Electrical Engineering 3BA3: Structure of Biological Materials

Solutions to Midterm Quiz #2 (2008)

1. **The *iron lung* is classified as:**
   a. **a negative pressure ventilator,**
   b. **an intracorporeal device,**
   c. **a bioartificial organ, or**
   d. **a continuous positive airway pressure ventilator.** **(5 pts)**

   The answer is **a. a negative pressure ventilator**. See slides 15–16 of Student Presentation #11.

2. **The major difficulty with utilizing DNA as the biomolecule in a biosensor is that:**
   a. **it has very low specificity,**
   b. **there is no way to detect its binding with the analyte,**
   c. **it could alter the genes of the patient, or**
   d. **it is difficult to keep it stable.** **(5 pts)**

   The answer is **d. it is difficult to keep it stable**. See slide 7 of Lecture #15.

3. ***Bioartificial* organs:**
   a. **may utilize human or nonhuman cells,**
   b. **must be housed extracorporeally,**
   c. **can utilize only human cells, or**
   d. **are fully tissue engineered.** **(5 pts)**

   The answer is **a. may utilize human or nonhuman cells**. See slide 3 of Lecture #16.

4. ***Synthetic blood vessels* have been most successful for replacing:**
   a. **capillaries,**
   b. **smaller diameter blood vessels,**
   c. **larger diameter blood vessels, or**
   d. **the cerebral arteries.** **(5 pts)**

   The answer is **c. larger diameter blood vessels**. See slides 23 & 27 of Student Presentation #9 (and also slide 15 of Lecture #4).

5. The main benefit of *intravenous* drug delivery is that:

a. it can be easily self administered,

b. there is no risk of infection,

c. 100% of the drug makes it into the circulation, or

d. there is no risk of overdose. (5 pts)

The answer is **c. 100% of the drug makes it into the circulation**. See slide 5 of Lecture #14.

6. Fluid flow is referred to as *inviscid* if:

a. the fluid density is constant,

b. pressure variations in the fluid are due only to gravitational forces,

c. the shear-stress versus shear-rate relationship is nonlinear, or

d. the effect of the fluid's viscosity on its flow is negligible. (5 pts)

The answer is **d. the effect of the fluid's viscosity on its flow is negligible**. See slide 8 of Lecture #10.

7. Synthetic blood substitutes currently in clinical use (or trials) are aimed primarily at replicating the function of:

a. platelets,

b. white blood cells,

c. red blood cells, or

d. plasma proteins. (5 pts)

The answer is **c. red blood cells**. See slides 25–59 of Student Presentation #10.

8. Peritoneal dialysis is used to treat patients with:

a. peritonitis,

b. emphysema,

c. periodontitis, or

d. kidney disease. (5 pts)

The answer is **d. kidney disease**. See slides 33–46 of Student Presentation #15.

9. The enzyme glucose oxidase can be used in a glucose biosensor to catalyze a chemical reaction between blood glucose and oxygen. Explain at least two different ways in which the rate of that chemical reaction can be measured and transduced in a biosensor. (15 pts)

The chemical reaction that glucose oxidase (GOD) catalyzes is:

$$\text{D-glucose} + O_2 \xrightleftharpoons{\text{GOD}} \text{D-glucono-1,5-lactone} + H_2O_2 \,.$$

One way to measure the rate of the reaction is to use a Clark oxygen electrode to measure the rate of reduction of oxygen amperometrically. As oxygen is consumed by the above reaction, the current measured will drop.

Another amperometric transducer that can be used is a platinum electrode that measures the rate of oxidation of the hydrogen peroxide (H_2O_2) that is produced by the above reaction.

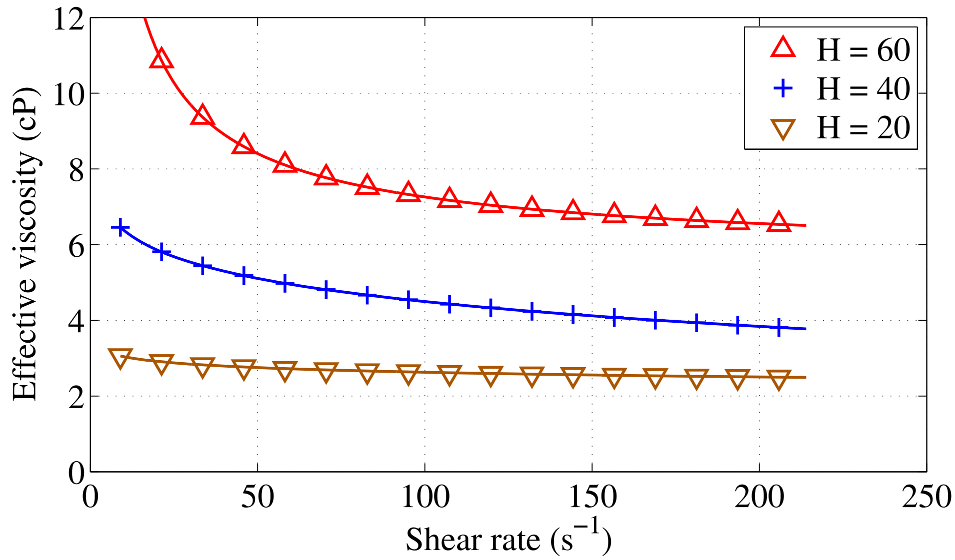
A potentiometric transducer that could be used is a pH sensor. In an aqueous solution the D-glucono-1,5-lactone (gluconic acid) forms D-gluconate + H^+, lowering the pH. A glass pH sensor can be used to measure the pH difference between the blood sample and an internal acidic solution, which generates an electrical potential between the internal and external solutions.

Alternatively, a calorimetric or thermometric transducer could measure the heat produced by the chemical reaction catalyzed by the GOD.

An optical biosensor that has been widely used in clinical glucose biosensors is to measure the hydrogen peroxide production using a test strip with a weakly coloured chromogen that reacts with the peroxide (with the enzyme *peroxidase* catalyzing this reaction) to produce a highly coloured dye. The colour change of the test strip can be assessed visually or with a reflectance meter.

10. ***Hematocrit*** **is the percentage by volume of blood that is occupied by red blood cells. Healthy adult females have a hematocrit of around** $H = 40$**. In an adult female suffering from anaemia (a low red blood cell count), the hematocrit may drop to** $H = 20$**, while a hematocrit of** $H = 60$ **would mean that she is suffering from polycythemia (a high red blood cell count).**

The effective viscosity of blood as a function of shear rate is known to change with hematocrit value H **as shown in the figure below. In addition, the blood density at human body temperature is known to vary as a function of hematocrit according to the equation** $\rho = 0.8H + 1015$ g/L**.**



Consider the blood flow in the artery of a female with a normal hematocrit. The flow in this artery is such that the shear rate is greater than $100\ \mathrm{s}^{-1}$ **for the majority of the blood, and the flow has a Reynolds number of 2500, such that it fluctuates between laminar and turbulent flow.**

a. Assuming that the average blood flow velocity and artery diameter do not change as a function of hematocrit, calculate the approximate value of the Reynolds number for flow in the artery for the following two cases:

i. The patient is suffering from anaemia, such that her hematocrit has fallen to 20.

ii. The patient is suffering from polycythemia, such that her hematocrit has risen to 60.

b. For each of these cases, state whether the blood flow is likely to be laminar or turbulent, providing your reasoning. (15 pts)

a. The Reynolds number is proportional to the ratio ρ/μ. Consequently, if the Reynolds number for $H = 40$ is 2500, then at a hematocrit of $H = 20$ the Reynolds number is:

$$\mathrm{Re}(H=20) = \mathrm{Re}(H=40) \times \frac{\rho(H=20)}{\rho(H=40)} \times \frac{\mu(H=40)}{\mu(H=20)}.$$

Likewise, for a hematocrit of $H = 60$:

$$\mathrm{Re}\left(H = 60\right) = \mathrm{Re}\left(H = 40\right) \times \frac{\rho\left(H = 60\right)}{\rho\left(H = 40\right)} \times \frac{\mu\left(H = 40\right)}{\mu\left(H = 60\right)}.$$

From the equation given for the blood density as a function of hematocrit, the densities at these three hematocrit values are $\rho\left(H = 20\right) = 0.8 \cdot 20 + 1015 = 1031\ \mathrm{g/L}$, $\rho\left(H = 40\right) = 1047\ \mathrm{g/L}$, and $\rho\left(H = 60\right) = 1063\ \mathrm{g/L}$.

Estimating the average effective viscosities for shear rates above $100\ \mathrm{s}^{-1}$ from the figure above gives values of $\mu\left(H = 20\right) \approx 2.5\ \mathrm{cP}$, $\mu\left(H = 40\right) \approx 4.1\ \mathrm{cP}$, and $\mu\left(H = 60\right) \approx 6.8\ \mathrm{cP}$.

Substituting these values in the equations above for the Reynolds numbers gives:

$$\mathrm{Re}\left(H = 20\right) \approx 2500 \times \frac{1031}{1047} \times \frac{4.1}{2.5} \approx 4037$$

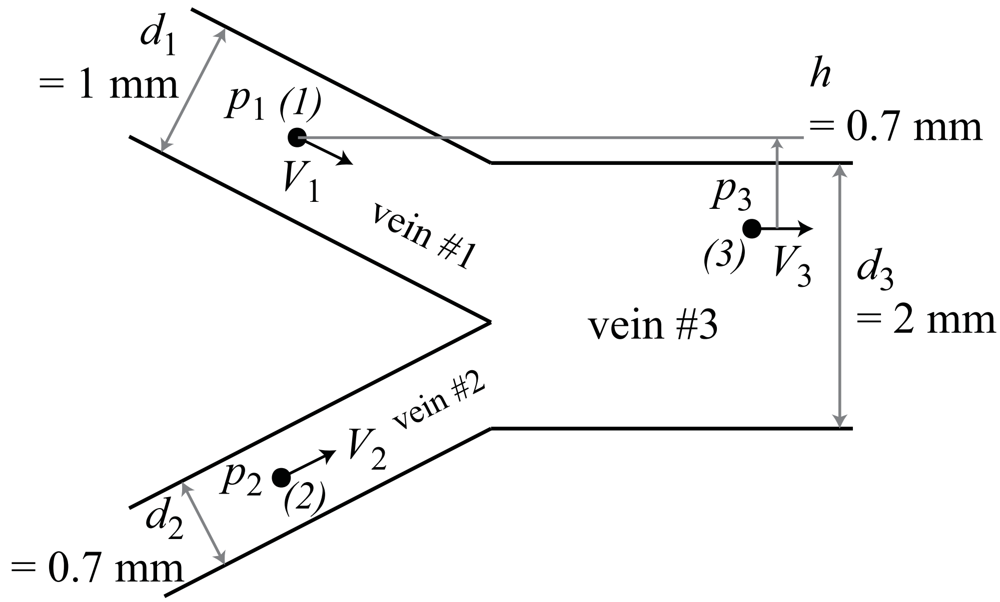
and

$$\mathrm{Re}\left(H = 60\right) \approx 2500 \times \frac{1063}{1047} \times \frac{4.1}{6.8} \approx 1530.$$

An equivalent method would be to find the value of the product Vd according to the formula $\mathrm{Re} = \rho V d / \mu$ using the values of $\mathrm{Re}\left(H = 40\right)$, $\rho\left(H = 40\right)$ and $\mu\left(H = 40\right)$ and then calculate the Reynolds number for $H = 20$ and $H = 60$ according to this formula.

b. Flow with a Reynolds number below 2000 is normally laminar, while flow with a Reynolds number above 4000 will almost always be turbulent. Consequently, in this case of anaemia, the percentage decrease in viscosity is greater than the percentage decrease in density, so the Reynolds number exceeds 4000, indicating turbulent flow. In contrast, for this case of polycythemia, the percentage increase in viscosity is greater than the percentage increase in density, so the Reynolds number falls well below 2000, and subsequently the flow should become exclusively laminar.

11. Consider the flow of blood from veins #1 and #2 into vein #3, as illustrated below.



Assume the following:

- i. **The veins are inelastic, such that their diameters are fixed at the values given in the figure.**
- ii. **The blood flow can be considered as steady Bernoulli flow.**
- iii. **The blood density is 1060 kg·m^{-3}, and acceleration due to gravity is 9.8 m·s^{-2}.**
- iv. **Points *(1)* and *(3)* lie on a streamline, and point *(1)* is 0.7 mm higher than *(3)*.**
- v. **The blood at point *(3)* has a 'gage' pressure (i.e., pressure relative to atmospheric pressure) of $p_3 = 2\ \text{kPa}$.**

Determine the following:

- a. **If the average flow velocity in vein #1 is $V_1 = 1\ \text{cm/s}$ and the average flow velocity in vein #2 is $V_2 = 3\ \text{cm/s}$, what is the average flow velocity in vein #3?**
- b. **Assuming that the blood flow velocity at point *(1)* is equal to the average velocity in that vein $(V_1 = 1\ \text{cm/s})$ and the blood flow velocity at point *(3)* (V_3) is equal to the average blood flow velocity in vein #3, as calculated in part a., what is the blood pressure at point *(1)* p_1?** **(15 pts)**

a. Due to conservation of mass, the volumetric flow rate in vein #3 must be equal to the sum of the volumetric flow rates in veins #1 and #2.

The volumetric flow rate in vein #1 is:

$$Q_1 = V_1 A_1 = V_1 \cdot \pi \left(\frac{d_1}{2}\right)^2 = 0.01 \cdot \pi \left(\frac{1\times10^{-3}}{2}\right)^2 = \pi \cdot 2.5\times10^{-9} \ \ \mathrm{m}^3 \cdot \mathrm{s}^{-1},$$

and the volumetric flow rate in vein #2 is:

$$Q_2 = V_2 A_2 = V_2 \cdot \pi \left(\frac{d_2}{2}\right)^2 = 0.03 \cdot \pi \left(\frac{0.7\times10^{-3}}{2}\right)^2 = \pi \cdot 3.675\times10^{-9} \ \ \mathrm{m}^3 \cdot \mathrm{s}^{-1},$$

giving:

$$Q_3 = Q_1 + Q_2 = \pi \cdot 6.175\times10^{-9} \ \ \mathrm{m}^3 \cdot \mathrm{s}^{-1}.$$

The average fluid velocity in vein #3 is then:

$$V_3 = Q_3 / A_3 = \frac{Q_3}{\pi (d_3/2)^2} = \frac{\not{\pi} \cdot 6.175\times10^{-9}}{\not{\pi} \cdot (2\times10^{-3}/2)^2} = 6.175\times10^{-3} \ \ \mathrm{m}\cdot\mathrm{s}^{-1} \quad \text{or} \quad 0.6175 \ \ \mathrm{cm}\cdot\mathrm{s}^{-1}.$$

b. The equation for steady Bernoulli flow along the streamline from point *(1)* to point *(3)* is:

$$p_1 + \rho\frac{V_1^2}{2} + \rho g z_1 = p_3 + \rho\frac{V_3^2}{2} + \rho g z_3.$$

Letting the vertical displacement be referenced to point *(3)*, i.e., $z_3 = 0$, and substituting in the known values, this equation becomes:

$$p_1 + 1060 \cdot \frac{(0.01)^2}{2} + 1060 \cdot 9.8 \cdot 0.7\times10^{-3} = 2\times10^3 + 1060 \cdot \frac{(6.175\times10^{-3})^2}{2} + 0$$

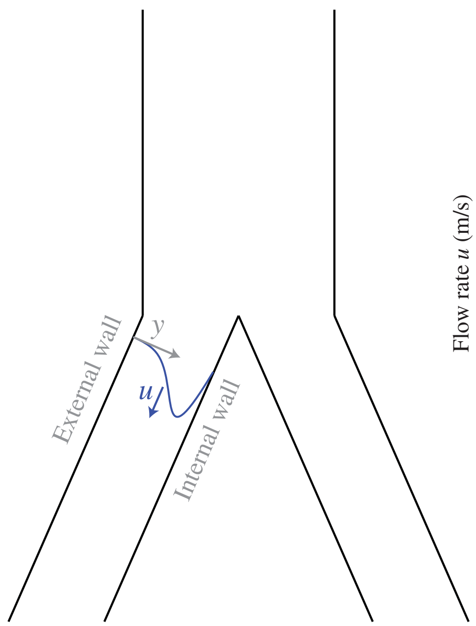
$$\Rightarrow p_1 = 2\times10^3 + 1060 \cdot \frac{(6.175\times10^{-3})^2}{2} - 1060 \cdot \frac{(0.01)^2}{2} - 1060 \cdot 9.8 \cdot 0.7\times10^{-3}$$
$$= 1.9927 \ \mathrm{kPa}.$$

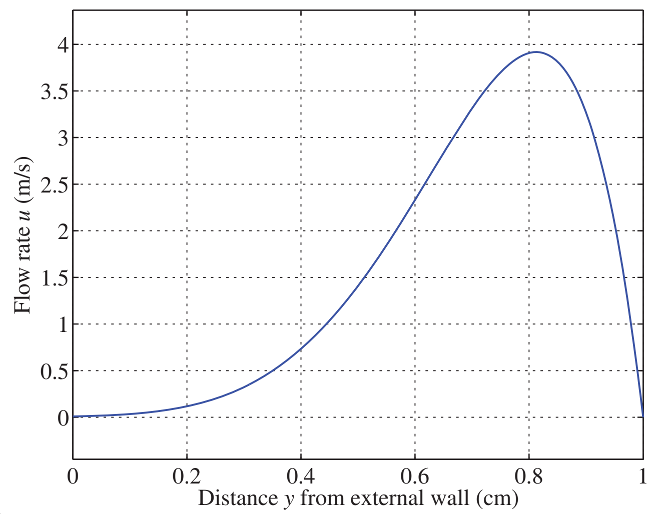
Note that the pressure at point *(1)* is somewhat smaller than the pressure at point *(3)*, as would be expected for Bernoulli flow, because point *(1)* is higher than point *(3)* and the flow is faster at point *(1)* than point *(3)*.

12. As depicted in the figure below, the flow rate u of air in a respiratory pathway of diameter 1 cm a short distance after a bifurcation can be described by the equation:

$$u = (1-y)\left(\frac{17}{2000} + \frac{27}{200}y + y^2 + \frac{19}{4}y^3 + \frac{29}{4}y^4 + \frac{127}{4}y^5\right),$$

where u has units of m/s and y is the distance from the "external" wall in units of cm in the plane of the bifurcation. (The "internal" wall here refers to the wall coming from the bifurcation point, while the "external" wall refers to the wall at the outside of the bifurcation, as indicated in the figure below). Assume that the air is a Newtonian fluid with viscosity $\mu = 0.018\ \mathrm{cP}$.





a. **Derive an expression for the air's shear stress τ as a function of lateral position y.**

b. **Is the shear stress greatest on the "internal" or the "external" wall of the airway? Provide the reasoning behind your answer.** **(15 pts)**

a. Expanding the equation for the flow rate gives:

$$\begin{aligned}
u &= (1-y)\left(\frac{17}{2000} + \frac{27}{200}y + y^2 + \frac{19}{4}y^3 + \frac{29}{4}y^4 + \frac{127}{4}y^5\right) \\
&= \frac{17}{2000} + \frac{27}{200}y + y^2 + \frac{19}{4}y^3 + \frac{29}{4}y^4 + \frac{127}{4}y^5 \\
&\quad - \frac{17}{2000}y - \frac{27}{200}y^2 - y^3 - \frac{19}{4}y^4 - \frac{29}{4}y^5 - \frac{127}{4}y^6 \\
&= \frac{17}{2000} + \frac{253}{2000}y + \frac{173}{200}y^2 + \frac{15}{4}y^3 + \frac{5}{2}y^4 + \frac{49}{2}y^5 - \frac{127}{4}y^6.
\end{aligned}$$

Taking the derivate with respect to y (and noting that u is in units of m/s, while y is in units of cm) gives that the flow rate as a function of y is:

$$\frac{\mathrm{d}u}{\mathrm{d}y} = \frac{253}{2000} + \frac{173}{100}y + \frac{45}{4}y^2 + 10y^3 + \frac{245}{2}y^4 - \frac{381}{2}y^5 \ \ \mathrm{m \cdot s^{-1} \cdot cm^{-1}}$$

$$= \frac{253}{20} + 173y + 1125y^2 + 1000y^3 + 12250y^4 - 19050y^5 \ \ \mathrm{s^{-1}}.$$

Converting the viscosity into SI units gives $\mu = 0.018\,\mathrm{cP} = 0.018\,\mathrm{mPa \cdot s} = 1.8\times10^{-5}\ \mathrm{Pa \cdot s}$. Thus, the shear stress as a function of y is:

$$\tau = \mu\frac{\mathrm{d}u}{\mathrm{d}y} = 1.8\times10^{-5}\cdot\left(\frac{253}{20} + 173y + 1125y^2 + 1000y^3 + 12250y^4 - 19050y^5\right)\ \mathrm{Pa}.$$

b. Evaluating the equation for τ at the external wall ($y = 0$) gives a value of $2.277\times10^{-4}\ \mathrm{Pa} = 0.2277\,\mathrm{mPa}$, while at the internal wall ($y = 1\,\mathrm{cm}$) the shear stress is $-0.0808\ \mathrm{Pa} = -80.8083\ \mathrm{mPa}$. Thus, the shear stress on the internal wall is much greater than the shear stress on the external wall.

This is consistent with the flow rate profile that is generated by the bifurcation: the fluid is flowing much faster along the internal wall than the external wall, but the fluid at each wall is subject to the no-slip condition, such that the shear rate is greater at the internal wall, giving a greater shear stress.