EE 4CL4 – Control System Design

Homework Assignment #3

1. A discrete-time system with input $u[k]$ and output $y[k]$ is described by the difference equation:

   $$y[k] - 0.8y[k-1] + 0.15y[k-2] = 0.2u[k-i].$$

   a. Build a state space model for $i = 0$.

   b. Repeat for $i = 1$. **(25 pts)**

2. Consider a feedback control loop of a plant with nominal model $G_o(s) = \frac{1}{(s+1)}$. Assume that the controller $C(s)$ is such that the complementary sensitivity is:

   $$T_o(s) = \frac{4}{(s+2)^2}.$$

   a. Show that the control loop is internally stable.

   b. Compute the controller transfer function $C(s)$.

   c. If the reference $r(t)$ is a unit step, compute the plant input $u(t)$. **(25 pts)**

3. In a nominal control loop, the sensitivity is given by:

   $$S_o(s) = \frac{s(s+4.2)}{s^2 + 4.2s + 9}.$$

   Assume that the reference $r(t)$ is a unit step and that the output disturbance is given by $d_o(t) = 0.5\sin(0.2t)$.

   Find an expression for the plant output $y(t)$ in the steady state. **(25 pts)**

4. Consider the following sets of plants and controllers with nominal models $G_o(s)$ and controllers $C(s)$. Assuming a one-degree-of-freedom unity control loop, use Routh's criterion to find the conditions for each of the controller's parameters under which the nominal feedback loop is stable.

   a. Nominal plant $G_o(s) = \frac{1}{(s+1)^4}$, with controller $C(s) = K$.

   b. Nominal plant $G_o(s) = \frac{1}{(s+1)(s+2)}$, with controller $C(s) = \frac{as+b}{s}$. **(25 pts)**