the transmitter, the goal of minimizing the total transmitted power subject to satisfying the users' MSE requirements can be formulated as

$$\min_{\mathbf{P}, \mathbf{B}, g_k} \|\text{vec}(\mathbf{P})\|^2 \quad (5a)$$

$$\text{s. t. } b_{kj} = 0, \quad j = k, \dots, K, \quad (5b)$$

$$\left\| \begin{bmatrix} g_k \mathbf{h}_k \mathbf{P} - \mathbf{m}_k - \mathbf{b}_k & g_k \sigma_k \end{bmatrix} \right\|^2 \leq \zeta_k, \quad 1 \leq k \leq K. \quad (5c)$$

Using the definitions

$$\underline{\mathbf{h}}_k = \begin{bmatrix} \operatorname{Re}\{\mathbf{h}_k\} & \operatorname{Im}\{\mathbf{h}_k\} \end{bmatrix}, \quad (6)$$

$$\underline{\mathbf{P}} = \begin{bmatrix} \operatorname{Re}\{\mathbf{P}\} & \operatorname{Im}\{\mathbf{P}\} \\ -\operatorname{Im}\{\mathbf{P}\} & \operatorname{Re}\{\mathbf{P}\} \end{bmatrix}, \quad (7)$$

$$\underline{\mathbf{b}}_k = \begin{bmatrix} \operatorname{Re}\{\mathbf{b}_k\}/g_k & \operatorname{Im}\{\mathbf{b}_k\}/g_k \end{bmatrix}, \quad (8)$$

$$\underline{\mathbf{m}}_k = \begin{bmatrix} \operatorname{Re}\{\mathbf{m}_k\} & \operatorname{Im}\{\mathbf{m}_k\} \end{bmatrix}, \quad (9)$$

$$f_k = 1/g_k, \quad (10)$$

where, by definition, $\operatorname{Im}\{\mathbf{m}_k\} = \mathbf{0}$, the problem in (5) can be formulated as a convex Second Order Cone Program (SOCP) [39]:¹

$$\min_{\substack{\underline{\mathbf{P}}, \underline{\mathbf{B}}, \\ f_k, t}} t \quad (11a)$$

$$\text{s. t. } \|\operatorname{vec}(\underline{\mathbf{P}})\| \leq t, \quad (11b)$$

$$\underline{b}_{kj} = 0, \quad j = k, \dots, K, k + K, \dots, 2K, \quad (11c)$$

$$\|[\underline{\mathbf{h}}_k \underline{\mathbf{P}} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \quad \sigma_{n_k}]\| \leq \sqrt{\zeta_k} f_k, \quad 1 \leq k \leq K. \quad (11d)$$

This problem can be efficiently solved using general purpose implementations of interior point methods (e.g., SeDuMi [40]), and can be easily modified to incorporate a variety of different power constraints; e.g., [41]. More importantly, the convex formulation in (11) enables us to derive probabilistically-constrained counterparts of the original design problem in (5) for the uncertainty models presented in the following section.

3. Channel Uncertainty Model

We consider an additive model for the uncertainty of each user's channel,

$$\underline{\mathbf{h}}_k = \hat{\underline{\mathbf{h}}}_k + \underline{\mathbf{e}}_k, \quad (12)$$

where $\hat{\underline{\mathbf{h}}}_k$ is the transmitter's knowledge of the k^{th} user's actual channel, $\underline{\mathbf{h}}_k$, and $\underline{\mathbf{e}}_k$ is the corresponding mismatch, which is assumed to be independent of $\hat{\underline{\mathbf{h}}}_k$. We will consider four statistical models for $\underline{\mathbf{e}}_k$:

¹For simplicity, in (11) and the subsequent formulations we have left the structural constraint on $\underline{\mathbf{P}}$ in (7) implicit.

- **Model-G:** In this model, the elements of $\underline{\mathbf{e}}_k$, $e_{k,\ell}$, are modeled as independent Gaussian distributed random variables with zero-mean and variance $\sigma_{e_{k,\ell}}^2$. This model is appropriate for communication schemes with uplink-downlink reciprocity, which allows transmitter to estimate the users' channels on the uplink, and uncorrelated channel coefficients.
- **Model-U:** In this model, the elements $e_{k,\ell}$ are modeled as independent uniform distributed random variables on the interval $[-\rho_{k,\ell}, \rho_{k,\ell}]$. This model is suitable for communication schemes in which the users employ a scalar quantizer to quantize their channel state information and feed it back to the transmitter.
- **Model-VG:** In this model, the elements $e_{k,\ell}$ are modeled as jointly Gaussian random variables with zero-mean and covariance matrix $\mathbf{\Sigma}_{\mathbf{e}_k}$. This model is suitable for communication schemes in which the transmitter estimates the users' channels on the uplink, and the coefficients of each user's channel are correlated.
- **Model-VU:** In this model, the vector $e_{k,\ell}$ is uniformly distributed over the volume of a given ellipsoidal region. This model is suitable for communication schemes in which the users employ a vector quantizer to quantize their CSI, because the quantization cells in the interior of the quantization region can often be approximated by ellipsoids. This model includes the case in which $\underline{\mathbf{e}}_k$ is uniformly distributed over a spherical volume, which is often appropriate when the channel coefficients are uncorrelated and of equal variance.

4. Transceiver Design with Probabilistic QoS

In this paper, we consider probabilistic quality of service constraints that are expressed in the form of an upper bound on the outage probability of each user's MSE_k constraint; cf. (11d). This is motivated by the relationship between the MSE_k and SINR_k in the case of perfect CSI; cf. Lemma 1. In particular, based on that lemma we have the following result for the case of imperfect CSI:

Lemma 2. *Given a probability distribution for the users' channels, if there exists a transceiver design $\mathbf{P}, \mathbf{B}, g_k$ that guarantees that $\Pr\{\text{MSE}_k \leq \zeta_k\} \geq 1 - \epsilon_k$, then that design guarantees that when the modulo operator at the receiver removes i_k , $\Pr\{\text{SINR}_k \geq \gamma_k\} \geq 1 - \epsilon_k$, where $\gamma_k = (1/\zeta_k) - 1$.*

Our goal now is to design a robust transceiver that minimizes the transmitted power necessary to ensure that the QoS constraint of the k^{th} user is satisfied with a probability of outage that is less than ϵ_k . Using the SOCP formulation in (11), this goal can be formulated as the following optimization problem:

$$\min_{\substack{\mathbf{P}, \mathbf{B}, \\ f_k, t}} t \quad (13a)$$

$$\text{s. t. } \|\text{vec}(\mathbf{P})\| \leq t, \quad (13b)$$

$$\underline{b}_{kj} = 0, \quad j = k, \dots, K, k + K, \dots, 2K, \quad (13c)$$

$$\Pr \left\{ \left\| \underline{\mathbf{h}}_k \mathbf{P} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \quad \sigma_{n_k} \right\| \leq \sqrt{\zeta_k} f_k \right\} \geq 1 - \epsilon_k, \quad 1 \leq k \leq K. \quad (13d)$$

The problem in (13) is a chance constrained optimization problem (e.g., [42]) in which the chance constraints are on the probability that randomly perturbed second order cone constraints hold; cf. (13d). In general, chance constrained optimization problems are challenging; to develop a computationally-tractable algorithm one must obtain an efficiently-computable representation of the chance constraints. In some problems, that is straightforward (e.g., [29]), but when the chance constraints involve more general conic constraints, such as second order cone constraints or semidefinite constraints, they are generically intractable [33, 43]. To overcome this intractability, we will construct deterministic convex design formulations that guarantee the satisfaction of the chance constraints. These (conservative) formulations are efficiently-solvable, and their size is independent of the outage probability of each chance constraint.

In the implementation of downlink transceiver designs that are based on CSI (both perfect and uncertain) the receivers must obtain information regarding the design decisions that the transmitter has made. In some linear transceiver systems that is done by performing a secondary “dedicated training” phase in which the receiver estimates the projection of the design precoding matrix onto its channel [18]. In other systems, a Bayesian approach is taken and the statistics receiver’s estimates are taken into account in the design [32]. In the systems considered herein, the receiver gains are computed jointly with the transmitter matrices, via $f_k = 1/g_k$, and the receivers are informed of the gains that they are to use.

5. Design Approach 1

In this section, we will develop a computationally-efficient conservative approach to the outage-based robust transceiver design problem in (13) for uncertainty models G and U in Section 3. To simplify the development we will first define the following set of arrow-structured matrices:

$$\mathbf{C}_{k,0}(\zeta_k) = \begin{bmatrix} \sqrt{\zeta_k} f_k & & [\hat{\mathbf{h}}_k \mathbf{P} - f_k \mathbf{m}_k - \mathbf{b}_k, & \sigma_{n_k}] \\ [\hat{\mathbf{h}}_k \mathbf{P} - f_k \mathbf{m}_k - \mathbf{b}_k, & \sigma_{n_k}]^T & (\sqrt{\zeta_k} f_k) \mathbf{I} & \end{bmatrix}, \quad (14)$$

$$\mathbf{C}_{k,\ell}(\theta_{k,\ell}) = \begin{bmatrix} 0 & & \theta_{k,\ell} [\mathbf{m}_k \mathbf{P}, & 0] \\ \theta_{k,\ell} [\mathbf{m}_k \mathbf{P}, & 0]^T & \mathbf{0} & \end{bmatrix}, \quad (15)$$

where

$$\theta_{k,\ell} = \begin{cases} \sigma_{e_{k,\ell}} & \text{for Model-G;} \\ \rho_{k,\ell} & \text{for Model-U.} \end{cases} \quad (16)$$

We will also define the following function of the outage probability for each of the uncertainty models

$$\lambda(\epsilon) = \begin{cases} \min_{0 < z < 0.5} \frac{\min(2\sqrt{2/z}, 10\sqrt{-\ln z})}{\min(1, \frac{1-\phi(z)}{\sqrt{-2\ln 2\epsilon})}} & \text{for Model-G;} \\ \min_{0 < z < 0.5} \frac{\min(2\sqrt{2/z}, 4 + 4\sqrt{\ln(2/z)})}{+4\sqrt{-\ln 2\epsilon - \ln(1-z)}} & \text{for Model-U.} \end{cases} \quad (17)$$

where $\phi(\cdot)$ is the inverse of cumulative probability distribution (CDF) of the standard Gaussian random variable. Using these definitions, we can state the following result.

Theorem 1. *Consider the robust transceiver design problem with probabilistic QoS guarantees in (13) under either Model-G or Model-U, and consider the definitions in (14), (15), and (17). For $\epsilon_k \in (0, 0.5)$, the optimal solution*

of the following semidefinite program (SDP)

$$\min_{\substack{t \\ \underline{\mathbf{P}}, \underline{\mathbf{B}} \\ f_k, t}} t \quad (18a)$$

$$\text{s.t. } \|\text{vec}(\underline{\mathbf{P}})\| \leq t, \quad (18b)$$

$$\underline{b}_{kj} = 0, \quad j = k, \dots, K, k + K, \dots, 2K, \quad (18c)$$

$$\mathbf{C}_k(\zeta_k, \theta_{k,\ell}, \lambda_k) = \begin{bmatrix} \frac{1}{\lambda_k} \mathbf{C}_{k,0}(\zeta_k) & \mathbf{C}_{k,1}(\theta_{k,1}) & \dots & \mathbf{C}_{k,2N_t}(\theta_{k,2N_t}) \\ \mathbf{C}_{k,1}(\theta_{k,1}) & \frac{1}{\lambda_k} \mathbf{C}_{k,0}(\zeta_k) & & \\ \vdots & & \ddots & \\ \mathbf{C}_{k,2N_t}(\theta_{k,2N_t}) & & & \frac{1}{\lambda_k} \mathbf{C}_{k,0}(\zeta_k) \end{bmatrix} \geq \mathbf{0}, \quad (18d)$$

1 ≤ k ≤ K,

where $\lambda_k = \lambda(\epsilon_k)$, is a conservative solution of (13) that guarantees that the probability of outage of the QoS constraint of each user is at most ϵ_k . \square

PROOF. See Appendix A.

The optimization problem in (18) can be efficiently solved using general purpose implementations of interior point methods, e.g., [40]. These implementations can exploit the block-arrow structure of the matrices in (18d) and the arrow structure of the constituent blocks. We also point out that it can be shown using (17) that $\lambda(\epsilon)$ is a decreasing function of ϵ . Therefore, as the desired outage probability of user k decreases, λ_k increases and hence the size of the feasible set described by the constraint in (18d) decreases. Consequently, as one might have intuitively expected, the transmitted power of the precoder design increases with decreasing outage probabilities.

By choosing the value of λ_k in (18) to be $\lambda(\epsilon_k)$, we guarantee that when the SDP is feasible, its optimal solution satisfies the corresponding QoS target at or below the specified outage probability, ϵ_k . This choice of λ_k has the advantage that it depends only on the uncertainty model, and not on the set of channel estimates $\{\hat{\mathbf{h}}_k\}_{k=1}^K$, and hence the values of λ_k in (18d) can be pre-computed and stored offline for different possible values of ϵ_k . However, this choice of λ_k is a conservative choice. We now develop an iterative algorithm that, for a given set of channel estimates, seeks values of λ_k that are smaller than $\lambda(\epsilon_k)$, and hence are less conservative.

For simplicity, we will consider the case in which the users may have different QoS requirements, ζ_k , but they have the same outage probability,

$\epsilon_k = \epsilon$. (The principles of our approach can be extended to the general case.) For $\epsilon_k = \epsilon$ all values of λ_k are the same, and for convenience we will denote this value by λ_{\max} ; i.e., $\lambda_{\max} = \lambda(\epsilon)$. We now consider whether there exists a value of $\lambda \in (0, \lambda_{\max})$ such that the precoder design in (18) satisfies the constraints in (13d). Guided by the monotonicity of $\lambda(\epsilon)$, we propose to search for such values of λ via a bisection search algorithm on $(0, \lambda_{\max})$. In each iteration, we solve (18) for the given λ and then use a statistical validation procedure to determine whether that solution meets or exceeds the required outage probability. The outcome of the validation procedure determines the interval of λ that is to be bisected in the next iteration. The iterative algorithm finds the smallest such value of λ within an accuracy of μ_λ , or declares that there is no such λ , in at most $\log_2(\lambda_{\max}/\mu_\lambda)$ iterations. Using the importance sampling technique described in [33], the statistical validation procedure can be constructed in an efficient and reliable manner. Furthermore, in the case of uncorrelated Gaussian uncertainties (Model-G) one can square both sides of the inequality inside the braces in (13d) and obtain a closed-form validation procedure in terms the noncentral chi-square distribution.

6. Design Approach 2

The approach in the previous section was based on a semidefinite program that is applicable to both uncertainty model G and uncertainty model U in Section 3. In the following theorem we provide an alternative approach that is tailored to the Gaussian uncertainty model (Model-G) and is based on the following second order cone program.

Theorem 2. *If λ_k is chosen such that $\epsilon_k = \sqrt{e} \lambda_k \exp(-\lambda_k^2/2)$, then the optimal solution of the second order cone program (SOCP)*

$$\min_{\substack{\mathbf{P}, \mathbf{B}, f_k, t, \\ \beta, \alpha_1, \dots, \alpha_K}} t \quad (19a)$$

$$s. t. \quad \|\text{vec}(\mathbf{P})\| \leq t, \quad (19b)$$

$$\underline{b}_{kj} = 0, \quad j = k, \dots, K, k + K, \dots, 2K, \quad (19c)$$

$$\|[\hat{\mathbf{h}}_k \mathbf{P} - f_k \mathbf{m}_k - \mathbf{b}_k, \quad \sigma_{n_k}]\| \leq \sqrt{\zeta_k} f_k - \lambda_k \beta_k, \quad 1 \leq k \leq K, \quad (19d)$$

$$\|[\sigma_{e_{k,\ell}} \mathbf{m}_\ell \mathbf{P}]\| \leq \alpha_{k,\ell} \quad 1 \leq k \leq K, \quad 1 \leq \ell \leq 2N_t, \quad (19e)$$

$$\|\alpha_k\| \leq \beta_k, \quad 1 \leq k \leq K, \quad (19f)$$

is a conservative solution of (13) under the assumptions of the Model-G. \square

PROOF. See Appendix B.

Similar to the previous section, the values of λ_k in Theorem 2 are independent of the channel estimates and can be pre-computed and stored offline, but they are conservative. When all the outage probabilities are equal, $\epsilon_k = \epsilon$, we will use an iterative algorithm analogous to that in the previous section to obtain less conservative values of λ for a given set of channel estimates.

7. Design Approach 3

The design approaches presented in Sections 5 and 6 are applicable to cases in which the coefficients of CSI uncertainty of each user are independent, as in the uncertainty models G and U. These approaches rely on conservative representations of chance constraints involving randomly perturbed second order cones when the random parameters are independent. While the existence of counterpart formulations when the random parameters are jointly distributed is still an open problem [33], we will adopt a different approach in order to obtain efficient design algorithms for the correlated uncertainty models, Model-VG and Model-VU. The proposed approach relies on characterizing a bounded region \mathbf{R}_k that contains $1 - \epsilon_k$ of the probability of each user's channel \mathbf{h}_k , and designing a robust transceiver that guarantees the satisfaction of the requested QoS level for each channel realization in this region. Such a design is guaranteed to satisfy the QoS constraints with a probability that is at least $1 - \epsilon_k$; e.g., [44].

Consider the CSI uncertainty model Model-VG, in which the uncertainty coefficients \mathbf{e}_k are jointly Gaussian with probability density function

$$f(\mathbf{e}_k) = \frac{1}{(2\pi)^{N_t} \det(\mathbf{\Sigma}_{\mathbf{e}_k})} \exp\left(-\frac{1}{2}\mathbf{e}_k^T \mathbf{\Sigma}_{\mathbf{e}_k}^{-1} \mathbf{e}_k\right). \quad (20)$$

The region $\mathcal{R}_k(\epsilon_k)$ that contains $1 - \epsilon_k$ of the probability density of \mathbf{e}_k is given by (see [45])

$$\mathcal{R}_k(\epsilon_k) = \{\mathbf{e}_k \mid \mathbf{e}_k^T \mathbf{\Sigma}_{\mathbf{e}_k}^{-1} \mathbf{e}_k \leq \lambda_k^2(\epsilon_k)\} = \{\mathbf{e}_k \mid \mathbf{e}_k = \mathbf{\Phi}_k \mathbf{u}, \quad \mathbf{u}^T \mathbf{u} \leq \lambda_k^2(\epsilon_k)\}, \quad (21)$$

where $\lambda_k^2(\epsilon_k) = \text{CDF}_{\chi_{2N_t}}^{-1}(1 - \epsilon_k)$ is the value of the inverse cumulative distribution function (CDF) of a Chi-square random with $2N_t$ degrees of freedom evaluated at $1 - \epsilon_k$, and $\mathbf{\Phi}_k = \mathbf{\Sigma}_{\mathbf{e}_k}^{\frac{1}{2}}$.

Now, consider the CSI uncertainty model Model-VU, in which the CSI error vector of the k^{th} user is uniformly distributed over the volume of the $2N_t$ -dimensional ellipsoid

$$\mathcal{E}_k = \{\underline{\mathbf{e}}_k | \underline{\mathbf{e}}_k^T \Psi_k^{-1} \underline{\mathbf{e}}_k \leq 1\} = \{\underline{\mathbf{e}}_k | \underline{\mathbf{e}}_k = \Phi_k \mathbf{u}, \quad \mathbf{u}^T \mathbf{u} \leq 1\}, \quad (22)$$

where $\Phi_k = \Psi_k^{\frac{1}{2}}$. The region $\mathcal{R}_k(\epsilon_k)$ that contains $1 - \epsilon_k$ of the probability density of $\underline{\mathbf{e}}_k$ is another ellipsoid that is aligned with \mathcal{E}_k , but with $1 - \epsilon_k$ of its volume. It is given by

$$\mathcal{R}_k(\epsilon_k) = \{\underline{\mathbf{e}}_k | \underline{\mathbf{e}}_k = \Phi_k \mathbf{u}, \quad \mathbf{u}^T \mathbf{u} \leq \lambda_k^2(\epsilon_k)\}, \quad (23)$$

where $\lambda_k^2(\epsilon_k) = \sqrt[2N_t]{1 - \epsilon_k}$. From (21) and (23), it is clear that the regions $\mathcal{R}_k(\epsilon_k)$ that contain $1 - \epsilon_k$ of the probability density for both CSI uncertainty models Model-VG and Model-VU have the same geometry, and are parameterized by

$$\lambda(\epsilon) = \begin{cases} \sqrt{\text{CDF}_{\chi^2_{2N_t}}^{-1}(1 - \epsilon)} & \text{for Model-VG;} \\ \sqrt[2N_t]{1 - \epsilon} & \text{for Model-VU.} \end{cases} \quad (24)$$

Our next step is to guarantee that each user's MSE constraint, cf. (11d), is satisfied for all $\underline{\mathbf{e}}_k \in \mathcal{R}_k(\epsilon_k)$, that is

$$\|[(\hat{\mathbf{h}}_k + \underline{\mathbf{e}}_k)\mathbf{P} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \quad \sigma_{n_k}]\| \leq \sqrt{\zeta_k} f_k \quad \forall \underline{\mathbf{e}}_k \in \mathcal{R}_k(\epsilon_k). \quad (25)$$

The constraint in (25) represents an infinite number of second order cone constraints, one for each $\underline{\mathbf{e}}_k \in \mathcal{R}_k(\epsilon_k)$. However, using an approach similar to that in [46] (see also [39]) one can show that this infinite set of constraints is satisfied if and only if there exists μ_k such that following Linear Matrix Inequality holds

$$\begin{bmatrix} \sqrt{\zeta_k} f_k - \mu_k & \mathbf{0} & [\hat{\mathbf{h}}_k \mathbf{P} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \quad \sigma_{n_k}] \\ \mathbf{0} & \mu_k \mathbf{I} & \lambda_k [\Phi_k \mathbf{P}, \quad \mathbf{0}] \\ [\hat{\mathbf{h}}_k \mathbf{P} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \quad \sigma_{n_k}]^T & \lambda_k [\Phi_k \mathbf{P}, \quad \mathbf{0}]^T & \sqrt{\zeta_k} f_k \mathbf{I} \end{bmatrix} \geq \mathbf{0}. \quad (26)$$

Using this result we can state the following theorem.

Theorem 3. Consider the robust transceiver design problem with probabilistic QoS guarantees in (13) under CSI uncertainty models VG and VU, and consider the definition in (24). The optimal solution of the following semidefinite program

$$\min_{\substack{\mathbf{P}, \mathbf{B}, \\ f_k, \mu_k, t}} t \quad (27a)$$

$$\text{s.t. } \|\text{vec}(\mathbf{P})\| \leq t, \quad (27b)$$

$$\underline{b}_{kj} = 0, \quad j = k, \dots, K, k + K, \dots, 2K, \quad (27c)$$

$$\left[\begin{array}{ccc} \sqrt{\zeta_k} f_k - \mu_k & \mathbf{0} & [\hat{\mathbf{h}}_k \mathbf{P} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \sigma_{n_k}] \\ \mathbf{0} & \mu_k \mathbf{I} & \lambda_k [\Phi_k \mathbf{P}, \mathbf{0}] \\ [\hat{\mathbf{h}}_k \mathbf{P} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \sigma_{n_k}]^T & \lambda_k [\Phi_k \mathbf{P}, \mathbf{0}]^T & \sqrt{\zeta_k} f_k \mathbf{I} \end{array} \right] \geq \mathbf{0}, \quad (27d)$$

$$1 \leq k \leq K, \quad (27d)$$

where $\lambda_k = \lambda(\epsilon_k)$, is a conservative solution of (13). \square

By choosing the value of λ_k in (27d) to be $\lambda(\epsilon_k)$ in (24), we guarantee that when the SDP in (27) is feasible, its optimal solution satisfies the corresponding QoS target at or below the specified outage probability, ϵ_k . As in the previous two approaches, this choice has the advantage that it depends only on the uncertainty model, and not on the channel estimates, but it is conservative. When the outage probabilities are equal, we will use an iterative algorithm that is similar to the one proposed in Section 5 to obtain less conservative values of λ for a given set of channel estimates.

8. Related Design Problems

In previous sections, we presented three deterministic approaches for the design of robust transceivers that minimize the total transmitted power subject to an outage-based QoS requirement of each user that involves its target MSE. In this section, we will briefly demonstrate how these approaches can be also applied to obtain efficiently-solvable design formulations to some related design problems.

8.1. Minimax Transceiver Design with Outage Constraints

The first problem is the design of a robust transceiver that minimizes the maximum MSE target among all users subject to each of the K users

achieving the MSE target with an outage probability of at most ϵ_k , $1 \leq k \leq K$, and a total power constraint on the transmitter, $\text{tr}(\mathbf{P}\mathbf{P}^H) \leq P_{\text{total}}$. By denoting the maximum allowable MSE for any user by ζ_0 , this design problem can be formulated as:

$$\min_{\substack{\mathbf{P}, \mathbf{B}, \\ f_k, \sqrt{\zeta_0}}} \sqrt{\zeta_0} \quad (28a)$$

$$\text{s. t. } \|\text{vec}(\mathbf{P})\| \leq \sqrt{2P_{\text{total}}}, \quad (28b)$$

$$\underline{b}_{kj} = 0, \quad j = k, \dots, K, k + K, \dots, 2K, \quad (28c)$$

$$\Pr \left\{ \left\| \begin{bmatrix} \mathbf{h}_k \mathbf{P} - f_k \mathbf{m}_k - \mathbf{b}_k, & \sigma_{n_k} \end{bmatrix} \right\| \leq \sqrt{\zeta_0} f_k \right\} \geq 1 - \epsilon_k, \quad 1 \leq k \leq K. \quad (28d)$$

Similar to the design problem in (13), the constraints in (28d) represent chance constraints that involve randomly perturbed second order cone (SOC) constraints. However, each SOC in (28d) contains a bilinear term, since $\sqrt{\zeta_0}$ is now an optimization variable. Nevertheless, the design approaches that were presented in the previous sections can still be used to obtain efficiently-solvable conservative formulations. For example, using the first design approach in Section 5, the core optimization problem is: Given appropriate values for λ_k ,

$$\min_{\substack{\mathbf{P}, \mathbf{B}, \\ f_k, \sqrt{\zeta_0}}} \sqrt{\zeta_0} \quad (29a)$$

$$\text{s. t. } \|\text{vec}(\mathbf{P})\| \leq \sqrt{2P_{\text{total}}}, \quad (29b)$$

$$\underline{b}_{kj} = 0, \quad j = k, \dots, K, k + K, \dots, 2K, \quad (29c)$$

$$\begin{bmatrix} \frac{1}{\lambda_k} \mathbf{C}_{k,0}(\zeta_0) & \mathbf{C}_{k,1}(\theta_{k,1}) & \dots & \mathbf{C}_{k,2N_t}(\theta_{k,2N_t}) \\ \mathbf{C}_{k,1}(\theta_{k,1}) & \frac{1}{\lambda_k} \mathbf{C}_{k,0}(\zeta_0) & & \\ \vdots & & \ddots & \\ \mathbf{C}_{k,2N_t}(\theta_{k,2N_t}) & & & \frac{1}{\lambda_k} \mathbf{C}_{k,0}(\zeta_0) \end{bmatrix} \geq \mathbf{0}, \quad 1 \leq k \leq K. \quad (29d)$$

The problem in (29) is quasi-convex (cf. [38]), and can be efficiently solved using standard techniques. Similarly, the core optimization problems obtained by applying the approaches in Sections 6 and 7 are also quasi-convex.

An alternative formulation of the core problem in (29) can be obtained by observing that the constraint in (29d) can be written as $\sqrt{\zeta_0} \frac{f_k}{\lambda_k} \mathbf{I} + \mathbf{D}_k \geq \mathbf{0}$,

where

$$\mathbf{D}_k = \begin{bmatrix} \bar{\mathbf{C}}_{k,0} & \mathbf{C}_{k,1} & \cdots & \mathbf{C}_{k,2N_t} \\ \mathbf{C}_{k,1} & \bar{\mathbf{C}}_{k,0} & & \\ \vdots & & \ddots & \\ \mathbf{C}_{k,2N_t} & & & \bar{\mathbf{C}}_{k,0} \end{bmatrix}, \quad (30)$$

$$\bar{\mathbf{C}}_{k,0} = \begin{bmatrix} 0 & [\hat{\mathbf{h}}_k \mathbf{P} - f_k \mathbf{m}_k - \mathbf{b}_k, \sigma_{n_k}] \\ [\hat{\mathbf{h}}_k \mathbf{P} - f_k \mathbf{m}_k - \mathbf{b}_k, \sigma_{n_k}]^T & \mathbf{0} \end{bmatrix}. \quad (31)$$

Using this observation, the design problem in (29) can be formulated as the minimization of the maximum generalized eigenvalue of a pair of symmetric matrices that depend affinely on the decision variables subject to linear matrix inequality (LMI) constraints—a class of problems for which some specialized algorithms are available; see [47, 48]. In our case the maximum generalized eigenvalue is $\sqrt{\zeta_0}$, and the LMI constraints can be used to express the power constraint in (29b) or any other power shaping constraint. Using similar observations, the core optimization problem in the third design approach can also be written as the minimization of a maximum generalized eigenvalue subject to LMI constraints.

8.2. Finding Maximum Possible Estimation Variance

In this section, we are interested in finding parameters that are related to the estimation methods and quantization schemes used to provide the transmitter with the users' CSI. In particular, we are interested in finding the maximum possible variance of the uncertainty in the users' channel coefficients for which there exists a feasible transceiver design that satisfies the outage-based QoS requirements. In the uncorrelated Gaussian uncertainty model (Model-G), this corresponds to finding the largest $\sigma_{e_{k,\ell}}^2$ for which such a transceiver exists. In a reciprocal channel with channel estimation on the uplink, knowledge of this maximum variance is particularly useful in the selection of the estimation algorithm used by the transmitter, and the parameters of that algorithm. The estimation algorithm and its parameters should be chosen so that the variance of the channel estimates lies well below the computed maximum variance, because for uncertainties larger than that a feasible transceiver cannot be found using the method of interest. In the uncorrelated uniform uncertainty model (Model-U), the problem corresponds to finding the largest value of $\frac{\rho_{k,\ell}^2}{3}$. This is useful in order to determine the

quality of the quantization scheme (and consequently the number of code-words) to be employed by the users in the quantization of their CSI prior to feed back to the transmitter.

If we denote by θ_0 the maximum allowable value of $\theta_{k,\ell}$ (cf. (16)), then by using the design approach proposed in Section 5, we can obtain the following quasi-concave design formulation:

$$\max_{\substack{\mathbf{P}, \mathbf{B}, \\ f_k, \theta_0}} \theta_0 \quad (32a)$$

$$\text{s. t. } \underline{b}_{kj} = 0, \quad j = k, \dots, K, k + K, \dots, 2K, \quad (32b)$$

$$\mathbf{C}_k(\zeta_k, \theta_0, \lambda_k) \geq \mathbf{0}, \quad 1 \leq k \leq K. \quad (32c)$$

Similarly, the design approaches in Sections 6 and 7 can be used to obtain alternative quasi-concave design formulations for the problem of finding the maximum allowable variance for the uncertainty in the channel coefficients.

9. Simulation Studies

In this section, we demonstrate the performance of the proposed design approaches for outage-based linear and Tomlinson-Harashima transceivers, and we will provide performance comparisons with the robust power loading approach (Robust PL) in [49], which provides a robust linear transmitter for bounded channel uncertainties that match the additive uncertainty model that we have considered; cf. (12).² The robust power loading approach has additional constraints on the structure of the precoder that require the specification of normalized columns of \mathbf{P} . We will use the zero-forcing beamforming vectors for the transmitter's knowledge of the channels, $\{\hat{\mathbf{h}}_k\}_{k=1}^K$; i.e., the columns of the pseudo-inverse of $\hat{\mathbf{H}}$.

In the following simulation studies, we will consider a broadcast channel with $N_t = 3$ transmit antennas and $K = 3$ users. We assume standard Rayleigh fading channels, in which the coefficients of the fading channel are modeled as being independent proper complex Gaussian random variables with zero mean and unit variance, and we set the noise variance of each

²As in the third approach proposed herein, by choosing the size of the bounded uncertainty region of each channel so that it contains $1 - \epsilon_k$ of the probability of a stochastic uncertainty model, one can use that method to guarantee the satisfaction of the probabilistic QoS constraints that we have considered.

user to one. In order to facilitate the comparisons between all the proposed approaches, the uncertainty in the transmitter’s estimates of the channel is modelled using the Gaussian i.i.d. model (Model-G) in Section 3. The QoS constraints are expressed in terms of SINR requirements and are translated into the related constraints on the MSE using the result of Lemma 2.

In contrast to linear precoding, Tomlinson-Harashima precoding inherently requires ordering of the symbols to be transmitted to the users, prior to precoding. Finding the optimal ordering requires an exhaustive search over all possible orderings, and instead of that we have implemented a suboptimal ordering method that is a generalization of the method in [50]. That method orders the users in a way that minimizes the sum of the reciprocals of the received SINRs when the precoder matrix \mathbf{P} is an identity matrix. In our generalization, the ordering selection criterion is minimizing the sum of the ratios of each user’s SINR requirement to its received SINR when $\mathbf{P} = \mathbf{I}$, a quantity that is proportional to the power necessary for each user to achieve its SINR requirement. (To estimate the received SINR for this ordering process, the transmitter uses its channel estimates, $\{\hat{\mathbf{h}}_k\}_{k=1}^K$, as if they were precise.)

9.1. Power Minimization Problem with Outage Constraints

In the first experiment, we randomly generated 1000 realizations of the set of channel estimates $\{\hat{\mathbf{h}}_k\}_{k=1}^K$ and examined the performance of each design approach for both linear and Tomlinson-Harashima (TH) transceivers in a scenario in which the variance of each element of \mathbf{e}_k is $\sigma_{e_{k,\ell}}^2 = 0.003$ and the QoS requirements are specified in terms of the 10%-outage SINR, and are all the same; i.e., $\gamma_k = \gamma$ and $\epsilon_k = \epsilon = 0.1$.

For each set of channel estimates and for each value of γ we determined whether each design approach is able to generate a transceiver (of finite power) that guarantees that the probabilistic QoS constraints are satisfied. In Fig. 3 we plot the percentage of channel realizations for which each approach generated such a transceiver against the users’ equal SINR requirement γ .

It can be seen from Fig. 3 that for both linear and TH transceivers the first design approach provides the probabilistic QoS guarantee to the largest percentage of the channel estimates and for largest range of SINR requirements γ . The second and third approaches follow, in that order, but the performance of all three approaches is quite similar. Although the first approach has the additional advantage that it is applicable to systems with uniformly distributed uncertainties, as well as those with Gaussian uncertainties, and

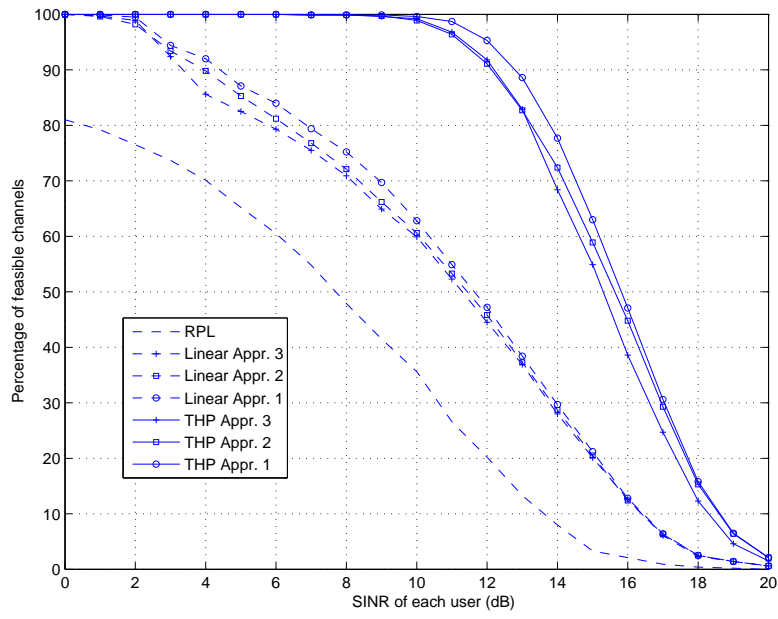


Figure 3: Percentage of channel realizations for which the probabilistic QoS requirement of an SINR of at least γ at an outage probability of at most 10% can be satisfied. The variance of each element of the channel uncertainty \mathbf{e}_k is $\sigma_{e_{k,\ell}}^2 = 0.003$.

although the third approach enables us to handle correlated uncertainties, the second approach is based on a second order cone program (SOCP) that is inherently easier to solve than semidefinite programs (SDPs) in the first and third approaches. Fig. 3 also shows that the proposed approaches to linear transceiver design extract a significant fraction of the performance gain of the proposed TH transceivers over the robust power loading approach.

In the second experiment, we selected all the channel estimates from the set of 1000 for which the proposed approaches generate TH transceivers that are able to provide the probabilistic SINR guarantee at $\gamma = 8$ dB with 10% outage. We calculated the average, over the 479 such channels, of the transmitted power required to achieve the probabilistic QoS guarantees and we have plotted the results for different values of γ in Fig. 4. The average transmitted power approaches infinity for a given value of γ when for one (or more) of the channel estimates the method under consideration cannot provide the probabilistic SINR guarantee with finite power. From Fig. 4 it can be observed that for both linear and TH transceivers, the first approach provides the slowest growth of the required transmission power with growth in the QoS requirement, followed by the second and third approaches, respectively. These approaches provide significant gains over the robust power loading approach to linear transceiver design.

9.2. Maximum Variance Problem

In this section, we compare the performance of the different approaches to finding the maximum tolerable variance of the uncertainty; cf. Section 8.2. We consider a scenario in which the variance of the elements of the channel uncertainty of each user's are equal, $\sigma_{e_{k,\ell}} = \sigma_e^2$, and the outage-based QoS requirement of each user is $\gamma_k = 10$ dB at a maximum outage of 10%, $\epsilon_k = 0.1$. In Fig. 5, we have plotted the percentage of the 1000 random channels from the first experiment for which each approach yielded a feasible transceiver with finite power against the equal variance σ_e^2 . From Fig. 5, we observe that for the case of TH transceivers the first of the proposed approaches provides significantly better performance than the second and third approaches, whereas for the case of linear transceivers the relative performance advantage of the first approach is smaller. All three of the proposed approaches yield linear transceivers that provide a significant gain over that designed using the robust power loading approach. An interesting observation from Fig. 5 is that the proposed TH transceivers are significantly more robust to small uncertainties than the linear transceivers.

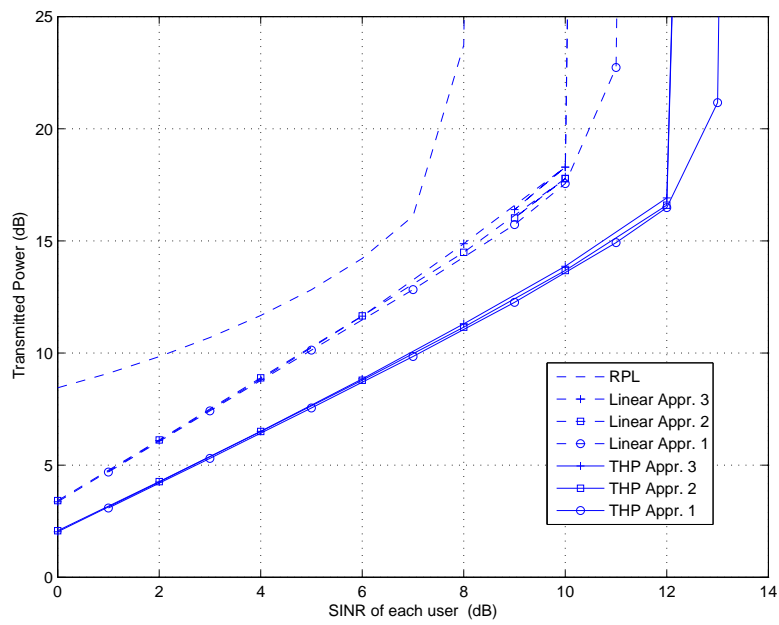


Figure 4: Average of the transmitted power against the users' equal QoS requirements of an SINR of γ at an outage probability of at most 10%. The average is performed over 479 of the 1000 channel realizations used in the first experiment.

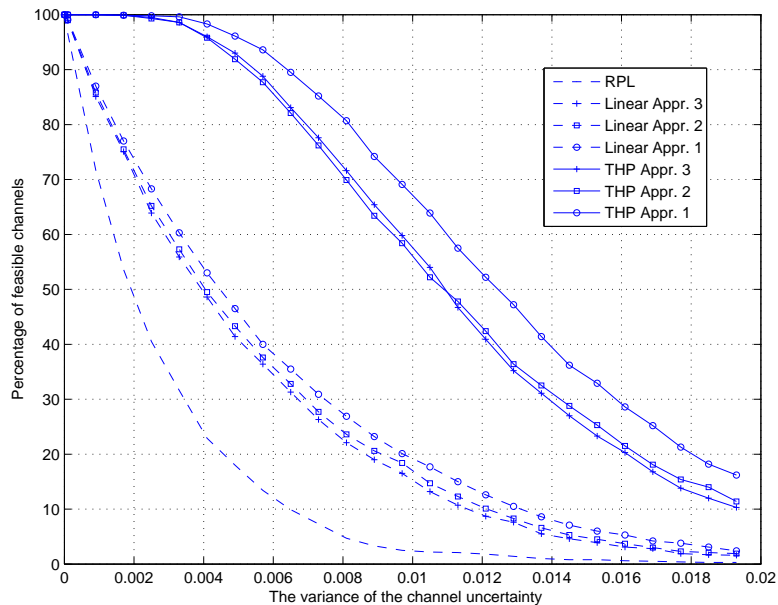


Figure 5: Percentage of channel realizations for which the probabilistic QoS requirement of an SINR of at least γ at an outage probability of at most 10% can be satisfied versus the variance of the entries of channel uncertainty \mathbf{e}_k , $\sigma_{e_{k,\ell}}^2$.

In the fourth experiment, we selected from the 1000 channels realizations those realizations for which all proposed approaches satisfied the outage-based QoS requirements for an uncertainty variance of up to 0.0025. We calculated the average transmitted power over these 405 channel realizations, and in Fig. 6 we have plotted this power against the variance of the channel uncertainty. It can be observed proposed approaches can significantly extend the range of variance of the stochastic channel uncertainty model for which the outage-based QoS requirements can be satisfied.

10. Conclusion

We have considered the design of Tomlinson-Harashima and linear transceivers for broadcast channels with probabilistically-constrained QoS requirements in the presence of uncertain channel state information (CSI) at the transmitter. The probabilistic QoS requirement of each user is formulated as a constraint on the maximum allowed outage probability of a mean square error measure of its received signal, and we demonstrated that these outage

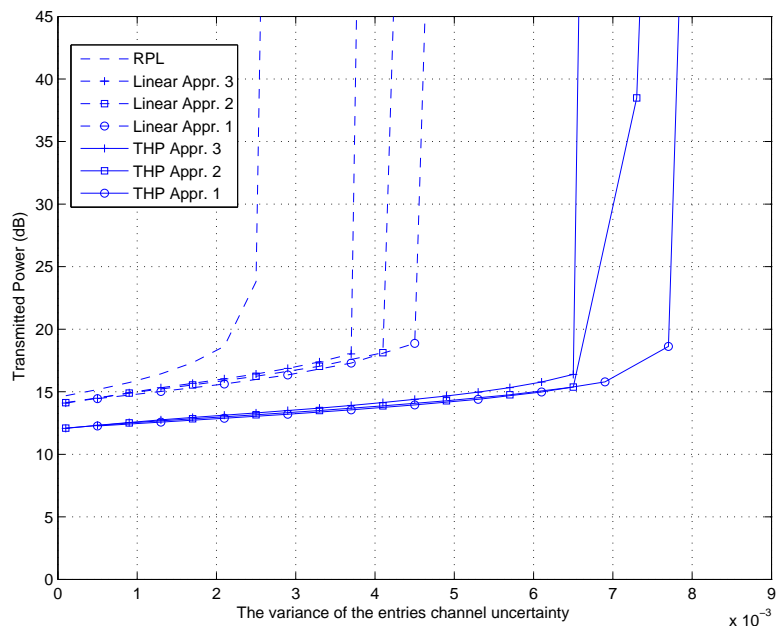


Figure 6: Average of the transmitted power versus the variance of the entries of channel uncertainty \mathbf{e}_k , $\sigma_{e_{k,\ell}}^2$. The users' QoS requirement is an SNR of at least 10 dB at an outage probability of at most 10%. The average is performed over 405 of the 1000 channel realizations used in the first experiment.

constraints are related to constraints on the outage in the received SINR sense. We considered four stochastic models for the uncertainty in the CSI, and we studied the design of robust transceivers so as to minimize the total transmitted power subject to the satisfaction of the QoS constraints with a maximum allowed outage probability. To overcome the intractability of this type of chance constrained problem, we presented three conservative design approaches based on efficiently-solvable deterministic convex design formulations that guarantee satisfaction of the probabilistic QoS constraints. Using these design approaches, we also presented computationally-efficient solutions to other related design problems. As demonstrated by the numerical studies, the proposed approaches can significantly expand the range of QoS requirements that can be satisfied in the presence of uncertainty in the CSI.

Appendix A. Proof of Theorem 1

Consider the MSE constraint of the k^{th} user, $\|[\underline{\mathbf{h}}_k \underline{\mathbf{P}} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \sigma_{n_k}]\| \leq \sqrt{\zeta_k} f_k$. Using Schur Complement Theorem [51], we can write this SOC constraint as an equivalent linear matrix inequality (LMI)

$$\mathbf{C}_k = \begin{bmatrix} \sqrt{\zeta_k} f_k & [\underline{\mathbf{h}}_k \underline{\mathbf{P}} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \sigma_{n_k}] \\ [\underline{\mathbf{h}}_k \underline{\mathbf{P}} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \sigma_{n_k}]^T & (\sqrt{\zeta_k} f_k) \mathbf{I} \end{bmatrix} \geq \mathbf{0}, \quad (\text{A.1})$$

Using the channel uncertainty model in (12), the LMI constraint in (A.1) can be written as

$$\begin{aligned} \mathbf{C}_k &= \begin{bmatrix} \sqrt{\zeta_k} f_k & [\underline{\mathbf{h}}_k \underline{\mathbf{P}} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \sigma_{n_k}] \\ [\underline{\mathbf{h}}_k \underline{\mathbf{P}} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \sigma_{n_k}]^T & (\sqrt{\zeta_k} f_k) \mathbf{I} \end{bmatrix} \quad (\text{A.2}) \\ &+ \sum_{\ell=1}^{2N_t} e_{k,\ell} \begin{bmatrix} 0 & [\underline{\mathbf{m}}_k \underline{\mathbf{P}}, 0] \\ [\underline{\mathbf{m}}_k \underline{\mathbf{P}}, 0]^T & \mathbf{0} \end{bmatrix}, \\ &= \mathbf{C}_{k,0} + \sum_{\ell=1}^{2N_t} \omega_{k,\ell} \mathbf{C}_{k,\ell}, \quad (\text{A.3}) \end{aligned}$$

where $\mathbf{C}_{k,0}$ and $\mathbf{C}_{k,\ell}$ were defined in (14) and (15), respectively, and $\omega_{k,\ell}$ are normalized uncertainty parameters. Under the assumptions of Model-G, $\omega_{k,\ell} = e_{k,\ell}/\sigma_{e_{k,\ell}}$ are i.i.d. standard Gaussian random variables, while under the assumptions of Model-U, $\omega_{k,\ell} = e_{k,\ell}/\rho_{k,\ell}$ are i.i.d. random variables that are uniformly distributed on $[-1, 1]$.

Now, the probabilistic constraint of the k^{th} user in (13d) can be written as

$$\Pr\left\{\mathbf{C}_{k,0} + \sum_{\ell=1}^{2N_t} \omega_{k,\ell} \mathbf{C}_{k,\ell} \geq \mathbf{0}\right\} \geq 1 - \epsilon_k. \quad (\text{A.4})$$

Since the considered probability distributions for $\omega_{k,\ell}$ are symmetric, (A.4) is equivalent to

$$\Pr\left\{-\mathbf{C}_{k,0} \leq \sum_{\ell=1}^{2N_t} \omega_{k,\ell} \mathbf{C}_{k,\ell} \leq \mathbf{C}_{k,0}\right\} \geq 1 - 2\epsilon_k. \quad (\text{A.5})$$

The advantage of the formulation in (A.5) is that we can obtain a deterministic (and efficiently-computable) linear matrix inequality that implies (A.5) using the following lemma from [33].

Lemma 3. *Let $\mathbf{C}_0, \mathbf{C}_1, \dots, \mathbf{C}_N$ be a set of symmetric matrices such that $\mathbf{C}_\ell = \mathbf{X}_\ell + \mathbf{y}_\ell \mathbf{a}_\ell^T + \mathbf{a}_\ell \mathbf{y}_\ell^T$ for arbitrary vectors \mathbf{y}_ℓ and \mathbf{a}_ℓ and a symmetric matrix \mathbf{X}_ℓ . If ω_ℓ are i.i.d. standard Gaussian random variables, and $\lambda = \min\left(2\sqrt{2/z}, 10\sqrt{-\ln z}\right) / \min\left(1, \frac{1-\phi(z)}{\sqrt{-2\ln \varepsilon}}\right)$, then the LMI*

$$\begin{bmatrix} \frac{1}{\lambda} \mathbf{C}_0 & \mathbf{C}_1 & \dots & \mathbf{C}_{k,N} \\ \mathbf{C}_1 & \frac{1}{\lambda} \mathbf{C}_{k,0} & & \\ \vdots & & \ddots & \\ \mathbf{C}_N & & & \frac{1}{\lambda_k} \mathbf{C}_0 \end{bmatrix} \geq \mathbf{0},$$

implies $\Pr\left\{-\mathbf{C}_0 \leq \sum_{\ell=1}^N \omega_\ell \mathbf{C}_\ell \leq \mathbf{C}_0\right\} \geq 1 - \varepsilon$. If ω_ℓ are i.i.d. uniform random variables on $[-1, 1]$, the same implication holds, but for $\lambda = \min\left(2\sqrt{2/z}, 4 + 4\sqrt{-\ln z}\right) + 4\sqrt{-\ln \varepsilon - \ln(1-z)}$. \square

By observing that all the matrices $\mathbf{C}_{k,0}$ and $\mathbf{C}_{k,\ell}$ in (14) and (15), respectively, satisfy the structural condition of Lemma 3, the proof of Theorem 1 can be completed by applying Lemma 3 to (A.5) and setting $\varepsilon = 2\epsilon_k$.

Appendix B. Proof of Theorem 2

Consider a constraint of the form $f(\mathbf{x}, \mathbf{y}) \leq 0$ in which $f(\mathbf{x}, \mathbf{y})$ is convex in \mathbf{x} for a given \mathbf{y} , convex in \mathbf{y} for a given \mathbf{x} , and satisfies $f(\mathbf{x}, k\mathbf{y}) =$

$|k|f(\mathbf{x}, \mathbf{y})$. Let the vector \mathbf{y} be randomly perturbed according to the model $\mathbf{y} = \mathbf{y}_0 + \sum_{\ell=1}^N \omega_\ell \mathbf{y}_\ell$, where ω_ℓ are i.i.d. standard Gaussian random variables. Under these assumptions, we can obtain a conservative representation of the probabilistic constraint $\Pr\{f(\mathbf{x}, \mathbf{y}) \leq 0\} \geq 1 - \varepsilon$ using the following lemma from [52].

Lemma 4. *Under the above conditions on $f(\mathbf{x}, \mathbf{y})$ and \mathbf{y} , the following set of constraints*

$$f(\mathbf{x}, \mathbf{y}_0) + \lambda\theta \leq 0, \quad (\text{B.1})$$

$$f(\mathbf{x}, \mathbf{y}_\ell) - \alpha_\ell \leq 0, \quad (\text{B.2})$$

$$f(\mathbf{x}, -\mathbf{y}_\ell) - \alpha_\ell \leq 0, \quad (\text{B.3})$$

$$\|\boldsymbol{\alpha}\|_{c_0} \leq \theta \quad (\text{B.4})$$

implies $\Pr\{f(\mathbf{x}, \mathbf{y}) \leq 0\} \geq 1 - \varepsilon$, where $\varepsilon = \sqrt{e/c_1} \lambda \exp(-\lambda^2/2c_2^2)$. The nature of the norm, c_0 , and the constants c_1 and c_2 are dependent on the function $f(\mathbf{x}, \mathbf{y})$. For a second order cone constraint with uncertainty parameters on the left hand side of the cone only, $c_0 = 2$ and $c_1 = c_2 = 1$. \square

By choosing $f(\mathbf{x}, \mathbf{y}) = \|\underline{\mathbf{h}}_k \mathbf{P} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \sigma_{n_k}\| - \sqrt{\zeta_k} f_k \leq 0$, and rewriting the uncertainty model as $\mathbf{y} = [\underline{\mathbf{h}}_k \quad \sigma_{n_k}] = [\hat{\underline{\mathbf{h}}}_k \quad \sigma_{n_k}] + \sum_{\ell=1}^{2N_t} \omega_\ell [\sigma_{e_{k,\ell}} \mathbf{m}_\ell \quad 0]$, Theorem 2 follows from an application of Lemma 4.

References

- [1] F. Rashid-Farrokhi, K. J. R. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1437–1450, Oct. 1998.
- [2] F. Rashid-Farrokhi, L. Tassiulas, and K. J. R. Liu, "Joint optimal power control and beamforming in wireless networks using antenna arrays," *IEEE Trans. Commun.*, vol. 46, no. 10, pp. 1313–1324, Oct. 1998.
- [3] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Antenna in Wireless Communications*, L. C. Godara, Ed. Boca Raton, FL: CRC Press, August 2001, ch. 18.
- [4] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Technol.*, vol. 53, pp. 18–28, Jan. 2004.

- [5] A. Wiesel, Y. Eldar, and S. Shamai, “Linear precoding via conic optimization for fixed MIMO receivers,” *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161–176, Jan. 2006.
- [6] M. Schubert and H. Boche, “A generic approach to QoS-based transceiver optimization,” *IEEE Trans. Commun.*, vol. 55, no. 8, pp. 1557–1566, Aug. 2007.
- [7] R. Hunger and M. Joham, “A complete description of the QoS feasibility region in the vector broadcast channel,” *IEEE Trans. Signal Process.*, vol. 58, no. 7, pp. 3870–3878, Jul. 2010.
- [8] Y. Huang and D. P. Palomar, “Rank-constrained separable semidefinite programming with applications to optimal beamforming,” *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 664–678, Feb. 2010.
- [9] C. Windpassinger, R. F. H. Fischer, T. Vencel, and J. B. Huber, “Precoding in multiantenna and multiuser communications,” *IEEE Trans. Wireless Commun.*, vol. 3, no. 4, pp. 1305–1316, Jul. 2004.
- [10] F. C.-H. Fung, W. Yu, and T. J. Lim, “Non-linear multi-user precoding for multi-antenna downlink channels using independent MSE constraints,” in *Proc. 22nd Biennial Symposium on Communications*, Kingston, Ontario, May 2004.
- [11] R. Doostnejad, T. Lim, and E. Sousa, “Joint precoding and beamforming design for the downlink in a multiuser MIMO system,” in *Wireless Mobile Computing, Networking, Commun.*, vol. 1, Montreal, Aug. 2005, pp. 153–159.
- [12] M. Joham and W. Utschick, “Ordered spatial Tomlinson-Harashima precoding,” in *Smart Antennas. State of the Art*, T. Kaiser, A. Bourdoux, H. Boche, J. R. Fonollosa, J. B. Andersen, and W. Utschick, Eds. Hindawi, 2005.
- [13] M. Schubert and H. Boche, “Iterative multiuser uplink and downlink beamforming under SINR constraints,” *IEEE Trans. Signal Process.*, vol. 53, no. 7, pp. 2324–2334, July 2005.

- [14] S. Shi, M. Schubert, and H. Boche, “Downlink MMSE transceiver optimization for multiuser MIMO systems: Duality and Sum-MSE minimization,” *IEEE Trans. Signal Process.*, vol. 55, no. 11, pp. 5436–5446, Nov. 2007.
- [15] L. Sanguinetti and M. Morelli, “Non-linear pre-coding for multiple-antenna multi-user downlink transmissions with different QoS requirements,” *IEEE Trans. Commun.*, vol. 6, no. 3, pp. 852 – 856, March 2007.
- [16] H. Weingarten, Y. Steinberg, and S. Shamai, “The capacity region of the gaussian multiple-input multiple-output broadcast channel,” *IEEE Trans. Inf. Theory*, vol. 52, no. 9, pp. 3936–3964, Sep. 2006.
- [17] N. Jindal, “MIMO broadcast channels with finite rate feedback,” *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 5045–5059, Nov. 2006.
- [18] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, “Multiuser MIMO achievable rates with downlink training and channel state feedback,” *IEEE Trans. Inf. Theory*, vol. 56, no. 6, pp. 2845–2866, Jun. 2010.
- [19] M. A. Maddah-Ali and D. Tse, “Completely stale transmitter channel state information is still very useful,” *IEEE Trans. Inf. Theory*, vol. 58, no. 7, pp. 4418–4431, Jul. 2012.
- [20] J. Xu, J. G. Andrews, and S. A. Jafar, “MISO broadcast channels with delayed finite-rate feedback: Predict or observe,” *IEEE Trans. Wireless Commun.*, vol. 11, no. 4, pp. 1456–1467, Apr. 2012.
- [21] D. Tsipouridou and A. P. Liavas, “On the sensitivity of the MIMO Tomlinson-Harashima precoder with respect to channel uncertainties,” *IEEE Trans. Signal Process.*, vol. 58, no. 4, pp. 2261–2272, Apr. 2010.
- [22] M. Botros Shenouda and T. N. Davidson, “Convex conic formulations of robust downlink precoder design with quality of service constraints,” *IEEE J. Sel. Topics Signal Process.*, vol. 1, no. 4, pp. 714–724, Dec. 2007.
- [23] G. Zheng, K.-K. Wong, and T.-S. Ng, “Robust linear MIMO in the downlink: A worst-case optimization with ellipsoidal uncertainty regions,” *EURASIP J. Adv. Signal Process.*, vol. 2008, no. 609018, Jul. 2008.

- [24] M. Botros Shenouda and T. N. Davidson, “Non-linear and linear broadcasting with QoS requirements: Tractable approaches for bounded channel uncertainties,” *IEEE Trans. Signal Process.*, vol. 57, no. 5, pp. 1936–1947, May 2009.
- [25] N. Vucic and H. Boche, “Robust QoS-constrained optimization of downlink multiuser MISO systems,” *IEEE Trans. Signal Process.*, vol. 57, no. 2, pp. 714–725, Feb. 2009.
- [26] M. Botros Shenouda and T. N. Davidson, “Probabilistically-constrained approaches to the design of the multiple antenna downlink,” in *Conf. Rec. 42nd Ann. Asilomar Conf. Signals, Systems, Computers*, Pacific Grove, CA, Oct. 2008, pp. 1120–1124.
- [27] K.-Y. Wang, T.-H. Chang, W.-K. Ma, A. M.-C. So, and C.-Y. Chi, “Probabilistic SINR constrained robust transmit beamforming: A Bernstein-type inequality based conservative approach,” in *Proc. Int. Conf. Acoust., Speech, Signal Processing*, Prague, May 2011, pp. 3080–3083.
- [28] B. Chalise, S. Shahbazpanahi, A. Czylik, and A. B. Gershman, “Robust downlink beamforming based on outage probability specifications,” *IEEE Trans. Wireless Commun.*, vol. 6, no. 10, pp. 3498–3503, 2007.
- [29] Y. Rong, S. A. Vorobyov, and A. B. Gershman, “Robust linear receivers for multi-access space-time block coded MIMO systems: A probabilistically constrained approach,” *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1560–1570, 2006.
- [30] R. F. H. Fischer, *Precoding and Signal Shaping for Digital Transmission*. New York: Wiley, 2002.
- [31] M. Botros Shenouda and T. N. Davidson, “Tomlinson-Harashima precoding for broadcast channels with uncertainty,” *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1380–1389, Sep. 2007.
- [32] F. A. Dietrich, P. Breun, and W. Utschick, “Robust Tomlinson-Harashima precoding for the wireless broadcast channel,” *IEEE Trans. Signal Process.*, vol. 55, no. 2, pp. 631–644, Feb. 2007.

- [33] A. Ben-Tal and A. Nemirovski, “On safe tractable approximations of chance constrained linear matrix inequalities,” *Math Oper. Res.*, vol. 34, no. 1, pp. 1–25, Feb. 2009.
- [34] A. P. Liavas, “Tomlinson-Harashima precoding with partial channel knowledge,” *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 5–9, Jan. 2005.
- [35] M. Payaró, A. Pascual-Iserte, A. I. Pérez-Neira, and M. Á. Lagunas, “Robust design of spatial Tomlinson-Harashima precoding in the presence of errors in the CSI,” *IEEE Trans. Wireless Commun.*, vol. 6, no. 7, pp. 2396–2401, Jul. 2007.
- [36] H. Khaleghi Bizaki and A. Falahati, “Tomlinson-Harashima precoding with imperfect channel state information,” *IET Commun.*, vol. 2, no. 1, pp. 151–158, Jul. 2008.
- [37] A. D. Dabbagh and D. J. Love, “Multiple antenna MMSE based down-link precoding with quantized feedback or channel mismatch,” *IEEE Trans. Commun.*, vol. 56, no. 11, pp. 1859–1868, Nov. 2008.
- [38] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge University Press, 2004.
- [39] M. Botros Shenouda and T. N. Davidson, “Non-linear and linear broadcasting with QoS requirements: Tractable approaches for bounded channel uncertainties,” *IEEE Trans. Signal Process.*, vol. 57, no. 5, pp. 1936–1947, 2009.
- [40] J. Sturm, “Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones,” *Optim. Methods Softw.*, vol. 11, pp. 625–653, 1999.
- [41] M. Botros Shenouda and T. N. Davidson, “On the design of linear transceivers for multi-user systems with channel uncertainty,” *IEEE J. Sel. Areas Commun.*, vol. 26, no. 6, pp. 1015–1024, Aug. 2008.
- [42] A. Prekopa, *Stochastic Programming*. Boston: Kluwer, 1995.
- [43] A. Nemirovski and A. Shapiro, “Convex approximations of chance constrained programs,” *SIAM J. Optim.*, vol. 17, no. 4, pp. 969–996, 2006.

- [44] A. Pascual-Iserte, D. P. Palomar, A. I. Pérez-Neira, and M. Á. Lagunas, “A robust maximin approach for MIMO communications with imperfect channel state information based on convex optimization,” *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 346–360, Jan. 2006.
- [45] Y. Bar-Shalom, X. R. Li, and T. Kirubarajan, *Estimation with application to tracking and navigation*. John Wiley, 2001.
- [46] A. Ben-Tal, A. Nemirovski, and C. Roos, “Robust solutions of uncertain quadratic and conic-quadratic problems,” *SIAM J. Optim.*, vol. 13, no. 2, pp. 535–560, Oct. 2002.
- [47] S. Boyd and L. El Ghaoui, “Method of centers for minimizing generalized eigenvalues,” *Linear Algebra Appl.*, vol. 188, no. 189, pp. 63–111, 1993.
- [48] Y. Nesterov and A. Nemirovskii, “An interior-point method for generalized linear-fractional programming,” *Math. Program.*, vol. 69, no. 1, pp. 177–204, 1995.
- [49] M. Payaro, A. Pascual-Iserte, and M. Lagunas, “Robust power allocation designs for multiuser and multiantenna downlink communication systems through convex optimization,” *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1390–1401, Sept. 2007.
- [50] C. Windpassinger, T. Vencel, and R. F. H. Fischer, “Precoding and loading for BLAST-like systems,” in *Proc. IEEE Int. Conf. Commun.*, vol. 5, Anchorage, May 2003, pp. 3061–3065.
- [51] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge University Press, 1985.
- [52] D. Bertsimas and M. Sim, “Tractable approximations to robust conic optimization problems,” *Math. Program.*, vol. 107, no. 1, 2006.