ELEC ENG 4CL4:

Control System Design

Notes for Lecture #6

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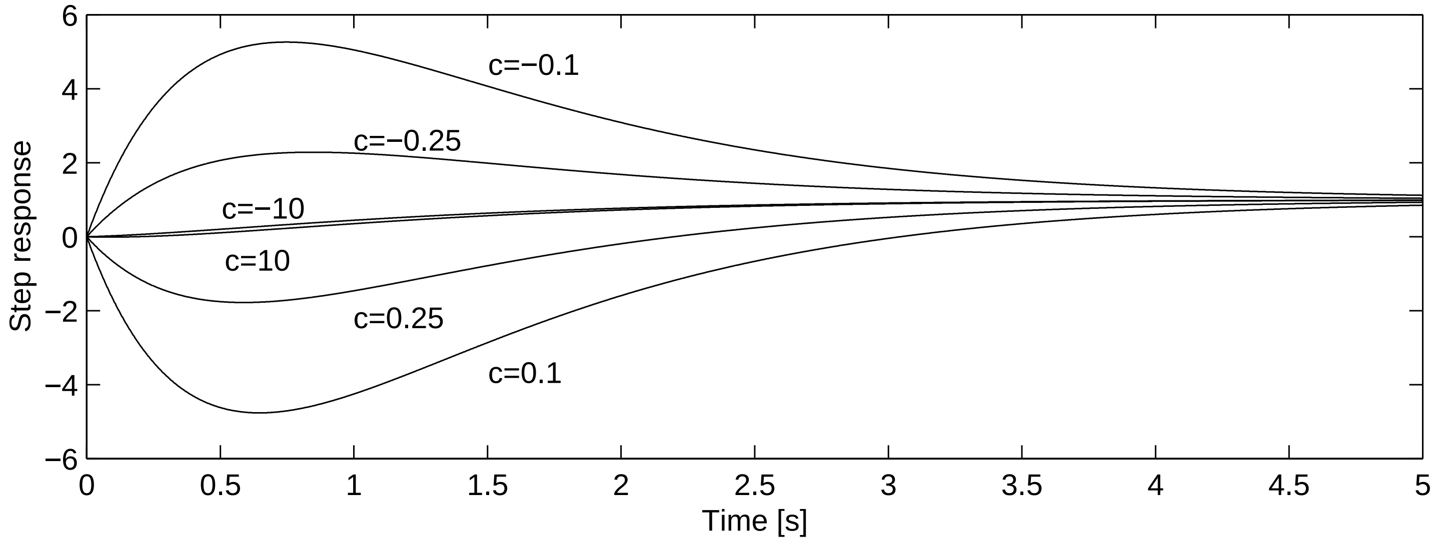
Zeros

The effect that zeros have on the response of a transfer function is a little more subtle than that due to poles. One reason for this is that whilst poles are associated with the states in isolation, zeros rise from additive interactions amongst the states associated with different poles. Moreover, the zeros of a transfer function depend on where the input is applied and how the output is formed as a function of the states.

Consider a system with transfer function given by

$$H(s) = \frac{-s + c}{c(s+1)(0.5s+1)}$$

Figure 4.6: *Effect of different zero locations on the step response*



These results can be explained as we show on the next slides.

Analysis of Effect of Zeros on Step Response

A useful result is:

Lemma 4.1: Let *H*(*s*) be a strictly proper function of the Laplace variable *s* with region of convergence $\Re\{s\} > -\alpha$. Denote the corresponding time function by *h*(*t*),

$$H(s) = \mathcal{L}\left[h(t)\right]$$

Then, for any z_0 such that $\Re\{z_0\} > -\alpha$, we have

$$\int_0^\infty h(t)e^{-z_0 t}dt = \lim_{s \to z_0} H(s)$$

Non minimum phase zeros and undershoot.

Assume a linear, stable system with transfer function $H(s)$ having unity d.c. gain and a zero at s=c, where $c \in \mathbb{R}^+$. Further assume that the unit step response, $y(t)$, has a settling time t_s (see Figure 4.3) i.e. $1+\delta \geq |y(t)| \geq 1-\delta (<<1), \forall t \geq t_s$. Then $y(t)$ exhibits an undershoot M_u which satisfies

$$M_u \geq \frac{1-\delta}{e^{ct_s}-1}$$

The lemma above establishes that, when a system has non minimum phase zeros, there is a trade off between having a fast step response and having small undershoot.

Slow zeros and overshoot. Assume a linear, stable system with transfer function $H(s)$ having unity d.c. gain and a zero at $s=c$, $c<0$. Define $v(t) = 1 - y(t)$, where $y(t)$ is the unit step response. Further assume that

A-1 The system has dominant pole(s) with real part equal to $-p$, $p>0$

A-2 The zero and the dominant pole are related by

$$\eta \triangleq \left|\frac{c}{p}\right| \ll 1$$

A-3 The value of δ defining the settling time (see Figure 4.3) is chosen such that there exists 0 < *K* which yields

$$|v(t)| < Ke^{-pt} \qquad \forall t \geq t_s$$

Then the step response has an overshoot which is bounded below according to

$$M_p \geq \frac{1}{e^{-ct_s} - 1}\left(1 - \frac{K\eta}{1-\eta}\right)$$