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SEQUENTIAL INTERFERENCE SUBTRACTION MULTI-USER TRANSCEIVERS: DESIGNS FOR GENERAL BOUNDED CHANNEL UNCERTAINTY MODELS

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ABSTRACT

We study the design of non-linear multi-user communication systems that implement sequential interference (pre-)subtraction, such as broadcast channels with Tomlinson-Harashima precoding (THP) and multiple access channels with decision feedback equalization (DFE), with an emphasis on the broadcast case. We consider scenarios with partial channel state information in which the channel uncertainty is deterministically bounded. We propose a general channel uncertainty model that embraces many bounded uncertainty regions, and we study the robust design of THP transceivers for the broadcast channel that minimize the maximum MSE over all set admissible channels. We show that the design problem is NP-hard, and we propose an iterative local optimization algorithm that is based on efficiently-solvable convex subproblems. We then generalize the robust designs to the case in which the channel uncertainty is described by the intersection of bounded regions. The robust design framework is also generalized to multiple access channels with DFE and bounded channel uncertainty. Simulation studies demonstrate that in presence of channel uncertainty, the proposed robust design framework can result in considerable improvement in the performance of THP-based transceivers for the broadcast channel.

1. INTRODUCTION

Tomlinson-Harashima precoding (THP) was originally proposed as a temporal non-linear pre-equalization technique for channels with inter-symbol-interference, in which it works by sequentially pre-subtracting the interference effect of previous symbols. The same principle can be applied at the base station of a downlink system in which independent data symbols are transmitted to decentralized users, in which the THP pre-subtracts the interference of previously precoded symbols that are intended for other users. The operation of the THP relies critically on the availability of channel state information (CSI) in order to accurately subtract the interference that otherwise would be created at each decentralized receiver. Based on the assumption of perfect CSI at the transmitter, several different approaches for designing TH precoders for broadcast channels have been proposed, including zero-forcing designs [1, 2, 3, 4], and minimum mean square error (MMSE) designs [5, 6].

In practical downlink scenarios, the CSI available at the base station is generally imperfect. In particular, in systems in which each user quantizes its channel information and feeds it back to the transmitter, e.g., [7, 8], the uncertainty in the CSI at the transmitter is mainly due to the effect of quantization errors. The resulting mismatch between the actual CSI and the transmitter's estimate of the CSI can result in a serious degradation of the performance of the downlink; e.g., [7]. Furthermore, CSI mismatch degrades the performance of Tomlinson-Harashima precoders in general [9]. In order to mitigate this performance degradation, in this paper we will explicitly incorporate robustness to CSI mismatch into our design. For downlink scenarios with limited feedback from the receivers, the transmitter can bound the CSI uncertainty using its knowledge of the quantization codebooks. For these scenarios, we propose a general channel uncertainty model that embraces many bounded uncertainty regions, and we consider the design of robust THP transceivers for the downlink that minimize the maximum MSE over all admissible channels. We show that the design

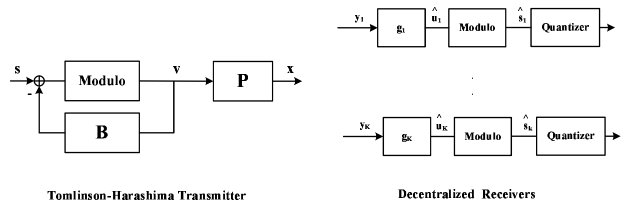

Figure 1: BC with Tomlinson-Harashima precoding.

problem is NP-hard, and we propose an iterative local optimization algorithm that is based on efficiently-solvable convex subproblems. We generalize the robust designs to the case in which the channel uncertainty is described by the intersection of bounded regions. We also consider the dual multiple access channel (MAC) with decision feedback equalization (DFE), and apply the proposed design approach to design robust transceivers for the MAC with bounded channel uncertainty. (The proposed designs include BC and MAC linear transceivers as special cases.) Simulation results show that in presence of channel uncertainty, the proposed robust approaches can result in significant improvement in the performance of THP-based transceivers for broadcast channels.

2. SYSTEM MODEL

We consider the broadcast channel (BC) with Tomlinson-Harashima precoding, and its dual, the multiple access channel (MAC) with decision feedback equalization. We will obtain robust minimax designs for these non-linear multi-user transceivers for deterministically bounded channel uncertainty models. Our development will start by describing the model of these two systems.

2.1 BC with Interference Pre-subtraction

We consider the downlink of a communication system with N_t antennas at the transmitter (base station) and K single-antenna receivers in which Tomlinson-Harashima (TH) precoding is used at the transmitter for multi-user interference pre-subtraction and spatial pre-equalization. In such schemes, the elements of the vector $\mathbf{v}$ in Fig. 1 are generated sequentially by computing $v_k = s_k - \sum_{j=1}^{k-1} B_{kj} v_j$, where s_k is the symbol intended to the k^{th} user which is chosen from a constellation whose Voronoi region is $\mathscr{V}$, and $\mathbf{B} \in \mathbb{C}^{K \times K}$ is a strictly lower triangular feedback matrix. To prevent v_k from growing outside $\mathscr{V}$, the modulo operation is then applied to each v_k. The vector $\mathbf{v}$ is subsequently linearly precoded using the feed forward matrix $\mathbf{P} \in \mathbb{C}^{N_t \times K}$ to generate the transmitted vector $\mathbf{x}$,

$$\mathbf{x} = \mathbf{P}\mathbf{v}. \tag{1}$$

We will assume that the elements of $\mathbf{s}$ are chosen from a square QAM constellation with cardinality M, and hence the Voronoi region $\mathscr{V}$ is a square of length D. Therefore, the modulo operation with respect to $\mathscr{V}$ corresponds to to performing separate modulo-D operations on the real and imaginary parts of v_k, and this is equivalent to the addition of the complex quantity $i_k = i_k^{re}\, D + j\, i_k^{imag}\, D$ to v_k, where $i_k^{re},\, i_k^{imag} \in \mathbb{Z}$, and $j = \sqrt{-1}$. Using this observation, we

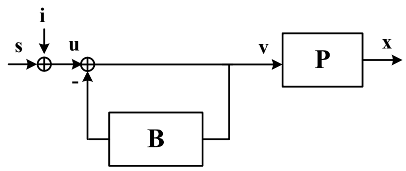

Figure 2: Equivalent linear model for the transmitter.

obtain the standard linearized model of the transmitter as shown in Fig. 2; e.g., [10]. For this equivalent model, the vector $\mathbf{v}$ is linearly related to the modified data vector $\mathbf{u} = \mathbf{s} + \mathbf{i}$,

$$\mathbf{v} = (\mathbf{I} + \mathbf{B})^{-1}\mathbf{u}. \tag{2}$$

As a result of the modulo operation, the elements of $\mathbf{v}$ are almost uncorrelated and uniformly distributed over the Voronoi region $\mathscr{V}$, [10, Th. 3.1]. Therefore, the elements of $\mathbf{v}$ will have slightly higher average energy than the input symbols of $\mathbf{s}$—something that is often called precoding loss [10]. For example, for square M-ary QAM we have $\mathrm{E}\{|v_k|^2\} = \frac{M}{M-1}\mathrm{E}\{|s_k|^2\}$ for $k = 2, \ldots, K$, and $\mathrm{E}\{|v_1|^2\} = \mathrm{E}\{|s_1|^2\}$, [10]. For moderate to large values of M this power increase can be neglected and the approximation $\mathrm{E}\{\mathbf{v}\mathbf{v}^H\} = \mathbf{I}$ is often used; e.g., [1, 11]. If we assume negligible precoding loss, the average transmitted power constraint can be written as $\mathrm{E}_{\mathbf{v}}\{\mathbf{x}^H\mathbf{x}\} = \mathrm{tr}(\mathbf{P}^H\mathbf{P}) \leq P_{\mathrm{total}}$.

The signal received by the k^{th} user, y_k, can be written as

$$y_k = \mathbf{h}_k\mathbf{x} + n_k, \tag{3}$$

where $\mathbf{h}_k \in \mathbb{C}^{1\times N_t}$ is a row vector representing the channel gains from the transmitting antennas to the k^{th} receiver, and n_k is the additive zero-mean white noise at the k^{th} receiver whose variance is σ_n^2. Collecting the received signals in the vector $\mathbf{y}$, we can write

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n},$$

where $\mathbf{H}$ is the broadcast channel matrix whose k^{th} row is $\mathbf{h}_k$, and $\mathbf{n}$ is the noise vector whose covariance matrix is $\mathrm{E}\{\mathbf{n}\mathbf{n}^H\} = \sigma_n^2\mathbf{I}$. Since the receivers operate independently, each receiver will process its received signal y_k using a single equalizing gain g_k to obtain the estimate, $\hat{u}_k = g_k y_k$, followed by a modulo operation to obtain $\hat{s}_k$. Assuming negligible precoding loss and that the vector $\mathbf{i}$ is eliminated by the receivers modulo operation, the error signal $\hat{u}_k - u_k$ is equivalent to $\hat{s}_k - s$, and can be used to define the mean square error,

$$\mathrm{MSE}_k = \|g_k\mathbf{h}_k\mathbf{P} - \mathbf{m}_k - \mathbf{b}_k\|^2 + \sigma_n^2|g_k|^2, \tag{4}$$

where $\mathbf{m}_k$ and $\mathbf{b}_k$ are the i^{th} row of $\mathbf{I}$ and $\mathbf{B}$, respectively. Similarly, the total MSE can be written as:

$$\mathrm{MSE} = \mathrm{tr}\{(\mathbf{GHP} - \mathbf{I} - \mathbf{B})^H(\mathbf{GHP} - \mathbf{I} - \mathbf{B})\} + \sigma_n^2\|\mathbf{g}\|^2, \tag{5}$$

where $\mathbf{g} = (g_1, \ldots, g_K)$ and $\mathbf{G} = \mathrm{Diag}(\mathbf{g})$.

2.2 Dual MAC with Interference Subtraction

By switching the roles of the transmitter and the receiver in the broadcast channel (BC), we obtain a dual multiple access channel (MAC) that consists of K transmitters, each with a single antenna, and a receiver with N_t antennas. The channel matrix between the transmitters and the receiver of the dual MAC is $\mathbf{H}^H$; e.g., [12]. Interference subtraction in the dual MAC is implemented using decision feedback equalization (DFE) in which detection starts with the K^{th} user, i.e., the matrix $\mathbf{B}^{\mathrm{MAC}}$ is an upper triangular matrix; see Fig 3. Because the transmitters in the dual MAC are decentralized and each have only one transmit antenna, linear precoding reduces to power loading:

$$x_k^{\mathrm{MAC}} = p_k^{\mathrm{MAC}} s_k^{\mathrm{MAC}}, \tag{6}$$

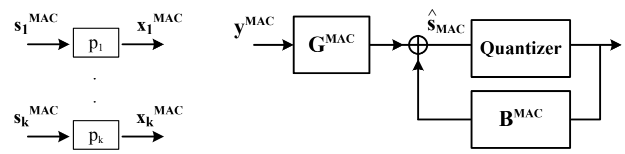

Figure 3: The Dual MAC with decision feedback equalization.

where s_k^{MAC} and x_k^{MAC} are the data symbol and the transmitted signal of the k^{th} transmitter. Without loss of generality, we will assume that $\mathrm{E}\{\mathbf{s}^{\mathrm{MAC}}\mathbf{s}^{\mathrm{MAC}H}\} = \mathbf{I}$. Hence, a total power constraint on all the transmitters can be written as $\sum_{k=1}^{K}|p_k^{\mathrm{MAC}}|^2 \leq P_{\mathrm{total}}$.

The vector of received signals $\mathbf{y}^{\mathrm{MAC}}$ is given by:

$$\mathbf{y}^{\mathrm{MAC}} = \mathbf{H}^H\mathbf{x}^{\mathrm{MAC}} + \mathbf{n}^{\mathrm{MAC}}, \tag{7}$$

where $\mathbf{n}^{\mathrm{MAC}}$ is the zero-mean receiver noise vector with $\mathrm{E}\{\mathbf{n}^{\mathrm{MAC}}\mathbf{n}^{\mathrm{MAC}H}\} = \sigma_n^2\mathbf{I}$. As shown in Fig. 3, the operation of the DFE can be represented by a feedforward matrix $\mathbf{G}^{\mathrm{MAC}} \in \mathbb{C}^{K\times N_r}$ and a strictly upper triangular feedback matrix $\mathbf{B}^{\mathrm{MAC}} \in \mathbb{C}^{K\times K}$. In this scenario, the detection of the k^{th} symbol is preceded by subtracting the effect of previously detected symbols. Assuming correct previous decisions, the input to the quantizer, $\hat{\mathbf{s}}^{\mathrm{MAC}}$, can be written as

$$\hat{\mathbf{s}}^{\mathrm{MAC}} = (\mathbf{G}^{\mathrm{MAC}}\mathbf{H}^H\mathbf{P}^{\mathrm{MAC}} - \mathbf{B}^{\mathrm{MAC}})\mathbf{s}^{\mathrm{MAC}} + \mathbf{G}^{\mathrm{MAC}}\mathbf{n}, \tag{8}$$

where $\mathbf{P}^{\mathrm{MAC}} = \mathrm{Diag}(p_1^{\mathrm{MAC}}, \ldots, p_K^{\mathrm{MAC}})$. Hence, the MSE associated with the estimation $\hat{s}_k^{\mathrm{MAC}}$ is

$$\mathrm{MSE}_k^{\mathrm{MAC}} = \|\mathbf{g}_k^{\mathrm{MAC}}\mathbf{H}^H\mathbf{P}^{\mathrm{MAC}} - \mathbf{m}_k - \mathbf{b}_k^{\mathrm{MAC}}\|^2 + \sigma_n^2\|\mathbf{g}_k^{\mathrm{MAC}}\|^2, \tag{9}$$

where $\mathbf{g}_k^{\mathrm{MAC}}$ and $\mathbf{b}_k^{\mathrm{MAC}}$ is the k^{th} row of $\mathbf{G}^{\mathrm{MAC}}$ and $\mathbf{B}^{\mathrm{MAC}}$, respectively.

In closing this section we point out that linear transceivers are a special subclass of the transceivers that we consider. In the BC case they correspond to THP transceivers in which the feedback matrix $\mathbf{B} = \mathbf{0}$, and in the MAC case they correspond to DFE transceivers in which $\mathbf{B}^{\mathrm{MAC}} = \mathbf{0}$; see Figs 1 and 3.

3. CHANNEL UNCERTAINTY MODELS

We consider the following additive uncertainty model for the CSI available at the transmitter:

$$\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_k, \tag{10}$$

where $\hat{\mathbf{h}}_k$ is the transmitter's estimate of $\mathbf{h}_k$, and $\mathbf{e}_k$ is the corresponding error. This can be equivalently written as $\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E}$, where $\mathbf{e}_k$ is the k^{th} row of $\mathbf{E}$. We will develop design formulations for robust transceivers channel uncertainty models that only assume that the error $\mathbf{e}_k$ is deterministically bounded. We will consider a general model for bounded uncertainty sets that is suitable for systems in which the channel state information is quantized at the receivers and fed back to the transmitter; e.g., [7]. In these systems, the transmitter can use its knowledge of the quantization codebook used by the k^{th} user to bound the error $\mathbf{e}_k$.

We will consider uncertainty sets of the form:

$$\mathscr{U}_k(\delta_k, \mathbf{\Phi}_k, \mathbf{Q}_k) = \{\mathbf{h}_k \,|\, \mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_k = \hat{\mathbf{h}}_k + \sum_{j=1}^{J} w_j\phi_k^{(j)},\ \mathbf{w}^T\mathbf{Q}_k\mathbf{w} \leq \delta_k^2\}. \tag{11}$$

This model enables us to treat several different uncertainty regions in a unified way. For example, it can model the following uncertainty sets:

- Ellipsoidal and Spherical Uncertainty Sets:
  By choosing $\mathbf{Q}_k = \mathbf{I}$, the uncertainty set in (11) describes an ellipsoidal uncertainty region around the channel estimate $\hat{\mathbf{h}}_k$. The spherical uncertainty set with center $\hat{\mathbf{h}}_k$ and radius δ_k is is the special case that arises when $\mathbf{\Phi}_k$, the matrix whose rows are $\phi_k^{(j)}$, is selected to be $\mathbf{I}$. If a vector quantizer is employed at the receivers, then the quantization cells in the interior of the quantization region can be often approximated by ellipsoids.
- Interval Uncertainty Sets:
  Interval constraints on an element of $\mathbf{h}_k$ can also be modeled as uncertainty sets of the form in (11). By taking ϕ_j to be the rows of $\mathbf{I}$ and $\mathbf{Q}_k$ to be the matrix whose only non-zero element is $Q_{ii} = 1$, then the uncertainty set in (11) models an interval constraint on the i^{th} entry of the error $\mathbf{h}_k$.

For uncertainty sets of the form in (11), robust downlink transceivers that employ Tomlinson-Harashima precoding at the base station will be presented in Section 4.

The above uncertainty model extends naturally to the case in which the uncertainty region for each $\mathbf{h}_k$ is described as the intersection of more than one uncertainty set $\mathscr{U}_k^\ell$ of the form (11). In that case, the uncertainty set is of the form

$$\tilde{\mathscr{U}}_k = \bigcap_{\ell=1}^{L} \mathscr{U}_k^\ell(\delta_k, \mathbf{\Phi}_k, \mathbf{Q}_k^\ell). \tag{12}$$

Note that there is no restriction in assuming that each $\mathscr{U}_k^\ell$ has the same uncertainty parameters δ_k and $\mathbf{\Phi}_k$, since $\mathbf{Q}_k^\ell$, in (11) can be chosen to accommodate different sizes and geometrical regions. Examples of constraint sets of the form in (12) include interval constraints on each of the entries of $\mathbf{h}_k$. In particular, if a simple scalar quantizer is employed at the receivers, the quantization region can be modeled using a set of interval constraints. Robust transceiver designs for the case of multiple intersecting uncertainty set will be presented in Section 4.1.

4. ROBUST THP TRANSCEIVER DESIGN: SINGLE UNCERTAINTY SET FOR EACH USER

In this section we present a robust transceiver design for the case in which each user's channel lies within a given uncertainty set $\mathscr{U}_k(\delta_k, \mathbf{\Phi}_k, \mathbf{Q}_k)$; cf. (11). Our goal is to jointly design the transmitter (i.e., $\mathbf{B}$ and $\mathbf{P}$), and the equalizing gains of the receivers, g_k, so as to minimize the worst-case MSE over all admissible channels $\mathbf{h}_k \in \mathscr{U}_k(\delta_k, \mathbf{\Phi}_k, \mathbf{Q}_k)$, subject to a total power constraint, and $\mathbf{B}$ being a strictly lower triangular matrix.[1] That is,

$$\min_{\mathbf{B},\mathbf{P},\mathbf{g}} \quad \max_{\mathbf{h}_k \in \mathscr{U}_k} \sum_{k=1}^{K} \|g_k \mathbf{h}_k \mathbf{P} - \mathbf{m}_k - \mathbf{b}_k\|^2 + \sigma_n^2 \|\mathbf{g}\|^2 \tag{13a}$$

$$\text{s. t.} \quad B_{ij} = 0 \quad 1 \le i \le j \le K, \tag{13b}$$

$$\|\text{vec}(\mathbf{P})\|^2 \le P_{\text{total}}. \tag{13c}$$

Our problem can be simplified by writing the this minimax problem as the following minimization problem

$$\min_{\mathbf{B},\mathbf{P},\mathbf{g},\mathbf{t}} \quad \sum_{k=0}^{K} t_k^2 \tag{14a}$$

$$\text{s.t.} \quad \|g_k \mathbf{h}_k \mathbf{P} - \mathbf{m}_k - \mathbf{b}_k\| \le t_k \quad \forall \mathbf{h}_k \in \mathscr{U}_k(\delta_k, \mathbf{\Phi}_k, \mathbf{Q}_k), \tag{14b}$$

$$\sigma_n \|\mathbf{g}\| \le t_0, \tag{14c}$$

$$B_{ij} = 0 \quad 1 \le i \le j \le K, \tag{14d}$$

$$\|\text{vec}(\mathbf{P})\|^2 \le P_{\text{total}}. \tag{14e}$$

[1] We have adopted the common implementation (e.g., [1, 2, 3, 4, 5, 6]) in which $\mathbf{P}, \mathbf{B}$ and g_k are jointly designed at the base station (using the available CSI), and the base station informs each receiver of the equalizing gain, g_k, that it is to use.

For each k, (14b) generates an infinite set of constraints, one for each $\mathbf{h}_k \in \mathscr{U}_k(\delta_k)$. However, each of these infinite sets of constraints can be precisely characterized by the following inequality [13, 14]:

$$\begin{bmatrix} t_k - \mu_k & \mathbf{0} & \mathbf{a}_k \\ \mathbf{0} & \mu_k \mathbf{Q}_k & \delta_k (g_k \mathbf{\Phi}_k \mathbf{P}) \\ \mathbf{a}_k{}^H & \delta_k (g_k \mathbf{\Phi}_k \mathbf{P})^H & t_k \mathbf{I} \end{bmatrix} \ge \mathbf{0}, \tag{15}$$

where $\mathbf{a}_k = g_k \hat{\mathbf{h}}_k \mathbf{P} - \mathbf{m}_k - \mathbf{b}_k$. Using the characterization in (15), the robust transceiver design can be formulated as:

$$\min_{\substack{\mathbf{B},\mathbf{P},\mathbf{g},\\ t,\mu,\alpha}} \quad \alpha \tag{16a}$$

$$\text{s.t.} \quad \left\| \begin{bmatrix} \sigma_n \mathbf{g} \\ t \end{bmatrix} \right\|^2 \le \alpha, \tag{16b}$$

$$\begin{bmatrix} t_k - \mu_k & \mathbf{0} & \mathbf{a}_k \\ \mathbf{0} & \mu_k \mathbf{Q}_k & \delta_k (g_k \mathbf{\Phi}_k \mathbf{P}) \\ \mathbf{a}_k{}^H & \delta_k (g_k \mathbf{\Phi}_k \mathbf{P})^H & t_k \mathbf{I} \end{bmatrix} \ge \mathbf{0}, \tag{16c}$$

$$B_{ij} = 0 \quad 1 \le i \le j \le K, \tag{16d}$$

$$\|\text{vec}(\mathbf{P})\|^2 \le P_{\text{total}}, \tag{16e}$$

where we have used the fact that the optimal value for t_0 is $\sigma_n \|\mathbf{g}\|$. The constraint in (16c) represents a set of K bilinear matrix inequalities and hence the optimization problem in (16) is NP hard [15]. However, given initial values for $\mathbf{P}$, $\mathbf{B}$ and $\mathbf{g}$, one can find a locally optimal solution by iteratively optimizing over $\mathbf{P}$ and $\mathbf{B}$ for fixed $\mathbf{g}$, and over $\mathbf{g}$ and $\mathbf{B}$ for fixed $\mathbf{P}$. Each of those problems is implicit in (16) and is a convex conic program that can be efficiently solved; e.g., [16]. One natural choice of the starting point for this iterative design would be the transceiver designed for the case in which the estimates $\hat{\mathbf{h}}_k$ are assumed to be the actual channels; e.g., [6].

The formulation in (13) employs a simple constraint on the transmitted power. However, other types of power constraints can be incorporated into the robust minimax transceiver design without compromising the convex conic nature of the steps in the proposed iterative algorithm. In particular, one can impose constraints on the power transmitted by each antenna, per-cell power constraints for distributed antenna arrays, and spatial masking constraints, see [17] for details.

4.1 Multiple Intersecting Uncertainties for Each User

The problem formulation in (13) can be generalized to the case in which the uncertainty region $\tilde{\mathscr{U}}_k$ for each $\mathbf{h}_k$ is described as the intersection of more than one uncertainty set of the form (11); cf. (12). In that case, the problem is at least as hard as the case of a single uncertainty set (the special case of (12) when $L = 1$). In particular, in the general case when $\mathscr{U}_k$ is replaced by $\tilde{\mathscr{U}}_k$ it is not possible to characterize the infinite set of constraints of the form in (14b) by a polynomial (in N_t) number of constraints [14]. Therefore, the number of constraints in the subproblems in an iterative local optimization algorithm analogous to to that described above for the problem in (16) grows faster than any polynomial in N_t. As a result, each of these subproblems is NP-hard, even though they remain convex. However, by adopting a conservative approach one can obtain an efficiently-solvable approximation to the problem with the uncertainty set in (12). This conservative approach involves enveloping (12) in a superset that can be described more efficiently, and then minimizing the maximum MSE in this superset. Using the superset characterization in [14] of sets of the form (12), it can be shown that the solution of robust transceiver design problem in (13) for the intersection of uncertainty sets in (12) is upper-bounded by the solution of the following optimization problem

$$\min_{\substack{\mathbf{B},\mathbf{P},\mathbf{g},\\ t,\mu_k^\ell,\alpha}} \quad \alpha \tag{17a}$$

$$\text{s.t.} \quad \left\| \begin{bmatrix} \sigma_n \mathbf{g} \\ t \end{bmatrix} \right\|^2 \leq \alpha, \tag{17b}$$

$$\begin{bmatrix} t_k - \sum_\ell \mu_k^\ell & \mathbf{0} & \mathbf{a}_k \\ \mathbf{0} & \sum_\ell \mu_k^\ell \mathbf{Q}_k^\ell & \delta_k(g_k \mathbf{\Phi}_k \mathbf{P}) \\ \mathbf{a}_k{}^H & \delta_k(g_k \mathbf{\Phi}_k \mathbf{P})^H & t_k \mathbf{I} \end{bmatrix} \geq \mathbf{0}, \tag{17c}$$

$$B_{ij} = 0 \quad 1 \leq i \leq j \leq K, \tag{17d}$$

$$\|\text{vec}(\mathbf{P})\|^2 \leq P_{\text{total}}, \tag{17e}$$

Similar to (16), a local optimal solution can be found by employing an alternative optimization algorithm that optimizes over $\mathbf{P}$ and $\mathbf{B}$ for fixed $\mathbf{g}$, and over $\mathbf{g}$ and $\mathbf{B}$ for fixed $\mathbf{P}$. In this conservative approach, those (convex) problems can be efficiently solved.

5. UPLINK MINIMAX ROBUST DESIGNS

The proposed design framework for minimax robust transceivers for the downlink is quite general and can be applied to uplink systems as well. In this section we will provide explicit formulations of the minimax robust designs for the dual MAC.

To derive the robust minimax design, we first observe that the MSE expression for the k^{th} user in the uplink is function is a function of all channels, not just its own. While these multiple sources of uncertainty can complicate the design, one can write the total MSE as

$$\text{MSE}^{\text{MAC}} = \sum_{k=1}^{K} \|\mathbf{G}^{\text{MAC}} \mathbf{h}_k^H p_k^{\text{MAC}} - \mathbf{m}_k^H - (\mathbf{b}_k^{\text{MAC}})^H\|^2 + \sigma_n^2 \text{tr}\{(\mathbf{G}^{\text{MAC}})^H \mathbf{G}^{\text{MAC}}\}, \tag{18}$$

where each term of the summation is subject to uncertainty from one source only. Using (18) and the analysis in Section 4, the uplink robust minimax design can be formulated as

$$\min_{\substack{\mathbf{B}^{\text{MAC}},\mathbf{G}^{\text{MAC}},\\ \mathbf{p}^{\text{MAC}},t,\mu,\beta}} \quad \beta \tag{19a}$$

subject to (19b)

$$\left\| \begin{bmatrix} \sigma_n \text{vec}(\mathbf{G}^{\text{MAC}}) \\ t \end{bmatrix} \right\|^2 \leq \beta, \tag{19c}$$

$$\begin{bmatrix} t_k - \mu_k & \mathbf{0} & (\mathbf{a}_k^{\text{MAC}})^H \\ \mathbf{0} & \mu_k \mathbf{Q}_k & \delta_k(p_k^{\text{MAC}} \mathbf{\Phi}_k \mathbf{G}^{\text{MAC}}) \\ \mathbf{a}_k^{\text{MAC}} & \delta_k(p_k^{\text{MAC}} \mathbf{\Phi}_k \mathbf{G}^{\text{MAC}})^H & t_k \mathbf{I} \end{bmatrix} \geq \mathbf{0}, \tag{19d}$$

$$B_{ij} = 0 \quad 1 \leq j \leq i \leq K, \tag{19e}$$

$$\|\mathbf{p}^{\text{MAC}}\|^2 \leq P_{\text{total}}, \tag{19f}$$

where $\mathbf{a}_k^{\text{MAC}} = \mathbf{G}^{\text{MAC}} \mathbf{h}_k^H p_k^{\text{MAC}} - \mathbf{m}_k^H - (\mathbf{b}_k^{\text{MAC}})^H$, and $\text{vec}(\cdot)$ is the operator that stacks the columns of matrix sequentially to produce a vector. Similarly, conservative formulations can be obtained for the robust uplink designs with multiple intersecting uncertainty sets for each channel. As was the case with the downlink, both problems are NP-hard, but one can employ a local iterative algorithm in which a convex conic program is solved at each iteration. In the formulation in (19), the power constraint is a constraint on the total power transmitted by the uses; cf. (19f). This constraint can be replaced by individual power constraints of the form $|p_k^{\text{MAC}}|^2 \leq P_{\text{total-k}}$ without disturbing the convex structure of the problem.

6. SIMULATION STUDIES

To compare the performance of the proposed designs with other existing approaches, we have simulated QPSK transmission over independent Rayleigh fading channels . We considered downlink scenarios with $N_t = 4$ and $K = 4$ users. The coefficients of the channel matrix $\mathbf{H}$ were modeled as being independent circularly symmetric complex Gaussian random variables with zero mean, and the performance was evaluated in terms of the average bit error rate (BER) over all users against the signal-to-noise-ratio, which is defined as $\text{SNR} = P_{\text{total}}/(K\sigma_n^2)$. All TH precoding strategies assume a given ordering of the users. Since finding an optimal ordering will involve an exhaustive search over $K!$ possible arrangements, a suboptimal ordering is usually employed. We chose the suboptimal ordering proposed for MMSE Tomlinson-Harashima transceiver design in [5], using the transmitter's channel estimate $\hat{\mathbf{H}}$. This ordering was used for all methods, including the proposed robust transceiver. We considered systems that use feedback to provide the transmitter with quantized version of the CSI, and we assumed that all K users employ the same vector quantization codebooks. In these feedback systems, the information available to the transmitter will include the users' codebooks and the statistics of the error resulting from the use of these codebooks. Since we assume that each user's channel is independent from the others, the transmitter can model the error matrix $\mathbf{E}$ as being zero mean with independent rows $\mathbf{e}_k$ and second order statistics given by $\text{E}\{\mathbf{E}\mathbf{E}^H\} = \varepsilon^2\mathbf{I}$. Thus, we have $\|\text{E}\{\mathbf{E}\mathbf{E}^H\}\| = \varepsilon^2$. To simulate quantization errors, we will generate matrices $\mathbf{E}$ such that the elements are independent and uniformly distributed such that $\text{E}\{\mathbf{E}\mathbf{E}^H\} = \varepsilon^2\mathbf{I}$. We will consider vector quantization schemes in which the transmitter employs a robust THP transceiver designed using spherical uncertainty regions $\|\mathbf{e}_k\| \leq \delta_k$. To estimate δ_k, we observe that an appropriate estimate of $\|\mathbf{E}\|$ can be ε, and since $\|\mathbf{E}\| \leq \sqrt{\sum_k \mathbf{e}_k^2}$, one can choose $\delta_k = \varepsilon/\sqrt{K}$.

In the first experiment, we compare the performance of the robust minimax Tomlinson-Harashima transceiver proposed in Section 4 with that of the zero-forcing Tomlinson-Harashima transceiver design (ZF-THP) in [3, 4], and the MMSE Tomlinson-Harashima transceiver design (MMSE-THP) in [5]. In Fig. 4, the performance of each method is plotted for values of $\varepsilon^2 = 0.03, 0.05$. It can be seen that the performance of the downlink with interference pre-subtraction is rather sensitive to the mismatch between the actual CSI and the transmitter's estimate of CSI. It can be also seen that while the effect of noise is dominant at low SNR, the channel uncertainty dominates at high SNR, where the proposed robust transceiver design performs significantly better than the other two approaches. Fig. 4 also shows that in the presence of channel uncertainty, both the ZF-THP and MMSE-THP designs have the same performance limit at high SNR. This is due to the fact that the MMSE method involves the addition of a regularization term whose value is inversely proportional to $P_{\text{total}}/(K\sigma_n^2)$; see [5].

In the second experiment, we simulate a scenario with two different sets of users' locations from the base station. The first two users are assumed to be close to the base station and their channel coefficients are generated using the above model but with variances equal to 10. The other two users are assumed to be farther from the base stations and their channel coefficients are generated using unit variance. We plot the average BER of all users in addition to the average BER of the two near users and the far users for value of $\varepsilon^2 = 0.1$. It can be seen from Fig. 5 that the advantage offered by using a robust design is even more significant in the case of the near users.

7. CONCLUSION

We have presented robust minimax designs for broadcast channels that employ Tomlinson-Harashima transceivers. The robust designs are based on a general uncertainty model that embraces different uncertainty regions. We also showed that the robust designs can be extended to multiple access channels transceivers that employ deci-

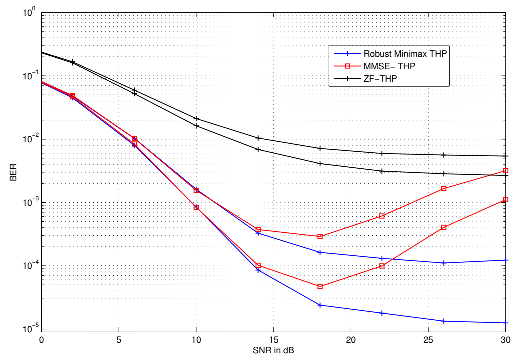

Figure 4: Comparison between the performance of the proposed robust minimax Tomlinson-Harashima transceiver, zero-forcing Tomlinson-Harashima transceiver design (ZF-THP) in [3, 4], and the MMSE Tomlinson-Harashima transceiver design (MMSE-THP) in [5] for values of channel uncertainty $\varepsilon^2 = 0.03, 0.05$ for a system with $N_t = 4$ and $K = 4$ using QPSK signalling. The upper performance curve of each method corresponds to channel uncertainty $\varepsilon^2 = 0.05$

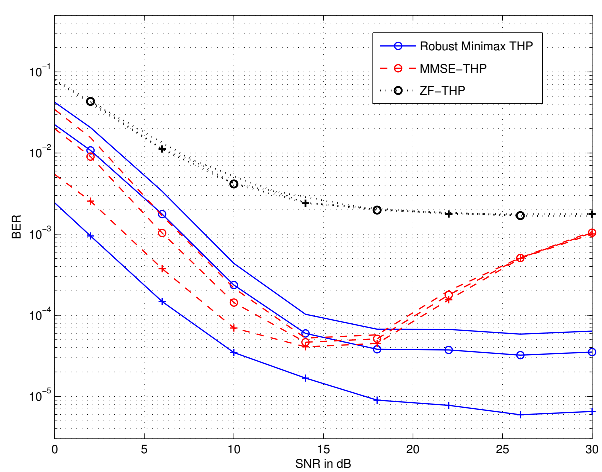

Figure 5: Comparison between the performance of the proposed robust minimax Tomlinson-Harashima transceiver, zero-forcing Tomlinson-Harashima transceiver design (ZF-THP) in [3, 4], and the MMSE Tomlinson-Harashima transceiver design (MMSE-THP) in [5] for values of channel uncertainty $\varepsilon^2 = 0.1$ for a system with $N_t = 4$ and $K = 4$ using QPSK signalling. The curves with (+) markers and no markers represent the average BER of the two near and the two far users, respectively.

sion feedback equalization. Simulation results showed that in presence of channel uncertainty, the proposed robust approaches can result in significant improvement in the performance of THP-based transceivers for broadcast channels.

REFERENCES

[1] C. Windpassinger, R. F. H. Fischer, T. Vencel, and J. B. Huber, "Precoding in multiantenna and multiuser communications," *IEEE Trans. Wireless Commun.*, vol. 3, no. 4, pp. 1305–1316, Jul. 2004.

[2] R. F. H. Fischer, C. Windpassinger, A. Lampe, and J. B. Huber, "MIMO precoding for decentralized receivers," in *Proc. Int. Symp. Inform. Theory*, Lausanne, Switzerland, Jul. 2002, p. 496.

[3] M. Joham, J. Brehmer, A. Voulgarelis, and W. Utschick, "Multiuser spatio-temporal Tomlinson-Harashima precoding for frequency selective vector channels," in *Proc. ITG Wkshp Smart Antennas*, Munich, Mar. 2004, pp. 208–215.

[4] R. Hunger, M. Joham, and W. Utschick, "Extension of linear and nonlinear transmit filters for decentralized receivers," in *Proc. European Wireless Conf.*, Nicosia, April 2005, pp. 40–46.

[5] M. Joham, J. Brehmer, and W. Utschick, "MMSE approaches to multiuser spatio-temporal Tomlinson-Harashima precoding," in *Proc. 5th Int. ITG Conf. Source and Channel Coding*, Erlangen, Jan. 2004, pp. 387–394.

[6] M. Schubert and S. Shi, "MMSE transmit optimization with interference pre-compensation," in *Proc. IEEE Veh. Technol. Conf.*, May-1 June 2005, vol. 2, pp. 845–849.

[7] N. Jindal, "MIMO broadcast channels with finite rate feedback," *IEEE Trans. Inform. Theory*, vol. 52, no. 11, pp. 5045–5059, Nov. 2006.

[8] P. Ding, D. J. Love, and M. D. Zoltowski, "Multiple antenna broadcast channels with shape feedback and limited feedback," *IEEE Trans. Signal Processing*, vol. 55, pp. 3417–3428, July 2007.

[9] W. Shi and R. Wesel, "The effect of mismatch on decision-feedback equalization and Tomlinson-Harashima precoding," in *Proc. Asilomar Conf. Signals, Syst., Comput.*, 1998.

[10] R. F. H. Fischer, *Precoding and Signal Shaping for Digital Transmission*, Wiley, New York, 2002.

[11] O. Simeone, Y. Bar-Ness, and U. Spagnolini, "Linear and nonlinear preequalization/equalization for MIMO systems with long-term channel state information at the transmitter," *IEEE Trans. Wireless Commun.*, vol. 3, no. 2, pp. 373–378, Mar. 2004.

[12] W. Yu, "Uplink-downlink duality via minimax duality," *IEEE Trans. Inform. Theory*, vol. 52, no. 2, pp. 361–374, 2006.

[13] M. Botros Shenouda and T. N. Davidson, "On the design of robust non-linear multi-user transceivers," In preparation.

[14] A. Ben-Tal, A. Nemirovski, and C. Roos, "Robust solutions of uncertain quadratic and conic-quadratic problems," *SIAM J. Optim.*, vol. 13, no. 2, pp. 535–560, Oct. 2002.

[15] O. Toker and H. Ozbay, "On the NP-hardness of solving bilinear matrix inequalities and simultaneous stabilization with static output feedback," in *Proc. IEEE American Control Conf.*, Seattle, 1995, pp. 2525–2526.

[16] J. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optim. Methods Softw.*, vol. 11, pp. 625–653, 1999.

[17] M. Botros Shenouda and T. N. Davidson, "Linear multiuser transceivers: Robustness via worst scenario MSE approach," *Proc. IEEE WCNC*, Las Vegas, April 2008.