COMP ENG 4TL4 – Digital Signal Processing

Homework Assignment #3

Submission deadline: 12 noon on Friday, October 31, 2003, in the designated drop box in CRL-101B (the CRL photocopying room).

1. Consider an LTI system that is stable and for which $H(z)$, the *z*-transform of the impulse response is given by:

$$H(z) = \frac{3}{1+\frac{1}{3}z^{-1}}.$$

   Suppose $x[n]$, the input to the system, is the unit step sequence $u[n]$.

   a. Find the output $y[n]$ by directly evaluating the discrete convolution of $x[n]$ and $h[n]$ in the time domain.

   b. Find the output $y[n]$ by calculating the inverse *z*-transform of $Y(z)$. **(20 pts)**

2. Sketch each of the following sequences and determine their *z*-transforms, including the region of convergence (ROC).

   a. $\sum_{k=-\infty}^{\infty} \delta[n-4k]$

   b. $\frac{1}{2}\left[e^{j\pi n} + \cos\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2} + 2\pi n\right)\right]u[n]$ **(20 pts)**

3. When the input to an LTI system is:

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n-1],$$

   the output is:

$$y[n] = 6\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{3}{4}\right)^n u[n].$$

   a. Find the transfer function $H(z)$ of the system. Plot the poles and zeros of $H(z)$ and indicate the ROC.

   b. Find the impulse response $h[n]$ of the system.

   c. Write the LCCD equation that characterizes the system.

   d. Is the system stable? Is it causal? **(40 pts)**

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4. Consider a real finite-length sequence $x[n]$ with Fourier transform $X\left(e^{j\omega}\right)$ and DFT $X[k]$. If:

$$\mathcal{Im}\{X[k]\}=0, \quad k=0,1,\ldots N-1,$$

can we conclude that:

$$\mathcal{Im}\left\{X\left(e^{j\omega}\right)\right\}=0, \quad -\pi \leq \omega \leq \pi ?$$

State your reasoning if your answer is yes. Give a counterexample if your answer is no. **(10 pts)**

5. Two finite-length sequences $x_1[n]$ and $x_2[n]$ are shown in the figure below. Sketch their N-point circular convolution for:

a. $N = 6$, and

b. $N = 10$. **(10 pts)**

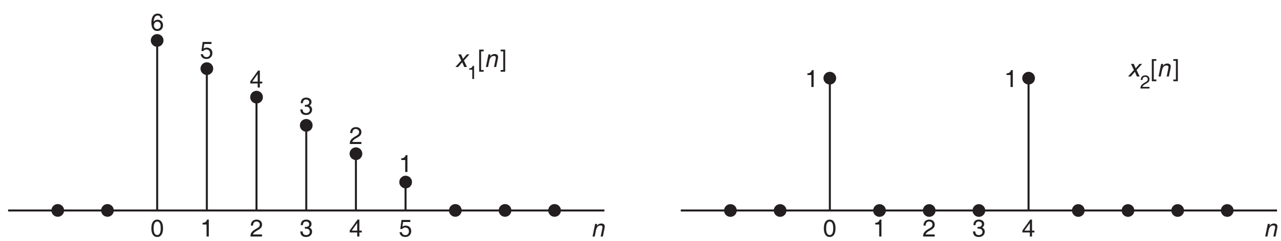

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TABLE 3.1 SOME COMMON z-TRANSFORM PAIRS

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
3. $-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
6. $-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
7. $na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
8. $-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
9. $[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\|z\| > 1$
10. $[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\|z\| > 1$
11. $[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\|z\| > r$
12. $[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\|z\| > r$
13. $\begin{cases} a^n, & 0 \le n \le N-1, \\ 0, & \text{otherwise} \end{cases}$	$\frac{1-a^Nz^{-N}}{1-az^{-1}}$	$\|z\| > 0$

TABLE 3.2 SOME z-TRANSFORM PROPERTIES

Property Number	Section Reference	Sequence	Transform	ROC
		$x[n]$	$X(z)$	R_x
		$x_1[n]$	$X_1(z)$	R_{x_1}
		$x_2[n]$	$X_2(z)$	R_{x_2}
1	3.4.1	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$	Contains $R_{x_1} \cap R_{x_2}$
2	3.4.2	$x[n-n_0]$	$z^{-n_0}X(z)$	R_x, except for the possible addition or deletion of the origin or ∞
3	3.4.3	$z_0^n x[n]$	$X(z/z_0)$	$\|z_0\|R_x$
4	3.4.4	$nx[n]$	$-z\frac{dX(z)}{dz}$	R_x, except for the possible addition or deletion of the origin or ∞
5	3.4.5	$x^*[n]$	$X^*(z^*)$	R_x
6		$\mathcal{R}e\{x[n]\}$	$\frac{1}{2}[X(z)+X^*(z^*)]$	Contains R_x
7		$\mathcal{J}m\{x[n]\}$	$\frac{1}{2j}[X(z)-X^*(z^*)]$	Contains R_x
8	3.4.6	$x^*[-n]$	$X^*(1/z^*)$	$1/R_x$
9	3.4.7	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	Contains $R_{x_1} \cap R_{x_2}$
10	3.4.8	Initial-value theorem:		
		$x[n]=0, \quad n<0$	$\lim_{z\to\infty} X(z) = x[0]$	