ELEC ENG 4CL4 – Control System Design

Homework Assignment #5

Submission deadline: 12 noon on Friday, April 2, 2004, in the designated drop box in CRL-101B (the CRL photocopying room).

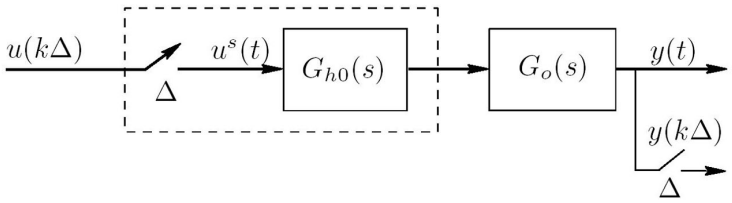
1. Consider the following recursive equation describing the relationship between the input $u[k]$ and the output $y[k]$ in a discrete-time (sampled-data) system:

   $$y[k] + 0.5y[k-1] = 1.5u[k-1].$$

   a. Determine the shift form (*z*-domain) transfer function $G_q(z)$ describing this system.

   b. Using the result from part a, compute the response of the system to the unit Kronecker delta $\delta_K[k]$. **(20 pts)**

2. Assume that in the block diagram:



   the continuous-time plant transfer function $G_o(s)$ is given by:

   $$G_o(s) = \frac{1}{(s+1)^2}.$$

   a. Compute the *discrete delta* transfer function from $u[k]$ to $y[k]$, $H_{o\delta}(\gamma)$, as a function of the sampling interval Δ.

   b. Verify that, if we make $\Delta \to 0$, then:

   $$\lim_{\Delta \to 0} H_{o\delta}(\gamma)\big|_{\gamma=s} = G_o(s).$$ **(30 pts)**

3. A continuous-time plant has the transfer function:

   $$G_o(s) = \frac{1}{(s+1)^2}.$$

   a. Synthesize a *minimal-prototype controller* $C_{1q}(z)$ and a *minimum-time dead-beat controller* $C_{2q}(z)$ for a sampling period $\Delta = 0.5$ s.

   b. For *each of these two controllers* placed in a one-d.o.f., unity-feedback control loop with the equivalent discrete-time (shift form) plant model $H_{oq}(z)$, evaluate the plant output $y[k]$ and the controller output $u[k]$ in response to a unit step reference. **(50 pts)**