Problem 3.2 p 61 (Goodwin, Graebe, Salgado)

3.2.1

$$v_o(t) = 8\, v_i^2(t)$$
$$= 8\,(v_{i1}(t) + v_{i2}(t))^2$$
$$= 8\,[v_{i1}^2(t) + 2(v_{i1}(t) \cdot v_{i2}(t)) + v_{i2}^2(t)]$$
$$= 8\, v_{i1}^2(t) + 16\, v_{i1}(t)\, v_{i2}(t) + 8\, v_{i2}^2(t)$$
$$\neq 8\, v_{i1}^2(t) + 8\, v_{i2}^2(t)$$

3.2.2

$$v_o(t) = 8\,[5 + \cos(100t)]^2 \qquad \cos^2(x) = \tfrac{1}{2}(1 + \cos(2x))$$
$$= 8\,[25 + 10\cos(100t) + \cos^2(100t)]$$
$$= 200 + 80\cos(100t) + \tfrac{8}{2} + \tfrac{8}{2}\cos(200t)$$
$$= 204 + 80\cos(100t) + 4\cos(200t)$$

3.2.3

$$v_o(t) = 8\, v_i^2(t)$$
$$\Delta v_o(t) = v_o(t) - v_{oQ}$$
$$\Delta v_i(t) = v_i(t) - v_{iQ}$$

$$f(x_0 + \Delta x) = f(\underset{\hookrightarrow Q}{x_0}) + \Delta x\, f'(x_0) + \tfrac{1}{2!}\Delta x^2 f''(x_0) + \ldots$$
$$f(x) = v_o(t), \quad x = v_i(t)$$

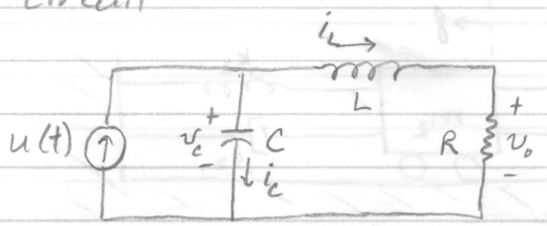
$$v_{oQ} = 8\, v_{iQ}^2$$

$$v_o(t) = v_{oQ} + \Delta v_i(t)[16\, v_{iQ}] + \tfrac{1}{2}\Delta v_i^2(t)\, 16 + \ldots$$
$$= 8\, v_{iQ}^2 + 16\, v_{iQ}(v_i(t) - v_{iQ}) + \tfrac{16}{2}(v_i(t) - v_{iQ})^2 + \ldots$$
$$v_o(t) - 8\, v_{iQ}^2 = \underbrace{v_o(t) - v_{oQ}}_{} = 16\, v_{iQ}(v_i(t) - v_{iQ}) + \cancel{8(v_i(t) - v_{iQ})^2} + \ldots$$

$$\Delta v_o(t) \simeq 16\, v_{iQ}\, \Delta v_i(t) \qquad \sim 0$$

dependence on operating point

Dorf + Bishop, Chapter 3
RLC circuit



The state of the system can be described by a set of state variables $(x_1, x_2) = (v_c(t), i_L(t))$

Not a unique set!

$$i_c = C\frac{dv_c}{dt} = +u(t) - i_L$$

$$L\frac{di_L}{dt} = -Ri_L + v_c$$

$$v_o = Ri_L(t) \quad \text{output}$$

Rewrite in terms of state variables:

$$\frac{dx_1}{dt} = -\frac{1}{C}x_2 + \frac{1}{C}u(t)$$

$$\frac{dx_2}{dt} = \frac{1}{L}x_1 - \frac{R}{L}x_2$$

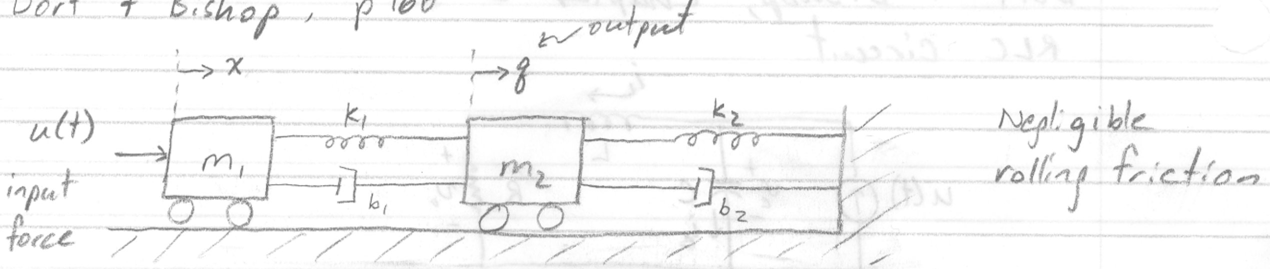
$$y_1(t) = v_o(t) = Rx_2$$

Rewrite in matrix form:

$$\dot{\underline{x}} = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R}{L} \end{bmatrix}\underline{x} + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix}u(t)$$

Dorf + Bishop, p 160

E 3.16



Find the state space representation of the system.

Governing equations:

$$m_1 \ddot{x} + k_1 (x - q) + b_1 (\dot{x} - \dot{q}) = u(t)$$

$$m_2 \ddot{q} + k_2 q + b_2 \dot{q} + b_1 (\dot{q} - \dot{x}) + k_1 (q - x) = 0$$

state variables:

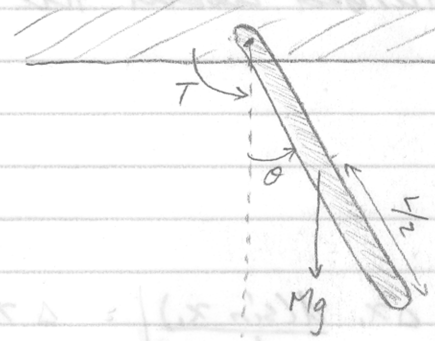
$$x_1 = x, \quad x_2 = \dot{x}, \quad x_3 = q, \quad x_4 = \dot{q}$$

Rewrite equations in matrix form in terms of state variables:

$$\underline{\dot{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{k_1}{m_1} & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{b_1}{m_2} & \frac{-(k_1+k_2)}{m_2} & \frac{-(b_1+b_2)}{m_2} \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = q(t) \longrightarrow y = [0 \quad 0 \quad 1 \quad 0] \, \underline{x}$$

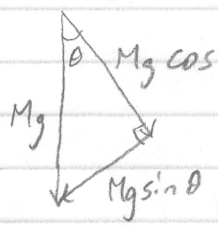
Nise, p155, Ex. 3.7



Assume mass is evenly distributed with the centre of mass at $\frac{L}{2}$

Find a state space representation of this system, then linearize the state equations about the pendulum's equilibrium point.

Free body diagram:



moment of inertia

$$\rightarrow \quad J\frac{d^2\theta}{dt^2} + \frac{MgL}{2}\sin\theta = T$$

Select state variables:

$$x_1 = \theta \quad , \quad x_2 = \frac{d\theta}{dt}$$

State Equations:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{MgL}{2J}\sin x_1 + \frac{T}{J} \quad \leftarrow \text{non-linear!}$$

Linearize state equations:

If x_1 + x_2 are perturbed about the equilibrium point,

$$x_1 = 0 + \Delta x_1$$

$$x_2 = 0 + \Delta x_2$$

Using Taylor series:

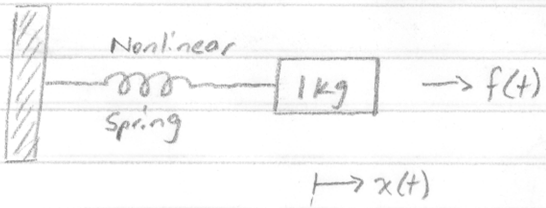
$$\sin x_1 = \sin 0 + \Delta x_1 \left.\frac{d(\sin x_1)}{dx_1}\right|_{x_1=0} = \Delta x_1$$

substitute back into state equations:

$$\Delta \dot{x}_1 = \Delta x_2$$

$$\Delta \dot{x}_2 = -\frac{MgL}{2J} \Delta x_1 + \frac{T}{J}$$

N.S. Nise SA Ex. 3.5, p 156



Given: $f_s(t) = 2x^2(t)$ spring force
$f(t) = 10 + \Delta f(t)$ applied force

$ma + T = f$

$$\frac{d^2x(t)}{dt^2} + 2x^2(t) = 10 + \Delta f(t) \quad \ldots \text{①}$$

Let $x(t) = x_0 + \Delta x(t)$

$$\frac{d^2(x_0 + \Delta x(t))}{dt^2} + 2(x_0 + \Delta x)^2 = 10 + \Delta f(t)$$

Use Taylor's series expansion to linearize $x^2(t)$:

$$x^2(t) \approx x_0^2 + \left.\frac{dx^2}{dx}\right|_{x_0} \Delta x + \ldots$$

$$(x_0 + \Delta x(t))^2 \approx x_0^2 + 2x_0 \Delta x \quad \ldots \text{②}$$

Sub ② into ①:

$$\frac{d^2(\overbrace{\cancel{x_0}}^{\text{constant}} + \Delta x(t))}{dt^2} + 2(x_0^2 + 2x_0 \Delta x) = 10 + \Delta f(t)$$

$$\frac{d^2 \Delta x(t)}{dt^2} + 4x_0 \Delta x(t) = 10 - 2x_0^2 + \Delta f(t)$$

At equilibrium, $F = 10 = 2x_0^2 \to x_0 = \sqrt{5}$

$$\therefore \frac{d^2 \Delta x(t)}{dt^2} + 4\sqrt{5}\, \Delta x(t) = \Delta f(t)$$

Select State Variables:

$$x_1 = \Delta x(t)$$

$$x_2 = \frac{d\Delta x(t)}{dt}$$

$$\therefore \frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = \frac{d^2 x(t)}{dt^2} = -4\sqrt{5}\,x_1 + \Delta f(t)$$

$$y(t) = x_1$$

$$\underline{\dot{x}} = \begin{bmatrix} 0 & 1 \\ -4\sqrt{5} & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta f(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x}$$