

# ELEC ENG 4CL4: Control System Design

Notes for Lecture #13  
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# Chapter 6

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## **Classical PID Control**

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This chapter examines a particular control structure that has become almost universally used in industrial control. It is based on a particular fixed structure controller family, the so-called PID controller family. These controllers have proven to be robust and extremely beneficial in the control of many important applications.

PID stands for:

- P** (*Proportional*)
- I** (*Integral*)
- D** (*Derivative*)

# Historical Note

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Early feedback control devices implicitly or explicitly used the ideas of proportional, integral and derivative action in their structures. However, it was probably not until Minorsky's work on ship steering\* published in 1922, that rigorous theoretical consideration was given to PID control.

This was the first mathematical treatment of the type of controller that is now used to control almost all industrial processes.

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\* Minorsky (1922) "Directional stability of automatically steered bodies", *J. Am. Soc. Naval Eng.*, 34, p.284.

# The Current Situation

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Despite the abundance of sophisticated tools, including advanced controllers, the Proportional, Integral, Derivative (PID controller) is still the most widely used in modern industry, controlling more than 95% of closed-loop industrial processes\*

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\* Åström K.J. & Hägglund T.H. 1995, “New tuning methods for PID controllers”, *Proc. 3rd European Control Conference*, p.2456-62; and Yamamoto & Hashimoto 1991, “Present status and future needs: The view from Japanese industry”, *Chemical Process Control, CPCIV, Proc. 4th International Conference on Chemical Process Control*, Texas, p.1-28.

# PID Structure

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Consider the simple SISO control loop shown in Figure 6.1:

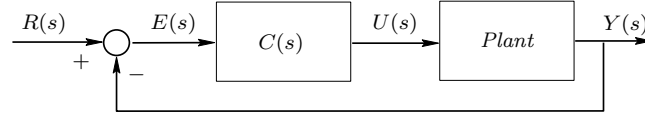


Figure 6.1: *Basic feedback control loop*

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The *standard form* PID are:

*Proportional only:*  $C_P(s) = K_p$

*Proportional plus Integral:*  $C_{PI}(s) = K_p \left( 1 + \frac{1}{T_r s} \right)$

*Proportional plus derivative:*  $C_{PD}(s) = K_p \left( 1 + \frac{T_d s}{\tau_D s + 1} \right)$

*Proportional, integral and derivative:*  $C_{PID}(s) = K_p \left( 1 + \frac{1}{T_r s} + \frac{T_d s}{\tau_D s + 1} \right)$

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An alternative *series* form is:

$$C_{series}(s) = K_s \left( 1 + \frac{I_s}{s} \right) \left( 1 + \frac{D_s s}{\gamma_s D_s s + 1} \right)$$

Yet another alternative form is the, so called, parallel form:

$$C_{parallel}(s) = K_p + \frac{I_p}{s} + \frac{D_p s}{\gamma_p D_p s + 1}$$