ELEC ENG 4CL4:

Control System Design

Notes for Lecture #22

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More General Effects of Open Loop Poles and Zeros

The results above depend upon the zeros of the various sensitivity functions at the origin. However, it turns out that zeros in the right half plane have an even more dramatic effect on achievable transient performances of feedback loops.

Lemma 8.3: Consider a feedback control loop having stable closed loop poles located to the left of $-\alpha$ for some $\alpha > 0$. Also assume that the controller has at least one pole at the origin. Then, for an uncancelled plant zero z_0 or an uncancelled plant pole η_0 to the right of the closed loop poles, i.e. satisfying $\Re\{z_0\} > -\alpha$ or $\Re\{\eta_0\} > -\alpha$ respectively, we have …..

(i) For a positive unit reference step or a negative unit step output disturbance, we have

$$\int_0^\infty e(t)e^{-z_0 t}dt = \frac{1}{z_0}$$

$$\int_0^\infty e(t)e^{-\eta_0 t}dt = 0$$

(ii) For a positive unit step reference and for z_0 in the right half plane, we have

$$\int_0^\infty y(t)e^{-z_0 t}dt = 0$$

(iii) For a negative unit step input disturbance, we have

$$\int_0^\infty e(t)e^{-z_0 t}dt = 0$$

$$\int_0^\infty e(t)e^{-\eta_0 t}dt = \frac{L(\eta_0)}{\eta_0 P(\eta_0)}$$

Observations

The above integral constraints show that (*irrespective of how the closed loop control system is designed*) the closed loop performance is constrained in various ways.

Specifically

(1) A real *stable* (LHP) zero to the right of all closed loop poles produces *overshoot* in the step response.

(2) A real *unstable* (RHP) zero always produces undershoot in the step response. The amount of undershoot grows as the zero approaches the origin.

(3) Any real open loop pole to the right of all closed loop poles will produce overshoot - in a one-degree-of-freedom control architecture.

We conclude that, to avoid poor closed loop transient performance:-

(1) The bandwidth should in practice be set less than the smallest non minimum phase zero.

(2) It is advisable to set the closed loop bandwidth greater than the real part of any unstable pole.

Example:

Consider a nominal plant model given by

$$G_o(s) = \frac{s - z_p}{s(s - p_p)}$$

The closed loop poles were assigned to {-1, -1, -1}. Then, the general controller structure is given by

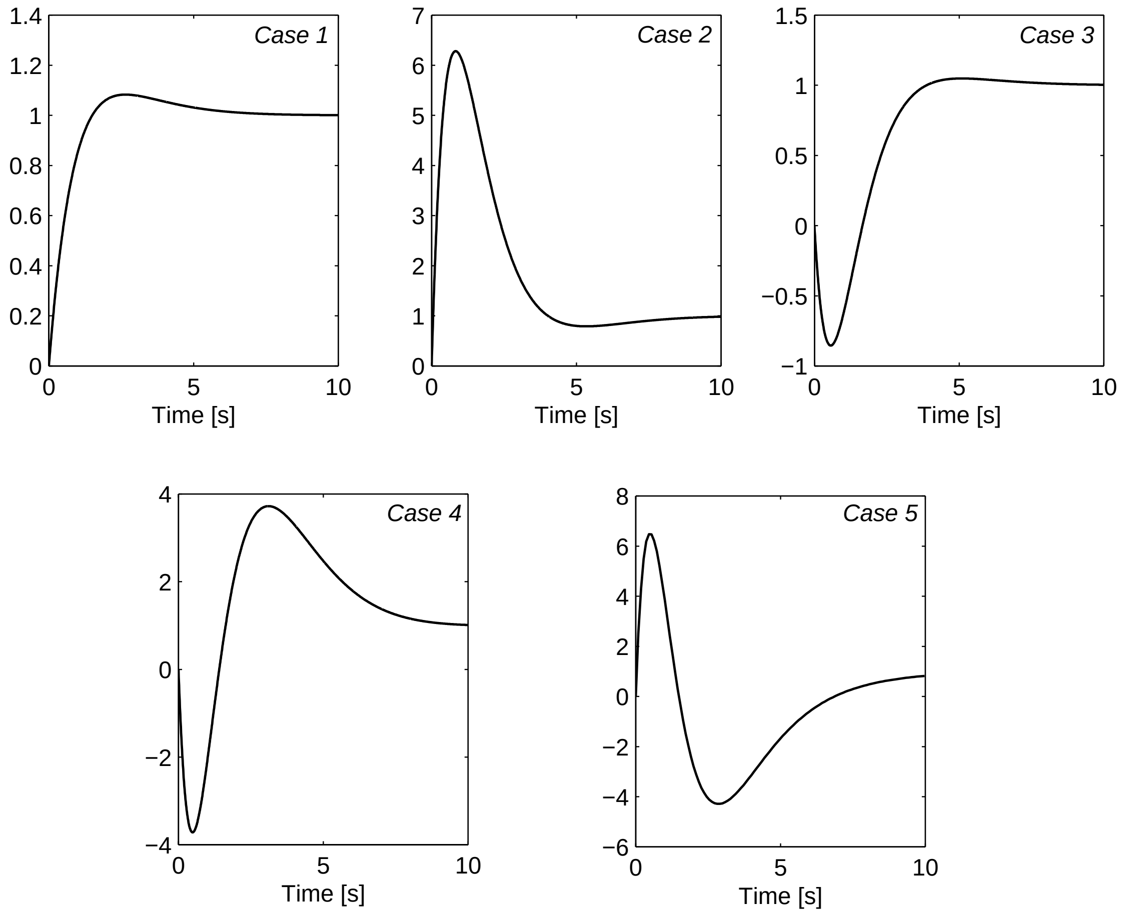
$$C(s) = K_c \frac{s - z_c}{s - p_c}$$

Five different cases are considered. They are described in Table 8.1.

Table 8.1: *Case description*

	Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**
	$p_p = -0.2$ $z_p = -0.5$	$p_p = -0.5$ $z_p = -0.1$	$p_p = -0.5$ $z_p = 0.5$	$p_p = 0.2$ $z_p = 0.5$	$p_p = 0.5$ $z_p = 0.2$
K_c	1.47	20.63	-3.75	-18.8	32.5
p_c	-1.33	18.13	-6.25	-22.0	29.0
z_c	-1.36	-0.48	-0.53	-0.11	0.15

Figure 8.3: *Plant output, y(t) for five different pole-zero configurations*



Case 1: (*Small stable pole*). A small amount of overshoot is evident as predicted.

Case 2: (*Very small stable zero*). Here we see a very large amount of overshoot, as predicted.

Case 3: (*Unstable zero, stable pole*). Here we see a significant amount of undershoot.

Case 4: (*Unstable zero, small unstable pole*). We observe significant undershot due to the RHP zero. We also observe significant overshoot due to the unstable open loop pole.

Case 5: (*Small unstable zero, large unstable pole*). We observe undershoot due to the RHP zero and overshoot due to the RHP pole. In this case, the overshoot is significantly larger than in Case 4, due to the fact that the unstable pole is further into the RHP.

Effect of Imaginary Axis Poles and Zeros

An interesting special case of Lemma 8.3 occurs when the plant has poles or zeros on the imaginary axis.

Consider a closed loop system as in Lemma 8.3, then for a unit step reference input:

(a) if the plant $G(s)$ has a pair of zeros at $\pm jw_0$, then

$$\int_0^\infty e(t)\cos\omega_0 t dt = 0$$

$$\int_0^\infty e(t)\sin\omega_0 t dt = \frac{1}{\omega_0}$$

(b) if the plant $G(s)$ has a pair of poles at $\pm jw_0$, then

$$\int_0^\infty e(t)\cos\omega_0 t dt = 0$$

$$\int_0^\infty e(t)\sin\omega_0 t dt = 0$$

where $e(t)$ is the control error, i.e. $e(t) = 1 - y(t)$

We see from the above formula that the maximum error in the step response will be very large if one tries to make the closed loop bandwidth greater than the position of the resonant zeros.

We illustrate by a simple example:

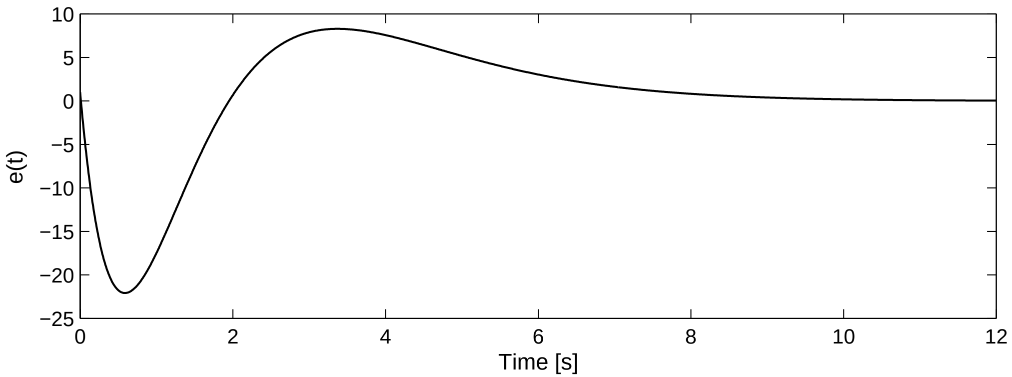
Example:

As a simple numerical example, consider a feedback control loop with complementary sensitivity transfer function given by

$$T(s) = \frac{100s^2 + 1}{s^3 + 3s^2 + 3s + 1}$$

Note that the closed loop poles are all at -1, while the zeros are at $\pm j0.1$. The simulation response of $e(t)$ for a unit step input is shown in Figure 8.5 on the next slide.

Figure 8.5: *Control error for a feedback loop with unit step reference and imaginary zeros*



imaginary zeros

We see that the maximum error in the transient response is very large. No fancy control methods can remedy this problem since it is fundamental. (*See the previous integral constraints*).