EE 4CL4 – Control System Design

Homework Assignment #10

1. The output $y(t)$ of a continuous-time system having a unit step input $u(t)$ is sampled every 1 second. The expression for the sampled sequence $\{y[k]\}$ is given by:

   $$y[k] = 0.5 - 0.5(0.6)^k \qquad \forall k \geq 0 .$$

   a. Determine $Y_q(z)$.

   b. Determine the transfer function from $U_q(z)$ to $Y_q(z)$.

   c. From the above result, derive the difference equation linking $\{y[k]\}$ to $\{u[k]\}$. **(25 pts)**

2. The transfer function of a sampled-data system (in delta form) is given by:

   $$G_\delta(\gamma) = \frac{\gamma + 0.5}{(\gamma + 0.1)(\gamma + 0.8)} .$$

   a. If $\Delta = 3.5$ s, is the system stable?

   b. Find the corresponding Z-transform function for $\Delta = 3.5$ s.

   c. Repeat parts a and b for $\Delta = 1.5$ s. **(25 pts)**

3. A continuous-time plant has a transfer function given by:

   $$G_o(s) = \frac{1}{(s+1)^2(s+2)} .$$

   a. Compute the location of the sampling zeros for $\Delta = 0.2$ s.

   b. How do the sampling zeros evolve when we vary Δ over the range [0.02 s, 2 s]? **(25 pts)**

4. A continuous-time plant has a transfer function given by:

   $$G_o(s) = \frac{-s+1}{(s+2)(s+1)} .$$

   a. Is there any sampling frequency at which no zero appears in the Z-domain transfer function (assuming a ZOH at the plant input)?

   b. Synthesize a minimal-prototype controller for $\Delta = 0.5$ s.

   c. Evaluate the control-loop performance to a unit step-output disturbance. **(25 pts)**