COMP ENG 4TL4 – Digital Signal Processing

Homework Assignment #2

**Submission deadline:** 12 noon on Friday, October 17, 2003, in the designated drop box in CRL-101B (the CRL photocopying room).

1. The impulse response of an LTI system is shown in Fig. 1 below. Determine and carefully sketch the response of this system to the input $x[n] = u[n-4]$. **(10 pts)**

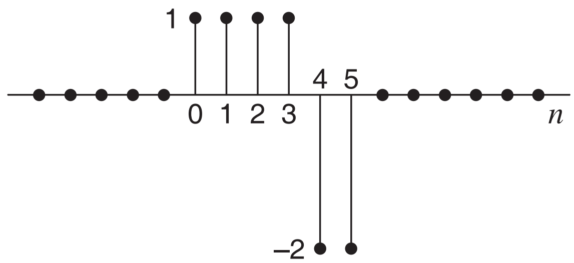


Figure 1: Impulse response $h[n]$ for Problem #1.

2. Consider the composite system shown in Fig. 2 below.
   a. Find the impulse response $h[n]$ of the overall system.
   b. Find the frequency response of the overall system. (Note: Tables of Fourier transform properties and Fourier transform pairs are given on the last page.)
   c. Specify a difference equation that relates the output $y[n]$ to the input $x[n]$.
   d. Is this system causal? Under what conditions would the system be stable? **(40 pts)**

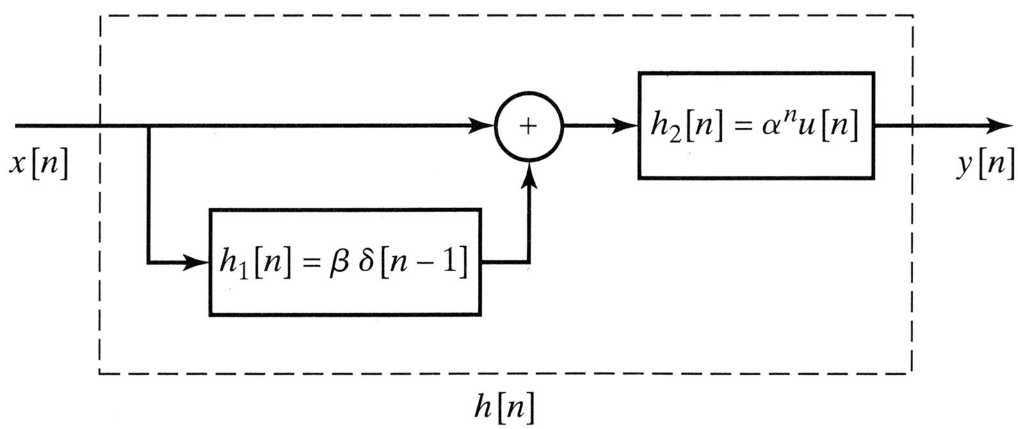


Figure 2: Composite system for Problem #2.

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3. A discrete-time sequence $x[n]$ has the Fourier transform $X(e^{j\omega})$ shown below in Fig. 3.

   a. If $x[n]$ is downsampled by a factor $M = 3$ to produce the sequence $x_d[n]$, sketch and label (with specific frequencies, amplitudes, etc.) the Fourier transform of the downsampled sequence $X_d(e^{j\omega})$ if $\omega_H = \pi/2$.

   b. What is the maximum value of $\omega_H$ that will avoid aliasing if $M = 4$? **(20 pts)**

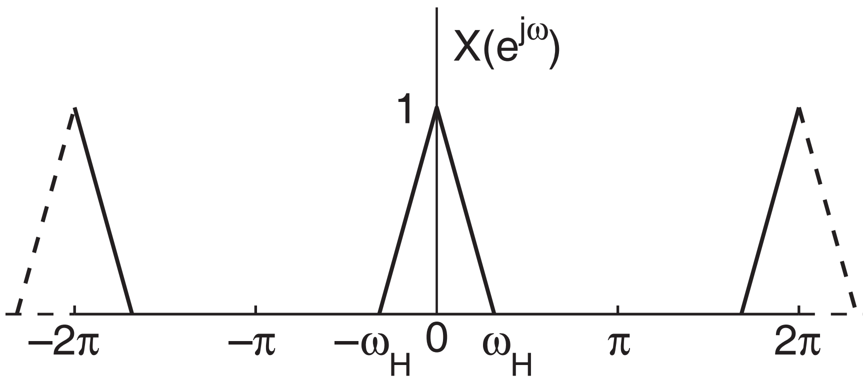


Figure 3: DTFT $X(e^{j\omega})$ for Problem #3.

4. A discrete-time speech waveform originally sampled at 16 kHz is to be added to a music track sampled at 44.1 kHz. The resampling system shown in Fig. 4 below is to be used to upsample the discrete-time speech sequence to 44.1 kHz.

   a. What are the minimum values of $L$ and $M$ that can be used, under the constraint that they are both integers?

   b. What gain and cutoff frequency are required for the discrete-time lowpass filter? **(10 pts)**

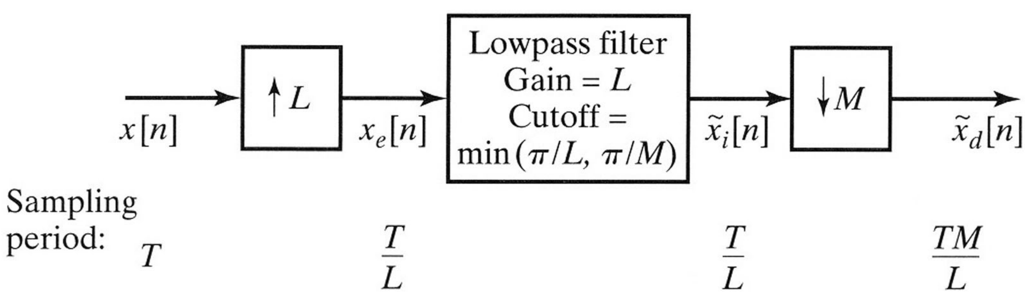


Figure 4: Resampling system for Problem #4.

5. An LTI system has the impulse response $h[n] = 4(-1/3)^n u[n]$. Use the Fourier transform to find the output of this system when the input is $x[n] = (1/2)^n\, u[n]$. (Note: Tables of Fourier transform properties and Fourier transform pairs are given on the last page.) **(20 pts)**

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**TABLE 2.2** FOURIER TRANSFORM THEOREMS

| Sequence<br>$x[n]$<br>$y[n]$ | Fourier Transform<br>$X(e^{j\omega})$<br>$Y(e^{j\omega})$ |
|---|---|
| 1. $ax[n] + by[n]$ | $aX(e^{j\omega}) + bY(e^{j\omega})$ |
| 2. $x[n - n_d]$ ($n_d$ an integer) | $e^{-j\omega n_d} X(e^{j\omega})$ |
| 3. $e^{j\omega_0 n} x[n]$ | $X(e^{j(\omega-\omega_0)})$ |
| 4. $x[-n]$ | $X(e^{-j\omega})$<br>$X^*(e^{j\omega})$ if $x[n]$ real. |
| 5. $nx[n]$ | $j\dfrac{dX(e^{j\omega})}{d\omega}$ |
| 6. $x[n] * y[n]$ | $X(e^{j\omega})Y(e^{j\omega})$ |
| 7. $x[n]y[n]$ | $\dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$ |

Parseval's theorem:

8. $$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

9. $$\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$$

**TABLE 2.3** FOURIER TRANSFORM PAIRS

| Sequence | Fourier Transform |
|---|---|
| 1. $\delta[n]$ | $1$ |
| 2. $\delta[n - n_0]$ | $e^{-j\omega n_0}$ |
| 3. $1$ $(-\infty < n < \infty)$ | $\displaystyle\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$ |
| 4. $a^n u[n]$ $(\lvert a\rvert < 1)$ | $\dfrac{1}{1 - ae^{-j\omega}}$ |
| 5. $u[n]$ | $\dfrac{1}{1 - e^{-j\omega}} + \displaystyle\sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$ |
| 6. $(n+1)a^n u[n]$ $(\lvert a\rvert < 1)$ | $\dfrac{1}{(1 - ae^{-j\omega})^2}$ |
| 7. $\dfrac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n]$ $(\lvert r\rvert < 1)$ | $\dfrac{1}{1 - 2r\cos\omega_p e^{-j\omega} + r^2 e^{-j2\omega}}$ |
| 8. $\dfrac{\sin\omega_c n}{\pi n}$ | $X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c, \\ 0, & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$ |
| 9. $x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$ | $\dfrac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$ |
| 10. $e^{j\omega_0 n}$ | $\displaystyle\sum_{k=-\infty}^{\omega} 2\pi\delta(\omega - \omega_0 + 2\pi k)$ |
| 11. $\cos(\omega_0 n + \phi)$ | $\displaystyle\sum_{k=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi}\delta(\omega + \omega_0 + 2\pi k)]$ |