COMP ENG 4TL4 – DIGITAL SIGNAL PROCESSING

Lab #3: Resampling, Reconstruction, Aliasing and the DFT

Objective:

To gain experience in resampling, reconstructing and analyzing digital signals within MATLAB. In particular, you will look at resampling, reconstruction and aliasing of 2-dimensional images and acoustic waveforms, and spectral analysis using the discrete Fourier transform (DFT).

Assessment:

- Your grade for this lab will be based on your ability to create and work with digital systems within MATLAB, and on your reporting of the results. The report should contain any mathematical calculations or derivations carried out, MATLAB plots of results with brief descriptions, the MATLAB code with which you obtained your results, and answers to specific questions below.
- Clearly label all plots and their axes (points for style will be deducted otherwise)
- DON'T COPY OTHERS BLINDLY!!
- Please attend the lab section to which you have been assigned.
- You should complete this lab with one lab partner. If there are an odd number of students, then one group of three will be created by the TA.
- Each pair of students should complete one lab report together, which is to be submitted one week from the date of the lab. Those students in the "At Home" section must submit their reports one week from the day on which they demonstrated their lab to Jeff Bondy.

Pre-lab:

- Carefully read through this lab description, so that you know what is required.
- Read through the lecture notes (and bring them with you) so that you know how to answer the questions.
- Familiarize yourself with the MATLAB commands that may be required for this lab – see the list at the end of this lab description for some hints.

1. Images, resampling and reconstruction:

So far in this course we have primarily been considering discrete-time signals and systems. However, many of the same principles apply to discrete-space signals and systems and can consequently be applied to images (2D or 3D).

The 2D image you will be using in this lab is a 256-level gray-scale portable networks graphic (.png) file taken by Jeff Bondy on a recent trip to Killarney National Park on Georgian Bay, `KillarneyPic.png`.

a. Import `KillarneyPic.png`, and report its size, bytes and class.

b. Most MATLAB functions require arrays to be of the 'double' class. Convert the imported picture into a double-class variable and display it in a MATLAB figure using the `imagesc()` function.

c. Set the colormap to `gray`. Next try a colormap of `(1-gray)`. What effect does this have on the image? Now return the colormap to `gray`.

d. You will be using this image to explore different resampling and reconstruction effects. Produce new images by resampling, and in some cases reconstructing, the original as follows:

   i. Impulsive sampling at 1/5 of the original rate (i.e., set 4 out of every 5 samples to zero);
   ii. Downsampling by a factor of 5 (i.e., discard 4 out of every 5 samples);
   iii. Zero-order hold reconstruction (back to the original rate) from the downsampled image created in part ii above; and
   iv. First-order hold reconstruction (linear interpolation—back to the original rate) from the downsampled image created in part ii above.

   This resampling and reconstruction must be done both horizontally and vertically. Since the X and Y directions are ORTHOGONAL, you can resample and reconstruct in each direction INDEPENDENTLY. Ensure that you have the expected number of rows and columns after each operation.

e. Of the 4 resampled/reconstructed images plotted in gray-scale, which one most closely resembles the original picture? Rank the images in order of FIDELITY compared to the original.

2. Introduction to the DFT:

The discrete Fourier transform (DFT) is <u>not</u> the same as the discrete-time Fourier transform from Lab #2. The main difference is in the resulting frequency resolution. When the DFT is computed for an array of length N, the frequency resolution is inherently determined. In contrast, the result of the DTFT has continuous or infinite frequency resolution and thus requires that frequencies be specified for the purpose of computation and plotting in MATLAB.

a. Create a MATLAB function for computing the DFT of an arbitrary length N input array `x`, for the set of frequency sampling points `k = 0:length(x)-1`.

   Hint: Either update one or two lines in your DTFT function to compute the DFT, or try writing a new function for the DFT that implements the matrix formulation of the DFT.

b. Use your function to compute the DFTs of the 5 rectangular signals of different lengths N:

$$x[n] = \begin{cases} 1, & 0 \le n \le 15 \\ 0, & otherwise \end{cases}$$

   with N = 16, 32, 64, 128 and 256.

c. Describe what happens to the magnitude of the output as the zero-padding at the tail end of the rectangular signal increases in length.

d. Use the function that you created in Lab #2 to compute and plot the DTFT of the same input set. What frequency vector `w` do you need to provide as input to the DTFT function to return the same outputs as the DFT?

e. Use the built-in MATLAB command for the Fast Fourier Transform `fft()` to compute the DFT for the input set. Compare the results of these three Fourier transform methods.

3. DFT Resolution:

a. Load the .wav file `aaa.wav`. This is a synthesized vowel /A/. What is the sampling frequency?

b. Plot the waveform (i.e., in time-domain) of the first 300 samples. You should see a natural repetition. What is the period of this natural repetition?

c. Take the magnitude of the DFT of one natural period and compare it with that of two natural periods. How does the spectrum change?

d. Zero-pad the same two signal pieces to 1024 points and plot the magnitude of the DFTs. From section 2 above, zero-padding should change the frequency sampling resolution. Is the frequency sampling resolution changing as you would expect?

e. Compute and plot the magnitude of the DFT of the signal windowed at 1, 2, 3, 4 and 5 times the natural period. Using your knowledge of the effects of windowing on spectral resolution, explain the change in the spectrum due to an increase in the number of samples used to compute the DFT.

4. Aliasing:

You will be using the same wavefile as section 3, `aaa.wav`.

a. Listen to the sound. Discard every second sample then duplicate the resulting array (i.e., concatenate the array at the end of itself) to make an `output.wav` which is the same length as `aaa.wav` resampled at ½ the original sampling rate. Play `output.wav`. Describe any differences that you hear.

b. Plot the magnitudes of the DFTs of `aaa.wav` and `output.wav`. Are there any similarities and/or differences in the spectrums?

c. Can you derive a prediction of the DFT of `output.wav` by shifting and combining the DFT of `aaa.wav` according to downsampling theory?

Potentially useful MATLAB commands

Note that this is not an exhaustive list! You are not required to incorporate all of these in your scripts.

help <topic>	helpwin	figure	plot	stem
hist	subplot	hold on	xlabel	ylabel
legend	title	function	clear	close
clc	imread	double	imagesc	colormap
conv	conv2	upsample	downsample	interp1q
interp2	repmat	if	for	end
size	zeros	ones	abs	sum
fft	fftshift	soundsc	wavread	wavwrite