COMP ENG 4TL4:

Digital Signal Processing

Notes for Lecture #19

Tuesday, October 21, 2003

6. DIGITAL FILTERS

6.1 What is Filtering?

Definition:

Digital filtering is just changing the frequency-domain characteristics of a given discrete-time signal.

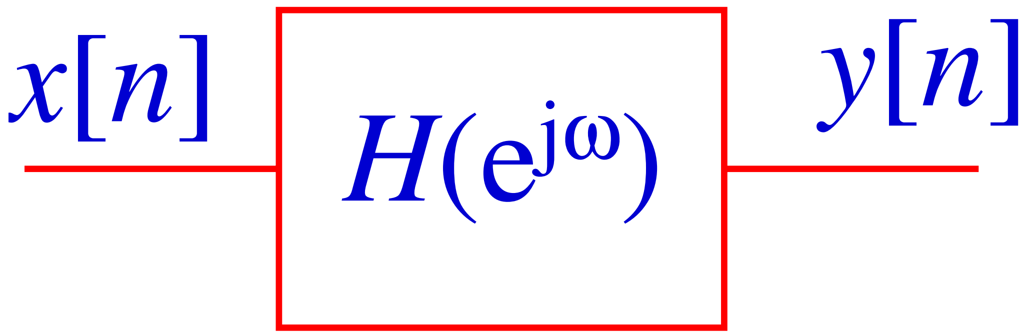
Filtering applications include:

- noise *suppression*
- *enhancement* of selected frequency ranges or edges in images
- bandwidth *limiting* (e.g., to prevent aliasing of digital signals or to reduce interference of neighboring channels in wireless communications)
- *removal* or *attenuation* of specific frequencies
- special operations like integration, differentiation, etc.

Recall causal LTI systems:

$$y[n] = \sum_{k=0}^{\infty} h[k]\, x[n-k] = x[n] * h[n]$$

$$Y\left(e^{j\omega}\right) = H\left(e^{j\omega}\right) X\left(e^{j\omega}\right)$$



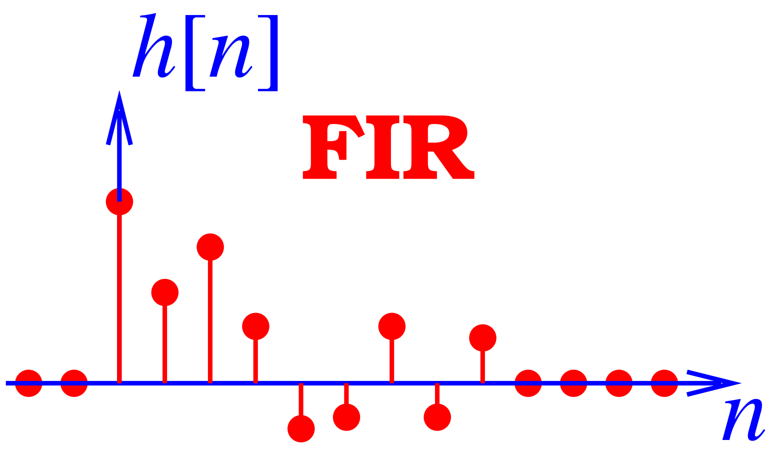
The impulse response values $\{h[0],\ h[1],\ h[2], \ldots\}$ can be interpreted as filter coefficients.

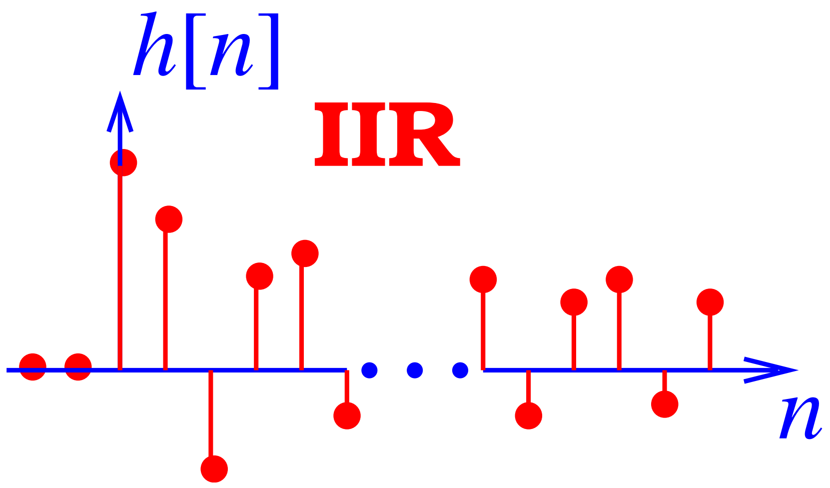
6.2 Finite and Infinite Impulse Responses

Definition:

If $h[n]$ is an *infinite duration* sequence, the corresponding filter is called an infinite impulse response (IIR) filter.

In turn, if $h[n]$ is a *finite duration* sequence, the corresponding filter is called a finite impulse response (FIR) filter.





A very general form of digital filter can be obtained from the familiar equation (recall LTI systems):

$$\sum_{k=0}^{N} a[k]\, y[n-k] = \sum_{k=0}^{M} b[k]\, x[n-k] \qquad \text{ARMA}$$

where $x[n]$ is the filter input signal and $y[n]$ is the filter output signal. As obtained in Lecture #13, the transfer function corresponding to this equation is the following rational function:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b[k]\, z^{-k}}{\sum_{k=0}^{N} a[k]\, z^{-k}} = \frac{B(z)}{A(z)}.$$

– If $N = 0$, the system is an FIR (nonrecursive) filter

– If $N > 0$, the system is an IIR (recursive) filter

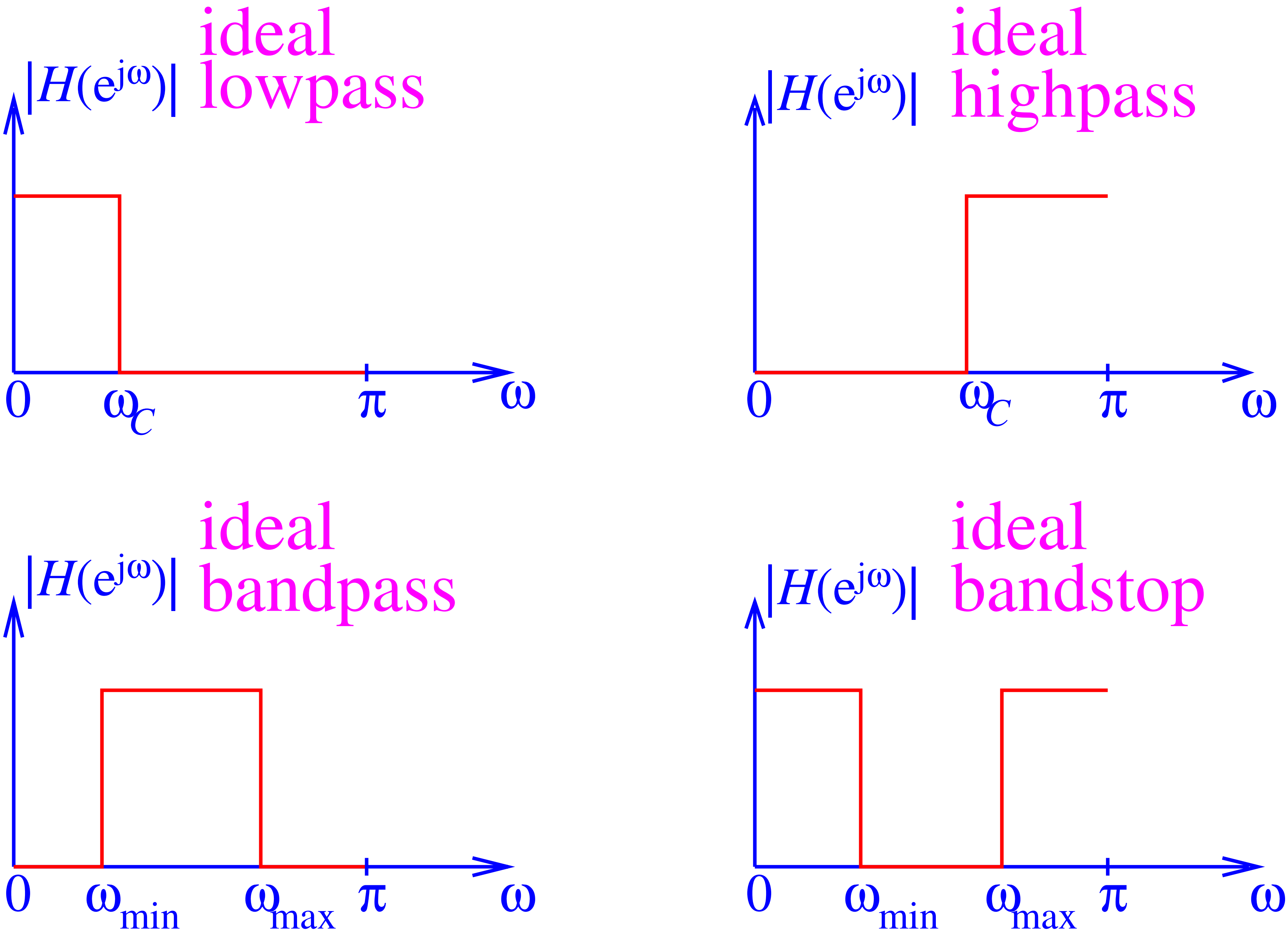
Note:

- The ARMA system equation allows us to design an IIR filter using a *finite number of filter coefficients*. For example, the (quite long) reverberant room impulse response from Lab #2 could be approximated by a substantially shorter set of IIR filter coefficients to simplify the computation of the system output.
- An IIR filter implemented via the ARMA system equation has $M+N+2$ filter coefficients, compared to $M+1$ coefficients for an FIR. Thus, an IIR filter of order $\max(M,N)$ has $N+1$ more degrees of freedom for fitting a desired frequency response than an FIR filter of order M. Consequently, an IIR filter of a particular order can have a sharper frequency response than an FIR filter of the same order.

6.3 Filter Specifications

Basic filter types:

- lowpass (LP) filters (to pass low frequencies from zero to a certain cut-off frequency ω_C and to block higher frequencies)
- highpass (HP) filters (to pass high frequencies from a certain cut-off frequency ω_C to π and to block lower frequencies)
- bandpass (BP) filters (to pass a certain frequency range $[\omega_{\mathrm{min}}, \omega_{\mathrm{max}}]$, which does not include zero, and to block other frequencies)
- bandstop (BS) filters (to block a certain frequency range $[\omega_{\mathrm{min}}, \omega_{\mathrm{max}}]$, which does not include zero, and to pass other frequencies)



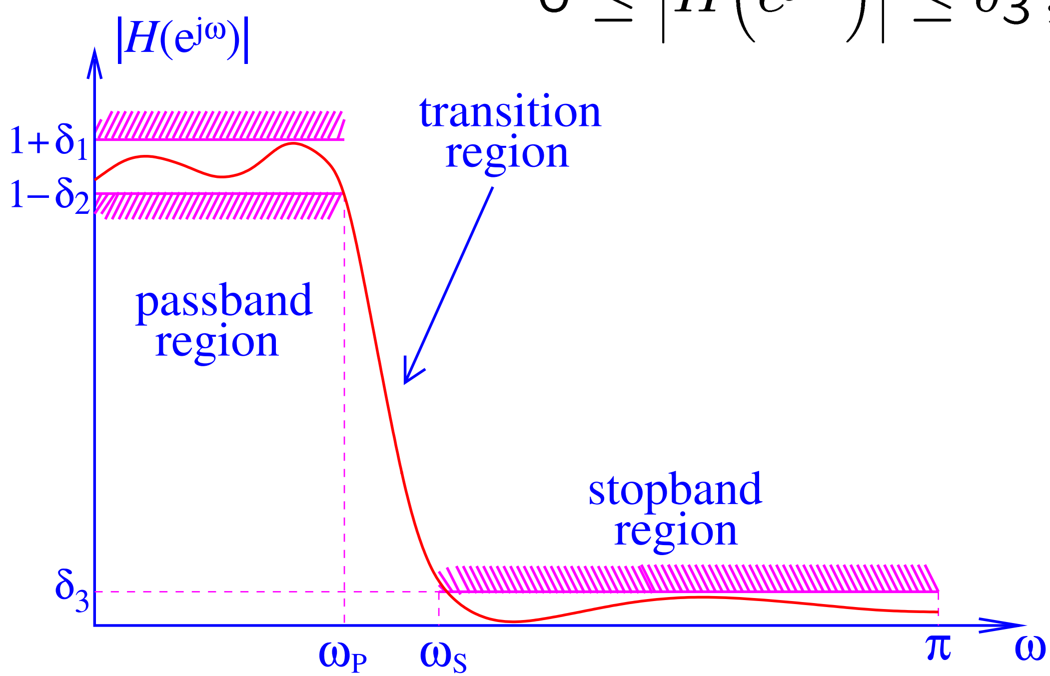
ideal
|H(e^jω)|
lowpass
0
ω_C
π
ω
ideal
|H(e^jω)|
highpass
0
ω_C
π
ω
ideal
|H(e^jω)|
bandpass
0
ω_min
ω_max
π
ω
ideal
|H(e^jω)|
bandstop
0
ω_min
ω_max
π
ω

Frequency responses of practical filters are not shaped in straight lines, i.e., they vary continuously as a function of frequency: they are neither exactly 1 in the passbands, nor exactly 0 in the stopbands.

Lowpass filter specifications:

$$1-\delta_2 \le \left|H\left(e^{j\omega}\right)\right| \le 1+\delta_1\,, \quad \omega \in [0, \omega_P];$$

$$0 \le \left|H\left(e^{j\omega}\right)\right| \le \delta_3\,, \qquad \omega \in [\omega_S, \pi]$$



Definition: The quantity $\max\{\delta_1,\delta_2\}$ is called passband (PB) ripple, and the quantity δ_3 is called stopband (SB) attenuation.

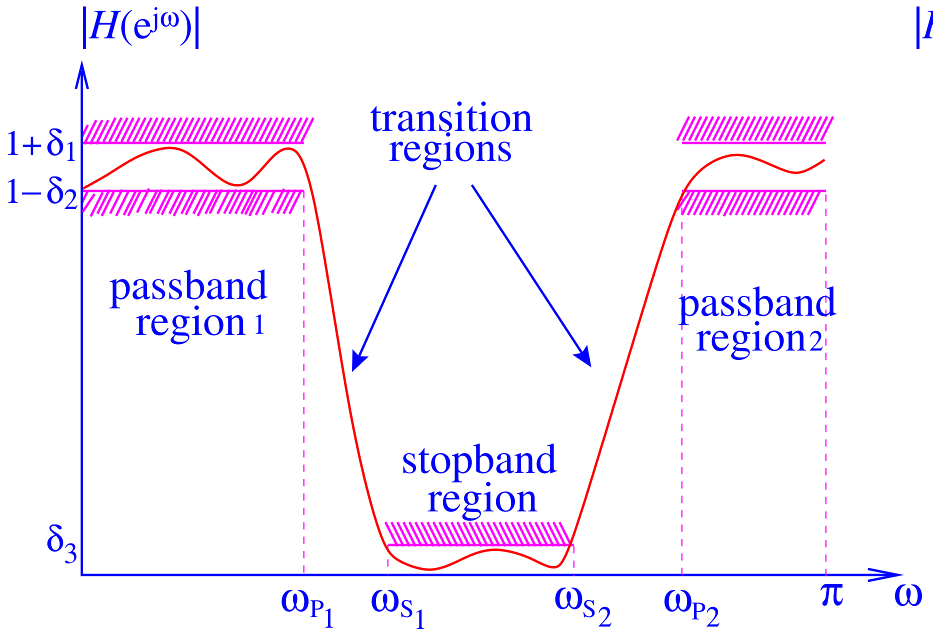
These filter parameters are usually specified in deciBels (dB):

$$A_P = \max\{20\log_{10}(1+\delta_1), -20\log_{10}(1-\delta_2)\} \leftarrow \text{PB ripple in dB}$$
$$A_S = -20\log_{10}\delta_3 \qquad \leftarrow \text{SB attenuation in dB}$$

Example: let $\delta_1 = \delta_2 = \delta_3 = 0.1 \Rightarrow$

$$A_P = \max\{0.828, 0.915\} \text{ dB} = 0.915 \text{ dB}$$
$$A_S = 20 \text{ dB}$$

BANDSTOP FILTER



BANDPASS FILTER