Smith Predictor Example

Given $G_0(s) = \dfrac{3e^{-s}}{(s+1)(s+3)}$ find $T_0(s)$

Requirements:

① The controller must achieve zero stedy-state error for step disturbances → forced integration

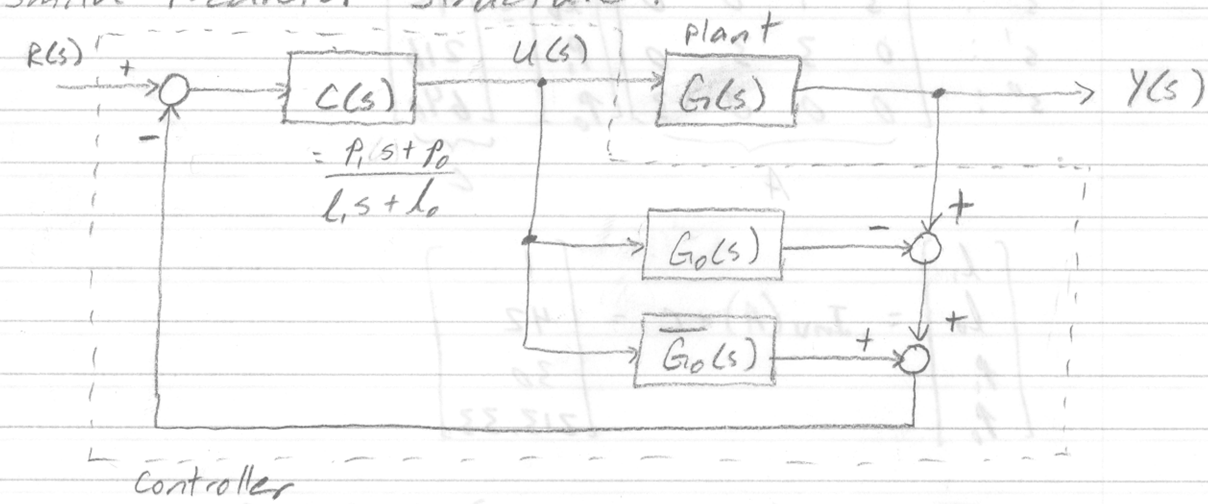
② Eliminate the plant pole at $s=-1$ because it is dominant (i.e. it is closest to the imaginary axis) → pole cancellation

③ Dominant closed-loop poles are at $s=-2.5 \pm 3.1225j$

Let: $\overline{G_0(s)} = \dfrac{3}{(s+1)(s+3)} = \dfrac{B_0}{A_0}$, the plant without delay.

$C(s)$ is the controller

Smith Predictor Structure:



$T_0(s) = e^{-s\tau}\, T_{zr}(s)$, $T_{zr}(s) = \dfrac{\overline{G_0(s)}\, C(s)}{1 + \overline{G_0(s)}\, C(s)}$

far away from
imaginary axis

$$A_{cl}(s) = (s^2+5s+16)(s+40)\,(s+1)$$

dominant poles
at $s = -2.5 \pm 3.1225j$

pole cancellation

$$A_{cl}(s) = s \cdot A_0(s)\,L(s) + B_0(s)\,P(s)\,(s+1)$$

forced integration

pole cancellation

$$\therefore\ s\cancel{(s+1)}(s+3)(l_1 s + l_0) + 3\cancel{(s+1)}(P_1 s + P_0) = \cancel{(s+1)}(s^2+5s+16)(s+40)$$

$$s(s+3)(l_1 s + l_0) + 3(P_1 s + P_0) = (s^2+5s+16)(s+40)$$

$$l_1 s^3 + l_0 s^2 + 3l_1 s^2 + 3l_0 s + 3P_1 s + 3P_0 = s^3 + 45s^2 + 216s + 640$$

$$l_1 s^3 + (l_0 + 3l_1)s^2 + (3l_0 + 3P_1)s + 3P_0 = s^3 + 45s^2 + 216s + 640$$

$$\begin{array}{c} s^3: \\ s^2: \\ s^1: \\ s^0: \end{array} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}}_{A} \begin{bmatrix} l_1 \\ l_0 \\ P_1 \\ P_0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 45 \\ 216 \\ 640 \end{bmatrix}}_{C}$$

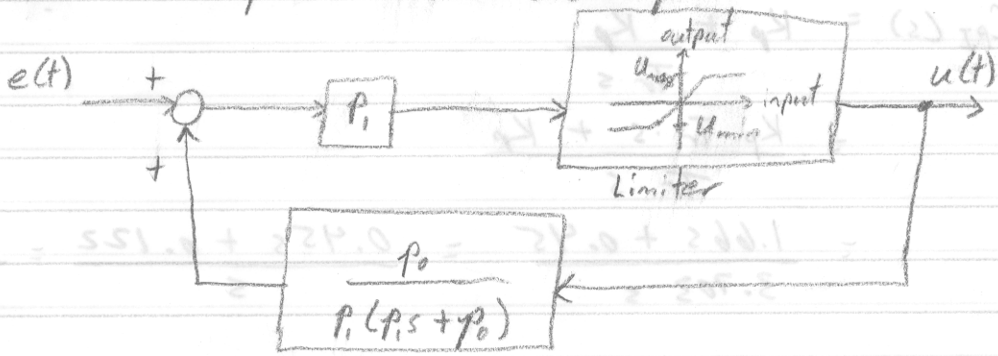
$$\begin{bmatrix} l_1 \\ l_0 \\ P_1 \\ P_0 \end{bmatrix} = \mathrm{Inv}(A) * C = \begin{bmatrix} 1 \\ 42 \\ 30 \\ 213.33 \end{bmatrix}$$

$$\therefore\ C(s) = \frac{(30s + 213.33)(s+1)}{s(s+42)} = \frac{\tilde{P}(s)}{\tilde{L}(s)}$$

$$\therefore\ T_0(s) = \frac{e^{-s} B_0(s)\,\tilde{P}(s)}{A_0(s)\,\tilde{L}(s) + B(s)\,\tilde{P}(s)} = \frac{3e^{-s}(30s + 213.33)\cancel{(s+1)}}{\cancel{(s+1)}(s+3)s(s+42) + 3(30s + 213.33)\cancel{(s+1)}}$$

$$= \frac{e^{-s}(90s + 640)}{s^3 + 45s^2 + 216s + 640}$$

Anti-Wind up / Limiter Example



Given $G_0(s) = \dfrac{-s+1}{s^2+s+1}$ design a PI controller with a limiter of $[u_{min}, u_{max}] = [-2, 2]$

Using the Z-N Oscillation Method:

$$K_c G_0(j\omega_c) = -1 \quad \text{for critical stability}$$

$$K_c \frac{(-j\omega_c+1)}{(j\omega_c)^2 + (j\omega_c) + 1} = -1$$

$$-K_c j\omega_c + K_c = -(j\omega_c)^2 - (j\omega_c) - 1$$
$$= (\omega_c^2 - 1) - j\omega_c$$

imag. : $K_c \omega_c = \omega_c \Longrightarrow K_c = 1$

real : $K_c = 1 = \omega_c^2 - 1$
$\omega_c = \sqrt{2}$ rad/s

$$\omega_c = 2\pi f_c = \frac{2\pi}{P_c} \Longrightarrow P_c = \frac{2\pi}{\omega_c} = 4.4429$$

for C_{PI} : $K_p = 0.45 K_c = 0.45$
$T_r = \dfrac{P_c}{1.2} = 3.703$

PTO →

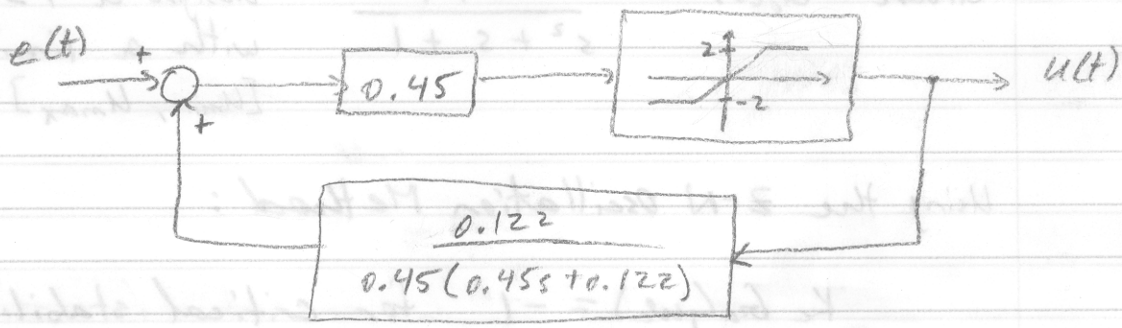
$$C_{PI}(s) = K_p + \frac{K_p}{T_r \cdot s}$$

$$= \frac{K_p\, T_r \cdot s + K_p}{T_r \cdot s}$$

$$= \frac{1.66s + 0.45}{3.703\, s} = \frac{0.45s + 0.122}{s} = \frac{P_1 s + P_0}{s}$$

$P_0 = 0.122$

$P_1 = 0.45$



$$\frac{U(s)}{E(s)} = \frac{P_1 s + P_0}{s} = C(s) = C_{PI}(s)$$

Problem 12.1, p 349

Find the z-transform for the discrete sequences that result from sampling the following signals at 1 Hz

a) $\mu(t) - \mu(t-3) = f(t)$ $\quad t = k\Delta$, $\Delta = 1$ s/sample

$f(k\Delta) = f[k]$

$f[k] = \mu[k] - \mu[k-3]$

$$Z\{f[k]\} = \frac{1 + z + z^2}{z^2} \qquad \text{ROC } |z| > 0$$

d) $f(t) = te^{-0.1t}\cos(t)$

$f[k] = k a^k \cos(k)$ $\quad$ where $a = e^{0.1}$ and $\theta = 1$

look-up tables:

$$a^k \cos(k\theta) \longleftrightarrow \frac{z(z - a\cos\theta)}{z^2 - 2az\cos\theta + a^2} \qquad \text{ROC } |z| > a$$

$$k\,g[k] = -z\frac{dG(z)}{dz}$$

$$\therefore F(z) = -z\frac{d}{dz}\left[\frac{z(z - a\cos 1)}{z^2 - 2az\cos 1 + a^2}\right]$$

$h'(\frac{f}{g}) = \frac{f'g - fg'}{g^2}$

$$= -z\left[\frac{(2z - a\cos 1)(z^2 - 2az\cos 1 + a^2) - (z^2 - az\cos 1)(2z - 2a\cos 1)}{(z^2 - 2az\cos 1 + a^2)^2}\right]$$

$$= \frac{-z\left[(\cancel{2z^3} - 4az^2\cos 1 + 2a^2 z - az^2\cos 1 + \cancel{2a^2 z\cos^2 1} - a^3\cos 1) - (\cancel{2z^3} - 2az^2\cos 1 - 2az^2\cos 1 + \cancel{2a^2 z\cos^2 1})\right]}{(z^2 - 2az\cos 1 + a^2)^2}$$

$$= \frac{az(z^2\cos 1 + a^2\cos 1 - 2az)}{(z^2 - 2az\cos 1 + a^2)^2}$$

Problem 12.2, p 349

Find the transfer function and compute the system's response to a unit kronecker delta.

a) $y[k] - 0.8\,y[k-1] = 0.4\,u[k-2]$

$Y_q(z) - 0.8 z^{-1} Y_q(z) = 0.4 z^{-2} U_q(z)$

$H_q(z) = \frac{Y_q(z)}{U_q(z)} = \frac{0.4 z^{-2}}{1-0.8z^{-1}} \cdot \frac{z^2}{z^2} = \frac{0.4}{z^2 - 0.8z}$

$\delta_K[k] \longleftrightarrow 1$

$Y_q(z) = H_q(z) U_q(z) = \frac{0.4}{z^2-0.8z} \cdot 1$

$= \frac{1}{z}\left[\frac{0.4}{z-0.8}\right] \cdot \frac{z}{z} = \frac{0.4z}{z^2(z-0.8)}$ ROC $|z| > 0.8$

$y_q[k] = 0.4\,\mu[k-2]\,0.8^{k-2}$

Problem 12.3, p349

Compute the step response for

$$H_q(z) = \frac{z-0.5}{z^2(z+0.5)}$$

$$Y_q(z) = H_q(z)\,U_q(z) \qquad U_q(z) = \frac{z}{z-1} \quad \text{step}$$

$$= \frac{z(z-0.5)}{z^2(z-1)(z+0.5)} \qquad \text{ROC } |z|>1$$

$$= \frac{z}{z^2}\cdot\frac{z-0.5}{(z-1)(z+0.5)} \qquad \text{partial fraction expansion:}$$

$$= \frac{z}{z^2}\cdot\left[\frac{A}{z-1}+\frac{B}{z+0.5}\right] = \frac{1}{z^2}\cdot\frac{Az+0.5A+Bz-B}{(z-1)(z+0.5)}$$

$$A+B=1 \qquad 0.5A-B=-0.5$$

$$B=2/3 \qquad 0.5A-1+A=-0.5$$

$$1.5A=0.5$$

$$A=1/3$$

$$Y_q(z) = z^{-2}\left[\frac{\frac{1}{3}z}{z-1}+\frac{\frac{2}{3}z}{z+0.5}\right]$$

$$Y_q[k] = \mu[k-2]\left(\tfrac{1}{3}+\tfrac{2}{3}(-0.5)^{k-2}\right)$$