Electrical Engineering EE 4CL4

Day Class Instructor: Dr. I. C. BRUCE
Duration of Examination: 1.5 Hours
McMaster University Midterm Examination February, 2003

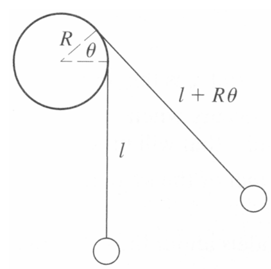
This examination paper includes ten (10) pages and five (5) questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions: Use of Casio *fx*-991 calculator only is allowed.
Each question is worth 25 points.
All five (5) questions may be answered, but the maximum total mark is capped at 100 points.
Some equations and tables that may assist you are provided on pages 3–10.

1. The figure below shows a simple pendulum system in which a cord is wrapped around a fixed cylinder. The motion of the system that results is described by the differential equation:

$$(l+R\theta)\ddot{\theta}+g\sin(\theta)+R\dot{\theta}^2=0,$$

where l is the length of the cord in the vertical (down) position and R is the radius of the cylinder.



a. Write the state space equations for this system.

b. Linearize the equation around the point $\theta=0$, $\dot{\theta}=0$, and show that for small values of θ the system equation reduces to an equation for a simple pendulum, that is, $\ddot{\theta}+(g/l)\theta=0$. **(25 pts)**

2. In a nominal control loop, the complimentary sensitivity is given by:

$$T_o(s)=\frac{1}{(s+1)(s+10)}.$$

If the system has an input disturbance $d_i(t) = 2\sin(0.5t)$, what does this input disturbance contribute to the plant input $u(t)$ in the steady state? **(25 pts)**

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3. Find the impulse response of the following linear transfer function:

$$H(s)=\frac{12}{(s+1)(s+3)(s+4)}.$$ **(25 pts)**

4. Find the range of values of K under which the controller:

$$C(s)=\frac{K(s+2)}{(s+10)}$$

stabilizes the unstable nominal plant model:

$$G_o(s)=\frac{1}{s(s-1)},$$

when placed together in a one-degree-of-freedom unity-feedback loop. **(25 pts)**

5. Consider a system having the following calibration and nominal models:

$$G(s)=F(s)\frac{1}{s-1} \qquad \text{and} \qquad G_o(s)=F(s)\frac{2}{s-2},$$

where F(s) is a proper, stable, and minimum-phase transfer function. Prove the following:

a. $G_\Delta(2)=-1$, i.e., the multiplicative modeling error equals −1 when s = 2.

b. The error sensitivity, $S_\Delta(s)\overset{\Delta}{=}\frac{1}{1+T_o(s)G_\Delta(s)}$, is unstable, having a pole at s = 2, where $T_o(s)$ is the complementary sensitivity of an internally stable control loop.

c. The achieved sensitivity $S(s)=S_\Delta(s)S_o(s)$ can be stable even though $S_\Delta(s)$ is unstable.

(25 pts)

THE END

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