OUTAGE-BASED DESIGNS FOR MULTI-USER TRANSCEIVERS

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ABSTRACT

We consider a broadcast channel with multiple antennas at the base station and single-antenna receivers, and we study transceiver design with Quality of Service (QoS) requirements in the presence of uncertain channel state information (CSI) at the transmitter. Each user's QoS requirement is formulated as an upper bound on the outage probability of the mean square error (MSE), and we demonstrate that these constraints imply bounds on the outage of the received signal-to-interference-plus-noise-ratio. Using this MSE framework, we provide a unified approach to the design of non-linear and linear transceivers that minimize the transmitted power required to satisfy the QoS constraints. We present three conservative design approaches that yield (deterministic) convex and efficiently-solvable design formulations that guarantee the satisfaction of the QoS constraints, and we propose computationally-efficient algorithms that can reduce the level of conservatism in the initial formulations.

***Index Terms*—** MIMO Systems, Broadcast Channel, Quality of Service (QoS), Outage-based Design.

1. INTRODUCTION

We consider the design of (non-linear) Tomlinson-Harashima (TH) transceivers and linear transceivers for the downlink of a narrowband cellular system with multiple transmit antennas at the base station and single antenna receivers. Motivated by the increasing demand for low latency interactive services, we seek to minimize the total transmitted power required to satisfy (physical layer) quality of service (QoS) constraints specified by the users. For the case in which the transmitter has perfect channel state information (CSI), this problem has received significant interest, e.g., [1, 2], but in practical downlink systems, the transmitter's CSI is subject to a variety of sources of imperfection.

One approach to obtaining robustness to channel uncertainty is to consider a bounded model for the error in the CSI and to constrain the design so that QoS requirements are satisfied for all channels admitted by this model, e.g., [1, 3]. In this paper, we consider an alternative approach that assumes a stochastic model for the channel uncertainty, and we design the transceivers so as to minimize the total transmitted power subject to satisfaction of probabilistic QoS requirements. We formulate each of these requirements as a constraint on the maximum allowed outage probability of the mean square error (MSE) in each user's received signal (with respect to a specified target MSE), and we demonstrate that these outage constraints imply corresponding constraints on the outage of the decision-point signal-to-interference-plus-noise-ratio (SINR). Using this MSE framework, we provide a unified approach to the design of non-linear and linear transceivers with probabilistic QoS constraints.

We consider four stochastic models for the channel uncertainty which cover a wide range of systems with uncertain CSI. We formulate the design problem as a chance constrained optimization problem, in which each chance constraint involves a randomly perturbed second order cone (SOC) constraints. Chance constraints that involve this type of cone are generally intractable [4], and (conservative) approaches that guarantee that the chance constraints are satisfied are usually considered. We present three conservative design approaches that yield (deterministic) convex and efficiently-solvable design formulations that guarantee the satisfaction of the probabilistic QoS constraints, and we propose computationally-efficient algorithms that can reduce the level of conservatism in the initial formulations. A related approach to the design of a linear downlink transceiver was considered in [5], but in that work the uncertainty in the CSI is modelled indirectly, via the channel covariance matrix. In our approach, the uncertainty is modelled directly.

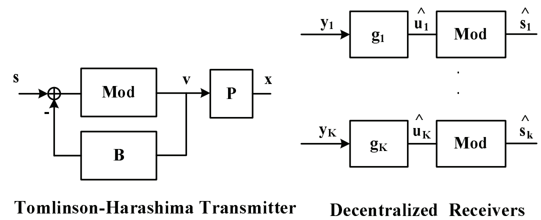

Fig. 1. Downlink with THP transceiver.

2. SYSTEM MODEL

We consider a narrowband downlink with N_t antennas at the transmitter and K users, each with one antenna. Tomlinson-Harashima precoding (THP) is used at the transmitter for spatial multi-user interference pre-subtraction (e.g., [6]), and each user employs a scalar equalizer g_k; see Fig. 1. Hence the design parameters are the THP feedforward and feedback matrices, $\mathbf{P}$ and $\mathbf{B}$, and $\{g_k\}_{k=1}^K$.

The vector $\mathbf{s} \in \mathbb{C}^K$ in Fig. 1 contains the data symbol destined for each user, and we assume that s_k is chosen from a square QAM constellation with cardinality M and that $\mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}$. The Voronoi region $\mathcal{V}$ of the constellation is a square of side length D. The transmitter's modulo operation with respect to $\mathcal{V}$ can be modelled as the addition of the complex quantity $i_k = i_k^{re}\ D + j\ i_k^{imag}\ D$ to v_k, where $i_k^{re},\ i_k^{imag} \in \mathbb{Z}$, and $j = \sqrt{-1}$. Using this observation, we obtain the standard linearized model, e.g., [6], of the transmitter, see Fig. 2, for which

$$\mathbf{v} = (\mathbf{I} + \mathbf{B})^{-1}\mathbf{u}, \tag{1}$$

where the modified data symbols $\mathbf{u} = \mathbf{i} + \mathbf{s}$. The elements of $\mathbf{v}$ are uncorrelated and uniformly distributed over the Voronoi region $\mathcal{V}$, [6, Th. 3.1], and hence will have slightly higher average energy than the input symbols $\mathbf{s}$. For moderate to large values of M this power increase can be neglected and $\mathrm{E}\{\mathbf{v}\mathbf{v}^H\} = \mathbf{I}$ is often used; e.g., [6]. Hence, the average transmitted power constraint can be written as

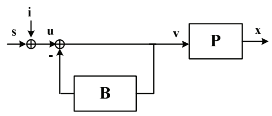

Fig. 2. Equivalent linear model for the transmitter.

$\text{E}_{\mathbf{v}}\{\mathbf{x}^H\mathbf{x}\} = \text{tr}(\mathbf{P}^H\mathbf{P})$.

The signals received at each user, y_k, can be written as

$$y_k = \mathbf{h}_k\mathbf{x} + n_k = \mathbf{h}_k\mathbf{P}(\mathbf{I}+\mathbf{B})^{-1}\mathbf{u} + n_k, \qquad (2)$$

where $\mathbf{h}_k \in \mathbb{C}^{1\times N_t}$ contains the channel gains from the transmitting antennas to the k^{th} receiver, and n_k represents zero-mean additive white noise with variance is $\sigma_{n_k}^2$. The equalizing gain g_k is used to obtain an estimate of the modified data symbol $\hat{u}_k = g_k\mathbf{h}_k\mathbf{P}(\mathbf{I}+\mathbf{B})^{-1}\mathbf{u} + g_k n_k$ of u_k, and then $\hat{s}_k$ is obtained via a modulus operation. We can define the error signal $\hat{u}_k - u_k = (g_k\mathbf{h}_k\mathbf{P} - \mathbf{m}_k - \mathbf{b}_k)\mathbf{v} + g_k n_k$, where $\mathbf{m}_k$ and $\mathbf{b}_k$ are the k^{th} rows of the matrices $\mathbf{I}$ and $\mathbf{B}$, respectively. When the integer i_k is correctly removed by the receiver's modulo operation, this error signal is equivalent to $\hat{s}_k - s_k$, and the Mean Square Error (MSE) of the k^{th} user can be written as

$$\text{MSE}_k = \big\|\big[g_k\mathbf{h}_k\mathbf{P} - \mathbf{m}_k - \mathbf{b}_k, \quad g_k\sigma_k\big]\big\|^2. \qquad (3)$$

Linear transceivers are a special subclass of the model herein, with $\mathbf{B} = \mathbf{0}$ and $i_k = 0$, and hence $u_k = s_k$.

3. TRANSCEIVER DESIGN WITH QOS: PERFECT CSI

As discussed in the Introduction, the QoS requirement of user k will take the form of an upper bound on MSE_k. This has the advantage that it enables unified treatment of linear and non-linear transceivers. Furthermore, guaranteeing an upper bound on the MSE implies a guaranteed lower bound on the decision point SINR [7]:

Lemma 1. *For any given set of channels $\{\mathbf{h}_k\}_{k=1}^K$, if there exists a transceiver design $\mathbf{P}, \mathbf{B}, g_k$ that guarantees that $\text{MSE}_k \le \zeta_k$, then that design also guarantees that $\text{SINR}_k \ge (1/\zeta_k) - 1$.*

Using the definitions

$$\underline{\mathbf{h}}_k = \big[\text{Re}\{\mathbf{h}_k\} \quad \text{Im}\{\mathbf{h}_k\}\big], \qquad (4)$$

$$\underline{\mathbf{P}} = \begin{bmatrix} \text{Re}\{\mathbf{P}\} & \text{Im}\{\mathbf{P}\} \\ -\text{Im}\{\mathbf{P}\} & \text{Re}\{\mathbf{P}\} \end{bmatrix}, \qquad (5)$$

$$\underline{\mathbf{b}}_k = \big[\text{Re}\{\mathbf{b}_k\}/g_k \quad \text{Im}\{\mathbf{b}_k\}/g_k\big], \qquad (6)$$

and $f_k = 1/g_k$, $\underline{\mathbf{m}}_k = \big[\text{Re}\{\mathbf{m}_k\}, \quad \text{Im}\{\mathbf{m}_k\}\big]$, the design of the transceiver components (assuming perfect CSI) so as to minimize the total transmitted power subject to satisfying the users' MSE requirements, ζ_k, can be formulated as an (efficiently-solvable) convex Second Order Cone Program (SOCP) [7]:

$$\min_{\underline{\mathbf{P}}, \underline{\mathbf{B}}, f_k, t} \quad t \qquad (7a)$$

$$\text{s. t.} \quad \big\|\text{vec}\big(\underline{\mathbf{P}}\big)\big\| \le t, \qquad (7b)$$

$$\underline{b}_{kj} = 0, \quad j = k, \ldots, K, k+K, \ldots, 2K, \qquad (7c)$$

$$\big\|[\underline{\mathbf{h}}_k\underline{\mathbf{P}} - f_k\underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \quad \sigma_{n_k}]\big\| \le \sqrt{\zeta_k}f_k. \qquad (7d)$$

The formulation in (7) can easily accommodate a variety of additional power constraints, such as shaping constraints and per-antenna constraints. More importantly, it enables us to derive probabilistically-constrained counterparts of the perfect CSI problem for the uncertainty models presented below.

4. CHANNEL UNCERTAINTY MODEL

We consider an additive model of the CSI uncertainty

$$\underline{\mathbf{h}}_k = \hat{\underline{\mathbf{h}}}_k + \underline{\mathbf{e}}_k, \qquad (8)$$

where $\hat{\underline{\mathbf{h}}}_k$ is the transmitter's knowledge of $\underline{\mathbf{h}}_k$, the k^{th} user's actual channel, and $\underline{\mathbf{e}}_k$ is the corresponding mismatch. We will consider four statistical models for $\underline{\mathbf{e}}_k$:

Model-G The elements of $\underline{\mathbf{e}}_k$, $e_{k,\ell}$, are independent and Gaussian with zero-mean and variance $\sigma_{e_{k,\ell}}^2$. This model is appropriate for systems with uplink-downlink reciprocity, which allows transmitter to estimate the users' channels on the uplink; specifically those in which the coefficients of each user's channel are uncorrelated.

Model-U The elements $e_{k,\ell}$ are independent and uniformly distributed random variables on the interval $[-u_{k,\ell}, u_{k,\ell}]$. This model is suitable for systems in which the users employ a scalar quantizer to quantize their channel state information and feed it back to the transmitter.

Model-VG The elements $e_{k,\ell}$ are jointly Gaussian with zero-mean and covariance matrix $\mathbf{\Sigma}_{\mathbf{e}_k}$. This model is suitable when the transmitter estimates the users' channels on the uplink, and the coefficients of each user's channel are correlated.

Model-VU The vector $\underline{\mathbf{e}}_k$ is uniformly distributed over the volume of a given ellipsoid. This model is suitable for systems in which the users employ vector quantization.

5. TRANSCEIVER DESIGN WITH PROBABILISTIC QOS

Given the stochastic channel uncertainty models in Section 4, our goal is to design a robust transceiver that minimizes the transmitted power necessary to ensure that the QoS constraint of the k^{th} user is satisfied with a probability of outage that is less than ϵ_k. The QoS constraint is formulated in terms of MSE_k, and this is motivated by the following lemma, which is a direct consequence of Lemma 1.

Lemma 2. *Let $f(\mathbf{h}_1, \ldots, \mathbf{h}_K)$ be a probability distribution of the users' channels. If there exists a transceiver design $\mathbf{P}, \mathbf{B}, g_k$ that guarantees that $Pr\{\text{MSE}_k \le \zeta_k\} \ge 1 - \epsilon_k$, then that design guarantees that $Pr\{\text{SINR}_k \ge (1/\zeta_k) - 1\} \ge 1 - \epsilon_k$.*

Using (7), the design problem can be stated as:

$$\min_{\substack{\underline{\mathbf{P}}, \underline{\mathbf{B}}, \\ f_k, t}} \quad t \qquad (9a)$$

$$\text{s. t.} \quad \big\|\text{vec}\big(\underline{\mathbf{P}}\big)\big\| \le t, \qquad (9b)$$

$$\underline{b}_{kj} = 0, \quad j = k, \ldots, K, k+K, \ldots, 2K, \qquad (9c)$$

$$\text{Pr}\big\{\big\|[\underline{\mathbf{h}}_k\underline{\mathbf{P}} - f_k\underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \quad \sigma_{n_k}]\big\| \le \sqrt{\zeta_k}f_k\big\} \ge 1 - \epsilon_k. \qquad (9d)$$

This is a chance constrained optimization problem (e.g., [8]), in which (9d) represents the probability that a randomly perturbed second order cone constraint holds. In general, chance constrained optimization problems are quite challenging. The key step in the development of a computationally-tractable algorithm is to obtain an efficiently-computable representation of the chance constraints. In some simple problems, such a representation can be obtained in a straightforward manner (e.g., [9]), but when the chance constraints involve conic constraints, as is the case in (9d), they are generally intractable [4]. To circumvent this intractability, we will construct three deterministic and efficiently-solvable convex design formulations that guarantee the satisfaction of the chance constraints.

6. DESIGN FORMULATION I

First, we consider Models G and U. We define

$$\mathbf{A}_{k,0} = \begin{bmatrix} \sqrt{\zeta_k} f_k & \mathbf{a}_{k,0} \\ \mathbf{a}_{k,0}{}^T & (\sqrt{\zeta_k} f_k)\mathbf{I} \end{bmatrix}, \quad (10)$$

$$\mathbf{A}_{k,\ell} = a_{k,\ell} \begin{bmatrix} 0 & [\mathbf{m}_k \underline{\mathbf{P}}, \quad 0] \\ [\mathbf{m}_k \underline{\mathbf{P}}, \quad 0]^T & \mathbf{0} \end{bmatrix}, \quad (11)$$

where $\mathbf{a}_{k,0} = [\hat{\underline{\mathbf{h}}}_k \underline{\mathbf{P}} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k, \quad \sigma_{n_k}]$, and for Model-G $a_{k,\ell} = \sigma_{e_{k,\ell}}$ and for Model-U $a_{k,\ell} = u_{k,\ell}$. We also define

$$\lambda_k = \begin{cases} \min\limits_{0<z<0.5} \dfrac{\min\{2\sqrt{2/z},\ 10\sqrt{-\ln z}\}}{\min\{1, \frac{1-\phi(z)}{\sqrt{-2\ln 2\epsilon_k}}\}} & \text{Model-G;} \\ \min\limits_{0<z<0.5} \min\{2\sqrt{2/z},\ 4+4\sqrt{\ln(2/z)}\} \\ \qquad +4\sqrt{-\ln 2\epsilon_k - \ln(1-z)} & \text{Model-U.} \end{cases} \quad (12)$$

where $\phi(\cdot)$ is the inverse of the CDF of the standard Gaussian random variable $N(0,1)$. Using these definitions, we have:

Theorem 1. *Consider the robust transceiver design problem with probabilistic QoS guarantees in (9) for Model-G and Model-U, and the definitions in (10), (11), and (12). For $\epsilon_k \in (0, 0.5)$, the optimal solution of the following semidefinite program (SDP)*

$$\min_{\underline{\mathbf{P}},\, \underline{\mathbf{B}},\, f_k,\, t} \quad t \quad (13a)$$

$$\text{s. t.} \quad \|\text{vec}(\underline{\mathbf{P}})\| \leq t, \quad (13b)$$

$$\underline{b}_{kj} = 0, \quad j = k, \ldots, K, k+K, \ldots, 2K, \quad (13c)$$

$$\begin{bmatrix} \frac{1}{\lambda_k}\mathbf{A}_{k,0} & \mathbf{A}_{k,1} & \ldots & \mathbf{A}_{k,2N_t} \\ \mathbf{A}_{k,1} & \frac{1}{\lambda_k}\mathbf{A}_{k,0} & & \\ \vdots & & \ddots & \\ \mathbf{A}_{k,2N_t} & & & \frac{1}{\lambda_k}\mathbf{A}_{k,0} \end{bmatrix} \geq \mathbf{0} \quad (13d)$$

is a conservative solution of (9) that guarantees that the probability of outage of the QoS constraint of each user is at most ϵ_k. □

Proof. The proof is based on the Schur-Complement Theorem, a sequence of algebraic transformations, and an application of Theorem 5.2 in [4], but is omitted for reasons of space. □

The SDP in (13) can be efficiently solved, and typical implementations can exploit the block-arrow structure of the matrices in (13d) and the arrow structure of the constituent blocks. It can be verified from (12) that as ϵ_k decreases, λ_k increases, and the size of the feasible set of (13d) decreases. Hence, as one might expect, the transmitted power increases with decreasing outage probabilities.

6.1. Alternative design of λ

An advantage of choosing λ_k according to (12) is is that if the SDP in (13) is feasible, its optimal solution is guaranteed to satisfy the corresponding QoS target at or below the specified outage probability, ϵ_k. Furthermore, λ_k can be computed offline. However, this choice may be conservative. In this section, we will describe an iterative algorithm that seeks values of λ_k that are smaller than those in (12), and hence require less transmission power and expand the range of achievable QoS requirements.

For simplicity, we will consider the case in which the users have the same outage probability, and hence all λ_k are the same. Given the monotonicity of λ with respect to ϵ, we will use a bisection search. In each iteration, we solve (13) for the given λ and then use a statistical validation procedure to determine whether that solution satisfies (9d). The outcome of the validation procedure determines the interval of λ that is to be bisected in the next iteration. The statistical validation procedure can be constructed (e.g., [4]) very efficiently compared to conventional Monte-Carlo techniques, and with arbitrary high reliability.

7. DESIGN FORMULATION II

In this section, we will present an alternative design formulation that provides probabilistic QoS guarantees for Model-G.

Theorem 2. *If λ_k is chosen such that $\epsilon_k = \sqrt{e}\ \lambda_k\ \exp(-\lambda_k^2/2)$, then the optimal solution of the following SOCP*

$$\min_{\substack{\underline{\mathbf{P}}, \underline{\mathbf{B}}, f_k, \\ \boldsymbol{\alpha}_k, t, \boldsymbol{\theta}}} \quad t \quad (14a)$$

$$\text{s. t.} \quad \|\text{vec}(\underline{\mathbf{P}})\| \leq t, \quad (14b)$$

$$\underline{b}_{kj} = 0, \quad j = k, \ldots, K, k+K, \ldots, 2K, \quad (14c)$$

$$\|[\hat{\underline{\mathbf{h}}}_k \underline{\mathbf{P}} - f_k \underline{\mathbf{m}}_k - \underline{\mathbf{b}}_k,\ \sigma_{n_k}]\| \leq \sqrt{\zeta_k} f_k - \lambda_k \theta_k, \quad (14d)$$

$$\|[\sigma_{e_{k,\ell}} \mathbf{m}_l \underline{\mathbf{P}}]\| \leq \alpha_{k,\ell},\ 1 \leq k \leq K,\ 1 \leq \ell \leq 2N_t \quad (14e)$$

$$\|\boldsymbol{\alpha}_k\| \leq \theta_k, \qquad 1 \leq k \leq K \quad (14f)$$

is a conservative solution of (9) for Model-G. □

Proof. The proof involves a sequence of algebraic transformations and application of Theorem 1 in [10]. □

As in Section 6, λ_k can be pre-computed, and when all $\epsilon_k = \epsilon$ an iterative algorithm analogous to that in Section 6.1 can be used to obtain less conservative values of λ.

8. DESIGN FORMULATION III

The designs in Sections 6 and 7 are applicable to uncertainty models with i.i.d. components. The existence of counterparts for dependent uncertainties is still an open problem [4]. In this section, we will adopt a different approach in which we characterize a bounded region $\mathbf{R}_k$ that contains $1-\epsilon_k$ of the probability of each user's channel $\underline{\mathbf{h}}_k$, and design a robust transceiver that guarantees the satisfaction of the requested QoS for each channel realization in this region. Such a design satisfies the QoS constraints with a probability that is at least $1-\epsilon_k$; see also [11].

For Model-VG, the channel uncertainty $\underline{\mathbf{e}}_k$ is Gaussian distributed with zero-mean and covariance $\mathbf{\Sigma}_{\mathbf{e}_k}$, and hence the region $\mathcal{R}_k^{\text{VG}}(\epsilon_k)$ that contains $1-\epsilon_k$ of the probability of $\underline{\mathbf{e}}_k$ is

$$\mathcal{R}_k^{\text{VG}}(\epsilon_k) = \{\underline{\mathbf{e}}_k | \underline{\mathbf{e}}_k = \mathbf{\Phi}_k \mathbf{u},\ \mathbf{u}^T\mathbf{u} \leq \lambda_k^2 = \text{CDF}_{\chi_{2N_t}}^{-1}(1-\epsilon_k)\}, \quad (15)$$

where $\text{CDF}_{\chi_{2N_t}}^{-1}(\cdot)$ is the inverse CDF of a Chi-square random variable with $2N_t$ degrees of freedom, and $\mathbf{\Phi}_k = \mathbf{\Sigma}_{\mathbf{e}_k}^{1/2}$. For

Model-VU, the CSI error is uniformly distributed over the ellipsoid $\mathcal{E}_k = \{\underline{\mathbf{e}}_k | \underline{\mathbf{e}}_k = \mathbf{\Phi}_k \mathbf{u},\ \mathbf{u}^T\mathbf{u} \le 1\}$. Hence, the region $\mathcal{R}_k^{\mathrm{UG}}(\epsilon_k)$ that contains $1-\epsilon_k$ of the probability of $\underline{\mathbf{e}}_k$ is another ellipsoid that is aligned with $\mathcal{E}_k$, but with $1-\epsilon_k$ of its volume, namely

$$\mathcal{R}_k^{\mathrm{UG}}(\epsilon_k) = \{\underline{\mathbf{e}}_k | \underline{\mathbf{e}}_k = \mathbf{\Phi}_k \mathbf{u},\ \mathbf{u}^T\mathbf{u} \le \lambda_k^2 = \sqrt[N_t]{1-\epsilon_k}\}. \quad (16)$$

Our next step, is to guarantee that each user's MSE constraint is satisfied for all $\underline{\mathbf{e}}_k \in \mathcal{R}_k(\epsilon_k)$. This constraint represents an infinite number of second order cone constraints, one for each $\underline{\mathbf{e}}_k \in \mathbf{R}(\epsilon_k)$. Using the approach in [7] one can obtain the following exact Linear Matrix Inequality representation of the constraint

$$\begin{bmatrix} \sqrt{\zeta_k} f_k - \mu_k & \mathbf{0} & \mathbf{a}_{k,0} \\ \mathbf{0} & \mu_k \mathbf{I} & \lambda_k[\mathbf{\Phi}_k \underline{\mathbf{P}},\ \mathbf{0}] \\ {\mathbf{a}_{k,0}}^T & \lambda_k[\mathbf{\Phi}_k \underline{\mathbf{P}},\ \mathbf{0}]^T & \sqrt{\zeta_k} f_k \mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \quad (17)$$

where $\mathbf{a}_{k,0}$ is as defined in Section 6. Hence, Theorem 3.

Theorem 3. *If λ_k is chosen according to (15) [(16)], then the optimal solution of the following SDP*

$$\min_{\underline{\mathbf{P}},\, \underline{\mathbf{B}},\, f_k,\, t} \quad t \quad (18a)$$

$$s.\ t. \quad \|\mathrm{vec}(\underline{\mathbf{P}})\| \le t, \quad (18b)$$

$$\underline{b}_{kj} = 0, \quad j = k, \ldots, K, k+K, \ldots, 2K, \quad (18c)$$

$$\text{and (17)} \quad (18d)$$

is a conservative solution of (9) for Model-VG [Model-UG]. □

Once again, λ_k can be pre-computed, and when $\epsilon_k = \epsilon$, an iterative algorithm can be used to obtain less conservative values of λ.

9. SIMULATION STUDIES

We now demonstrate the performance of the designs formulations in Theorems 1, 2, and 3, and their counterparts that implement the algorithm in Section 6.1. We consider a broadcast channel with $N_t = 3$ transmit antennas, $K = 3$ users, and a Rayleigh fading channel model. The uncertainty is modelled using Model-G with $\sigma^2_{e_{k,\ell}} = 0.003$. We consider a scenario in which each user is to be provided with an SINR of $\gamma_k = \gamma$, with an outage probability of at most 10%. The QoS constraints in terms of SINR are translated to corresponding constraints on the MSE using Lemma 2.

For TH precoding, ordering of the users' channels is necessary. Optimal ordering requires an exhaustive search, and instead we implemented a generalization of the suboptimal ordering method in [12]. In our generalization, the ordering criterion is minimizing the sum of each user's SINR requirements divided by its received SINR when $\mathbf{P} = \mathbf{I}$; a quantity that is proportional to the power necessary for each user to achieve its SINR requirement.

We randomly generated 1000 realizations of the set of channel estimates $\{\hat{\mathbf{h}}_k\}_{k=1}^K$ and examined the performance of each design as the SINR requirement, γ, increases. For each set of channel estimates and for each value of γ we determine whether each design yields a transceiver (of finite power) that guarantees that the probabilistic QoS constraints are satisfied. In Fig. 3 we plot the percentage of channel realizations for which each design generated such a transceiver. It can be seen from the figure, that the first design with bisection search over λ provides the probabilistic QoS guarantee to the largest percentage of the channel estimates (and for largest range of SINR requirements γ). The second and third designs with bisection search provide the next best performances, respectively. We also

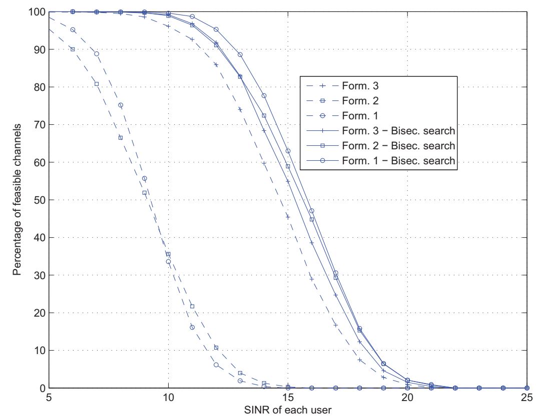

Fig. 3. Percentage of channel realizations for which the probabilistic QoS guarantee can be made, against the users' equal QoS requirements γ. The outage probability is $\epsilon_k = 0.1$.

observe the impact of the bisection search in improving the feasibility of each design. This is especially true for the first and second designs. While the first design has the advantage that it is applicable to uniformly distributed uncertainties, as well as Gaussian uncertainties, the second design is an SOCP problem and can be solved at lower computational cost the SDP in the first design.

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