Computer Engineering 4TL4: Digital Signal Processing

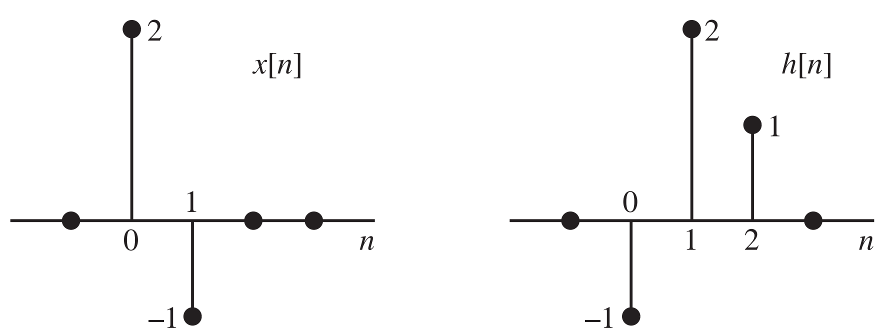
Day Class | Instructor: Dr. I. C. BRUCE
Duration of Examination: 1.5 Hours
McMaster University Midterm Examination | October, 2003

This examination paper includes seven (7) pages and six (6) questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions: Use of Casio *fx*-991 calculator only is allowed.
Each question is worth 20 points.
All six (6) questions may be answered, but the maximum total mark is capped at 100 points.
Some equations and tables that may assist you are provided on pages 4–7.

1. Consider a discrete-time linear time-invariant system with impulse response $h[n]$. Show that if the input $x[n]$ is a periodic sequence with period N (i.e., if $x[n]=x[n+N]$), then the output $y[n]$ is also a periodic sequence with period N. **(20 pts)**

2. Consider a system having the impulse response $h[n]$ and the input sequence $x[n]$ shown in the figure below.



   a. Use discrete-time *linear* convolution to find the system's response to this input.

   b. Find the 3-point *circular* convolution of $x[n]$ (for $n = 0, 1, 2$) with $h[n]$ (for $n = 0, 1, 2$).

   c. To what length N do you need to zero-pad $h[n]$ and $x[n]$ so that time-aliasing is avoided, i.e., the N-point circular convolution is identical to the linear convolution from part a? **(20 pts)**

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3. In Lab #1 you wrote a MATLAB function to implement a 3-bit uniform rounding quantizer, which did not produce a very high-fidelity output signal. Three options available to improve the quantizer *for the particular speech signal used in Lab #1* are:

   i. Increase the number of quantization levels;

   ii. Change to an optimal nonuniform quantizer; or

   iii. Pass the signal through a μ-law compander before performing the uniform quantization.

   Explain the advantages and disadvantages of each of these choices, and compare and contrast them with each other (i.e., describe the advantages and disadvantages relative to the other options). **(20 pts)**

4. A linear time-invariant system is described by the difference equation:

   $$y[n] = x[n] + 2x[n-1] + x[n-2].$$

   a. Determine $h[n]$, the impulse response of the system.

   b. Is this a stable system? Justify your answer.

   c. Determine $H(e^{j\omega})$, the frequency response of the system. Use trigonometric identities to obtain a simple expression for $H(e^{j\omega})$.

   d. Sketch the magnitude and phase responses of this system. **(20 pts)**

5. The continuous-time signal:

   $$x_c(t) = \cos(4000\pi t), \quad -\infty < t < \infty,$$

   is sampled with sampling period T to obtain the discrete-time signal:

   $$x[n] = \cos\left(\frac{\pi}{3}n\right), \qquad -\infty < n < \infty.$$

   a. Determine a choice for T that is consistent with this information.

   b. Is your choice for T unique? If so, explain why. If not, specify another choice of T that is consistent with the information given. **(20 pts)**

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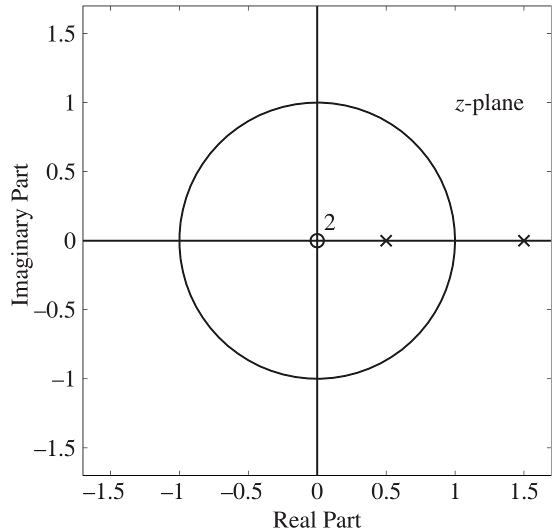
6. The z-domain transfer function $H(z)$ of a system has the pole-zero plot shown below.



a. Sketch the transfer function's region of convergence (ROC) if this system is known to be *stable*. What can you say about the system's impulse response $h[n]$ in this case?

b. Sketch the transfer function's ROC if this system is known to be *causal*. What can you say about the system's impulse response $h[n]$ in this case?

c. Sketch the transfer function's ROC if this system is known to be both *stable and causal*. What can you say about the system's impulse response $h[n]$ in this case? **(20 pts)**

THE END

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Tables and Other Information

Unit step function:

$$u[n] = \begin{cases} 0\,, & n < 0 \\ 1\,, & n \geq 0 \end{cases}$$

Unit impulse function:

$$\delta[n] = \begin{cases} 1\,, & n = 0 \\ 0\,, & n \neq 0 \end{cases}$$

Trigonometric identities:

$$e^{j(\omega n+\phi)} = \cos(\omega n + \phi) + j\sin(\omega n + \phi)$$

$$\cos(\omega n + \phi) = \frac{1}{2}\{e^{j(\omega n+\phi)} + e^{-j(\omega n+\phi)}\}$$

$$\sin(\omega n + \phi) = \frac{1}{2j}\{e^{j(\omega n+\phi)} - e^{-j(\omega n+\phi)}\}$$

Linear convolution:

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

***N*-point circular convolution:**

$$x[n] \circledast y[n] = \sum_{m=0}^{N-1} x[m]\, y[(n-m) \bmod N]$$

Autoregressive Moving Average (ARMA) difference equation:

$$\sum_{k=0}^{N} a[k]\, y[n-k] = \sum_{k=0}^{M} b[k]\, x[n-k]$$

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Discrete-Time Fourier Transform (DTFT):

$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} \qquad \text{DTFT}$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X\left(e^{j\omega}\right) e^{j\omega n}\, d\omega \qquad \text{inverse DTFT}$$

***z*–Transform:**

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n} \qquad z\text{–transform}$$

$$x[n] = \frac{1}{2\pi j} \oint X(z)\, z^{n-1}\, dz \qquad \text{inverse } z\text{–transform}$$

Discrete Fourier Transform (DFT):

$$X[k] = \sum_{n=0}^{N-1} x[n]\; e^{-j2\pi \frac{kn}{N}} \qquad \text{DFT}$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\; e^{j2\pi \frac{kn}{N}} \qquad \text{inverse DFT}$$

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TABLE 2.2 FOURIER TRANSFORM THEOREMS

Sequence $x[n]$ $y[n]$	Fourier Transform $X(e^{j\omega})$ $Y(e^{j\omega})$
1. $ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
2. $x[n - n_d]$ (n_d an integer)	$e^{-j\omega n_d} X(e^{j\omega})$
3. $e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$
4. $x[-n]$	$X(e^{-j\omega})$ $X^*(e^{j\omega})$ if $x[n]$ real.
5. $nx[n]$	$j \dfrac{dX(e^{j\omega})}{d\omega}$
6. $x[n] * y[n]$	$X(e^{j\omega}) Y(e^{j\omega})$
7. $x[n]y[n]$	$\dfrac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta}) Y(e^{j(\omega - \theta)}) d\theta$
Parseval's theorem:	
8. $\sum_{n=-\infty}^{\infty} \lvert x[n] \rvert^2 = \dfrac{1}{2\pi} \int_{-\pi}^{\pi} \lvert X(e^{j\omega}) \rvert^2 d\omega$	
9. $\sum_{n=-\infty}^{\infty} x[n] y^*[n] = \dfrac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) Y^*(e^{j\omega}) d\omega$	

TABLE 2.3 FOURIER TRANSFORM PAIRS

Sequence	Fourier Transform
1. $\delta[n]$	1
2. $\delta[n - n_0]$	$e^{-j\omega n_0}$
3. $1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi \delta(\omega + 2\pi k)$
4. $a^n u[n] \quad (\lvert a \rvert < 1)$	$\dfrac{1}{1 - ae^{-j\omega}}$
5. $u[n]$	$\dfrac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi \delta(\omega + 2\pi k)$
6. $(n + 1) a^n u[n] \quad (\lvert a \rvert < 1)$	$\dfrac{1}{(1 - ae^{-j\omega})^2}$
7. $\dfrac{r^n \sin \omega_p (n + 1)}{\sin \omega_p} u[n] \quad (\lvert r \rvert < 1)$	$\dfrac{1}{1 - 2r \cos \omega_p e^{-j\omega} + r^2 e^{-j2\omega}}$
8. $\dfrac{\sin \omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert \omega \rvert < \omega_c, \\ 0, & \omega_c < \lvert \omega \rvert \le \pi \end{cases}$
9. $x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{\sin[\omega(M + 1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
10. $e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\omega} 2\pi \delta(\omega - \omega_0 + 2\pi k)$
11. $\cos(\omega_0 n + \phi)$	$\sum_{k=-\infty}^{\infty} [\pi e^{j\phi} \delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi} \delta(\omega + \omega_0 + 2\pi k)]$

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TABLE 3.1 SOME COMMON z-TRANSFORM PAIRS

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3. $-u[-n-1]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
6. $-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
7. $na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
8. $-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
9. $[\cos\omega_0 n]u[n]$	$\dfrac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
10. $[\sin\omega_0 n]u[n]$	$\dfrac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
11. $[r^n\cos\omega_0 n]u[n]$	$\dfrac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
12. $[r^n\sin\omega_0 n]u[n]$	$\dfrac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
13. $\begin{cases} a^n, & 0 \le n \le N-1, \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$

TABLE 3.2 SOME z-TRANSFORM PROPERTIES

Property Number	Section Reference	Sequence	Transform	ROC
		$x[n]$	$X(z)$	R_x
		$x_1[n]$	$X_1(z)$	R_{x_1}
		$x_2[n]$	$X_2(z)$	R_{x_2}
1	3.4.1	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$	Contains $R_{x_1}\cap R_{x_2}$
2	3.4.2	$x[n-n_0]$	$z^{-n_0}X(z)$	R_x, except for the possible addition or deletion of the origin or ∞
3	3.4.3	$z_0^n x[n]$	$X(z/z_0)$	$\lvert z_0\rvert R_x$
4	3.4.4	$nx[n]$	$-z\dfrac{dX(z)}{dz}$	R_x, except for the possible addition or deletion of the origin or ∞
5	3.4.5	$x^*[n]$	$X^*(z^*)$	R_x
6		$\mathcal{R}e\{x[n]\}$	$\frac{1}{2}[X(z)+X^*(z^*)]$	Contains R_x
7		$\mathcal{J}m\{x[n]\}$	$\frac{1}{2j}[X(z)-X^*(z^*)]$	Contains R_x
8	3.4.6	$x^*[-n]$	$X^*(1/z^*)$	$1/R_x$
9	3.4.7	$x_1[n]*x_2[n]$	$X_1(z)X_2(z)$	Contains $R_{x_1}\cap R_{x_2}$
10	3.4.8	Initial-value theorem: $x[n]=0,\quad n<0$	$\lim_{z\to\infty} X(z)=x[0]$	