Problem 1

The system function of a stable discrete LSI system is

$$H(z) = \frac{z + 0.9}{(z + 1.1)(z - 0.95)}$$

a) Sketch the z-plane diagram showing all pole and zero locations. What is the ROC of $H(z)$?

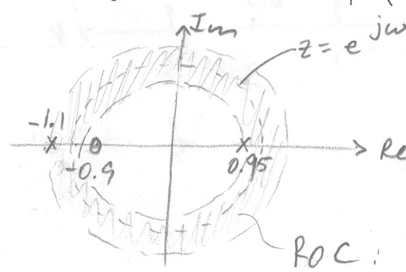
b) Determine the impulse response $h(n)$ of the system.

c) Determine the DTFT, $H(\omega)$ of the system. (Sketch approximately the magnitude $|H(\omega)|$ in the range $0 \leq \omega \leq 2\pi$)

d) Determine the 4-DFT $H(k)$ of the system and sketch $|H(k)|$.

a) zeros : $z = -0.9$

poles : $z = -1.1,\ 0.95$



ROC: $0.95 < |z| < 1.1$

↳ stable ∴ includes $|z| = 1$

b) $H(z) = \dfrac{A}{z+1.1} + \dfrac{B}{z-0.95} = \dfrac{A(z-0.95) + B(z+1.1)}{(z+1.1)(z-0.95)} = \dfrac{z+0.9}{(z+1.1)(z-0.95)}$

$Az - 0.95A + Bz + B(1.1) = z + 0.9$

$A + B = 1 \longrightarrow A + 0.82 + 0.86A = 1$

$1.1B - 0.95A = 0.9 \qquad 1.86A = 0.18$

$A = 0.097$

$B = \dfrac{0.9 + 0.95A}{1.1} = 0.82 + 0.86A$

$= 0.82 + 0.86(0.097) = 0.903$

$H(z) = \dfrac{0.097}{z+1.1} + \dfrac{0.903}{z-0.95} = \dfrac{0.097z^{-1}}{1+1.1z^{-1}} + \dfrac{0.903z^{-1}}{1-0.95z^{-1}}$

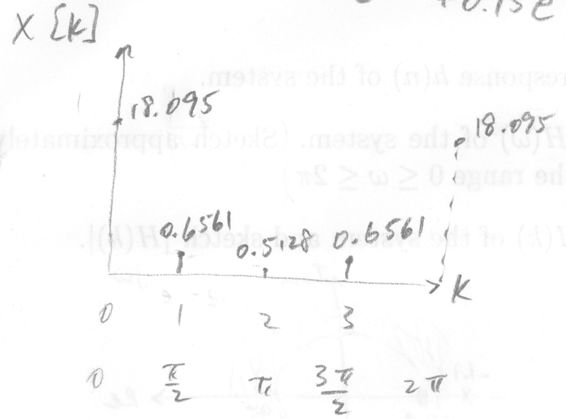
$h[n] = -0.097(-1.1)^{n-1}u(-n+1-1) + 0.903(0.95)^{n-1}u(n-1)$

c) DTFT $\longleftrightarrow$ $z = e^{j\omega}$

$$X(e^{j\omega}) = X(z)\Big|_{z=e^{j\omega}} = \frac{e^{j\omega} + 0.9}{(e^{j\omega}+1.1)(e^{j\omega}-0.95)} = \frac{e^{j\omega} + 0.9}{e^{2j\omega} + 0.15e^{j\omega} - 1.045}$$

d) $N=4 \rightarrow k = 0, 1, 2, 3$

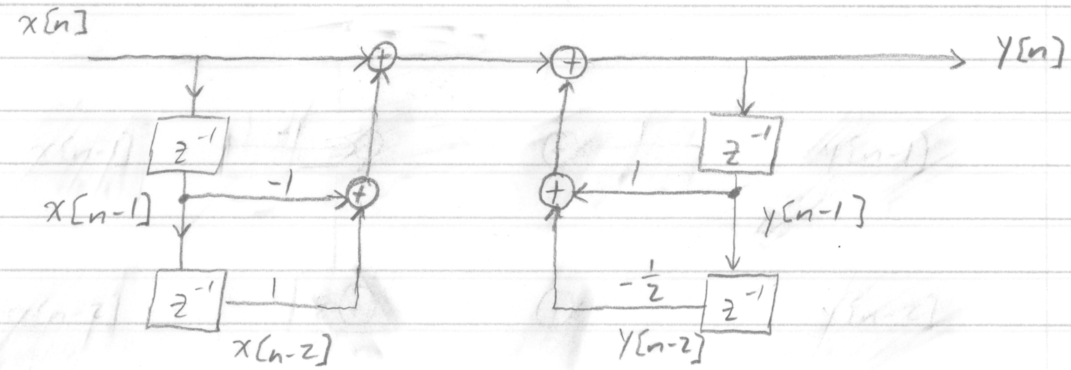
$$X[k] = X(e^{jk\frac{2\pi}{4}}) = \frac{e^{jk\frac{\pi}{2}} + 0.9}{e^{jk\pi} + 0.15e^{jk\frac{\pi}{2}} - 1.045}$$



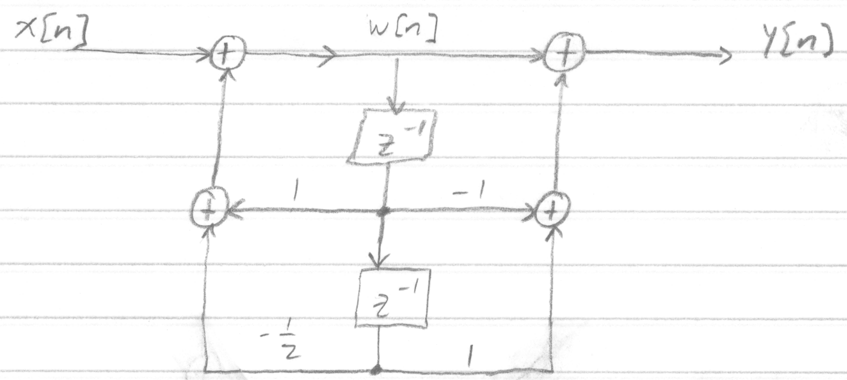
Problem 2 Obtain the direct form I, direct form II, cascade, and parallel structures for the system described by:

$$y[n] = y[n-1] - \frac{1}{2}y[n-2] + x[n] - x[n-1] + x[n-2]$$

direct form I:



direct form II / cascade / parallel



$$H(z) = \frac{1 - z^{-1} + z^{-2}}{1 - z^{-1} + \frac{1}{2}z^{-2}}$$

Consider the filter:

Problem 3 $$y[n] - 0.1\, y[n-1] = x[n] - 0.2\, x[n-1] + 0.1\, x[n-2]$$

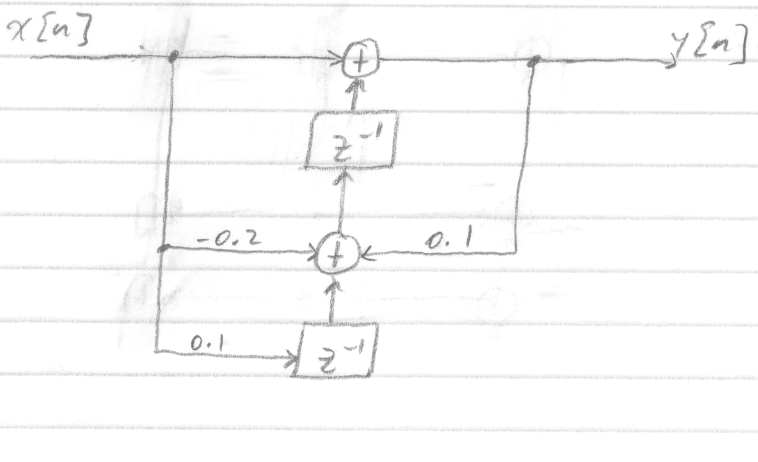
a) Is this an FIR or an IIR filter? Explain.
Obtain the filter transfer function and sketch a structural realization of the system.

$$Y(z)\,[1 - 0.1 z^{-1}] = X(z)\,[1 - 0.2 z^{-1} + 0.1 z^{-2}]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 0.2 z^{-1} + 0.1 z^{-2}}{1 - 0.1 z^{-1}}$$

$$\text{c.f. } H(z) = \frac{\sum_{k=0}^{M} b(k) z^{-k}}{\sum_{k=0}^{N} a(k) z^{-k}}$$

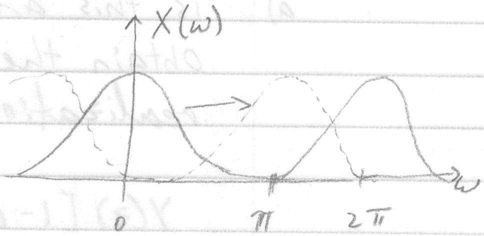
$\because N > 0$, filter is recursive
$\therefore$ it is has IIR



Problem 4 Show that $h_{hp}[n] = (-1)^n h_{lp}[n]$

Use frequency-shifting property of the Fourier Transform:

$$e^{j\omega_0 n} x[n] \longleftrightarrow X(e^{j(\omega-\omega_0)})$$



$$\omega_0 = \pi \longrightarrow e^{j\pi n} = (-1)^n$$

$$h_{hp}[n] = (-1)^n h_{lp}[n]$$