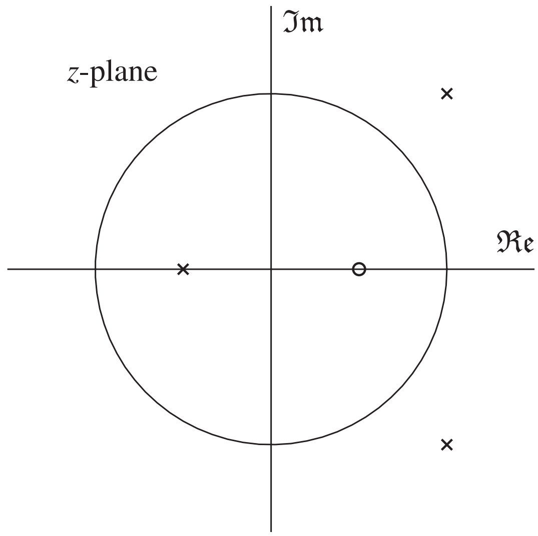
COMP ENG 4TL4 – Digital Signal Processing

Homework Assignment #4

Submission deadline: 12 noon on Friday, November 21, 2003, in the designated drop box in CRL-101B (the CRL photocopying room).

1. The figure below shows the pole-zero plot for a causal LTI system with a real-valued impulse response. Indicate which of the following properties apply to this system, justifying each answer:
   i. stable
   ii. FIR
   iii. minimum phase
   iv. all-pass
   v. generalized linear phase **(20 pts)**



2. A causal LTI system has the transfer function:

$$H(z) = \frac{\left(1-0.5z^{-1}\right)\left(1+4z^{-2}\right)}{1-0.64z^{-2}}.$$

   a. Find transfer functions for a *minimum-phase* system $H_1(z)$ and an *all-pass* system $H_{\text{ap}}(z)$ such that:

$$H(z) = H_1(z)H_{\text{ap}}(z).$$

   b. Sketch the pole-zero plots of $H(z)$, $H_1(z)$ and $H_{\text{ap}}(z)$, indicating their ROCs. **(20 pts)**

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3. Let $h_{\mathrm{lp}}[n]$ denote the impulse response of an FIR generalized linear-phase *lowpass* filter. The impulse response $h_{\mathrm{hp}}[n]$ of an FIR generalized linear-phase *highpass* filter can be obtained by the transformation:

$$h_{hp}[n] = (-1)^n h_{lp}[n].$$

If we wish $h_{\mathrm{hp}}[n]$ to be symmetric or antisymmetric, could we use a Type IV FIR generalized linear-phase filter for $h_{\mathrm{lp}}[n]$? Justify your answer. **(20 pts)**

4. Use the *bilinear transformation* IIR filter design method to design a discrete-time 2nd-order lowpass Butterworth filter with cutoff frequency $\omega_c = \pi/4$ radians, assuming a sampling frequency $f_s = 4$ kHz.

   a. Give details of each step of the design procedure and give the analog and digital filter transfer functions $H_c(s)$ and $H(z)$, respectively, making sure that you simplify your expression for $H(z)$ so that its numerator and denominator are either (i) products of factors in terms of the explicit poles and zeros of $H(z)$ or (ii) polynomials in descending negative powers of z.

   b. Does the assumed sampling frequency of $f_s = 4$ kHz have any effect on your expression for $H(z)$? Why or why not? **(20 pts)**

5. A causal LTI system has the transfer function:

$$H(z) = \frac{1 - \frac{1}{4}z^{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}.$$

Draw the block diagrams for this filter implemented in:

   a. direct form II (canonical form), and

   b. parallel form with 1st-order subsystems. **(20 pts)**