1. Given the network in Fig. 1,

(a) find the equations for $V_a(t)$ and $V_b(t)$.

(b) find the equations for $V_c(t)$ and $V_d(t)$

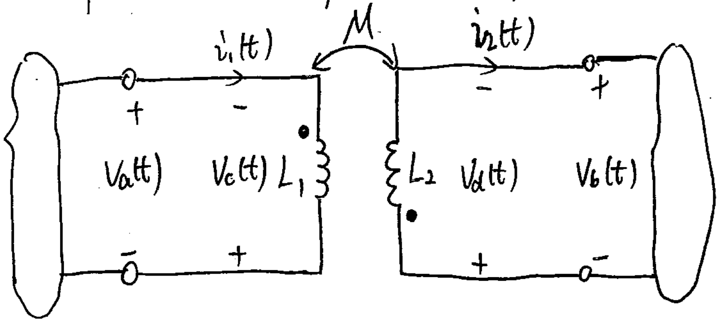


Fig. 1

Solution:

(a) $$V_a(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_b(t) = -M \frac{di_1}{dt} - L_2 \frac{di_2}{dt}$$

(b) $$V_c(t) = -V_a(t)$$

$$= -L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$V_d(t) = -V_b(t)$$

$$= M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

2. Find $\bar{V}_o$ in the network in Fig. 2

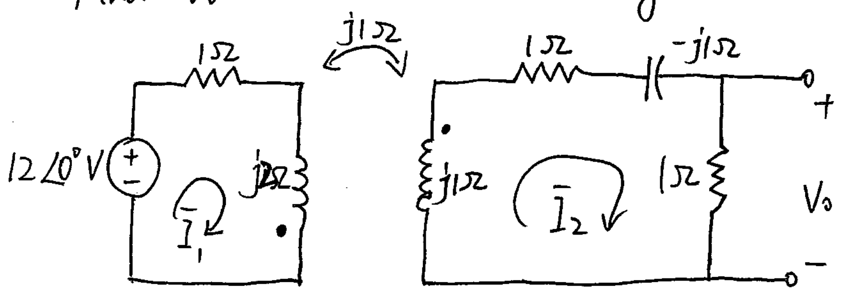


Fig. 2

Solution :

KVL for left network :

$$(1+j2)\bar{I}_1 + j1\,\bar{I}_2 = 12\angle 0^\circ$$

KVL for right network :

$$j1\,\bar{I}_1 + 3\bar{I}_2 = 0$$

$$\Longrightarrow \quad \bar{I}_1 = 5\angle -56.31^\circ \text{ A}$$

$$\bar{I}_2 = 1.66\angle -146.31^\circ \text{A}$$

$$\bar{V}_o = 2\bar{I}_2$$

$$= 2(1.66\angle -146.31^\circ)$$

$$= 3.32\angle -146.31^\circ$$

3. Find $\bar{V}_o$ in the network in Fig. 3

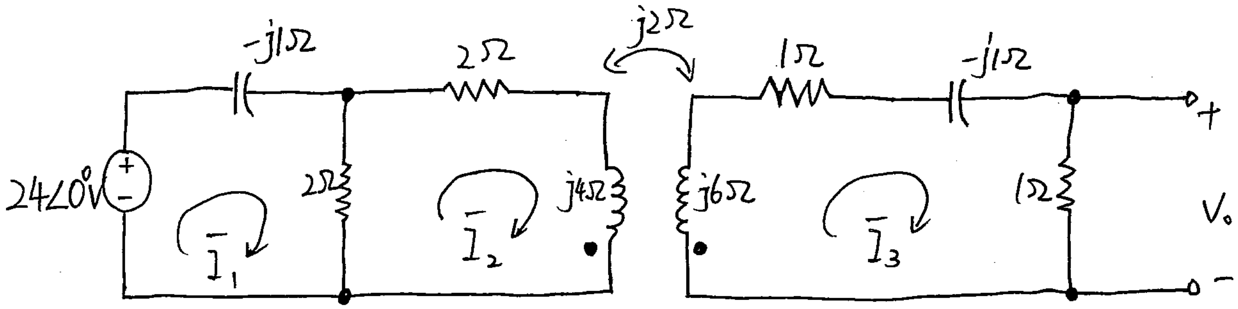


Fig. 3

Solution:

KVL: $(2-j1)\bar{I}_1 - 2\bar{I}_2 = 24\angle 0°$

KVL: $2(\bar{I}_2 - \bar{I}_1) + 2\bar{I}_2 + j4\bar{I}_2 - j2\bar{I}_3 = 0$

KVL: $j6\bar{I}_3 - j2\bar{I}_2 + (2-j1)\bar{I}_3 = 0$

$$\Longrightarrow \bar{I}_1 = 15.82\angle 21.14° A$$

$$\bar{I}_2 = 5.85\angle -16.61° A$$

$$\bar{I}_3 = 2.17\angle 5.19° A$$

$$\bar{V}_o = 1(\bar{I}_3)$$

$$= 2.17\angle 5.19° V$$

4. Find $\bar{V}_o$ in the network in Fig. 4

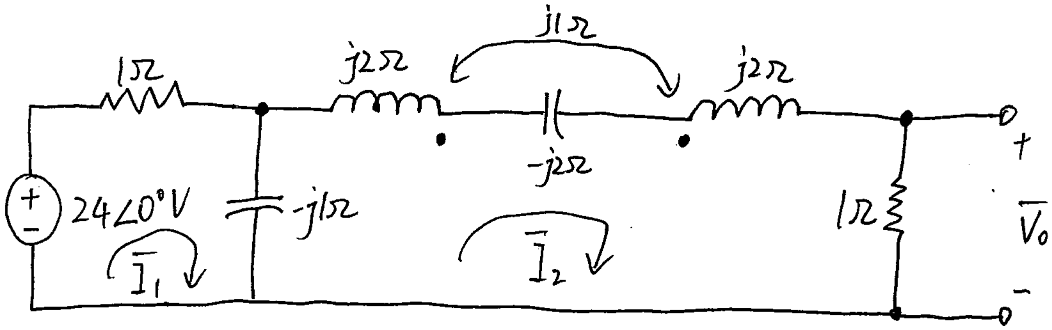


Fig. 4

Solution: KVL: $1\cdot\bar{I}_1 + (-j1)\cdot(\bar{I}_1 - \bar{I}_2) = 24\angle 0°$

KVL: $(-j1)(\bar{I}_2 - \bar{I}_1) + (j2 + j2 - j2 + 1)\bar{I}_2 - j1\bar{I}_2 - j1\bar{I}_2 = 0$

$$\Rightarrow \quad (1 - j1)\bar{I}_1 + j1\,\bar{I}_2 = 24\angle 0°$$

$$j1\bar{I}_1 + (1 - j1)\bar{I}_2 = 0$$

$$\Rightarrow \quad \bar{I}_1 = 15.18\angle 18.43° A$$

$$\bar{I}_2 = 10.73\angle -26.57° A$$

$$\bar{V}_o = 1\cdot(\bar{I}_2)$$

$$= 10.73\angle -26.57° \ V$$

5. Find V_o in the network in Fig. 5

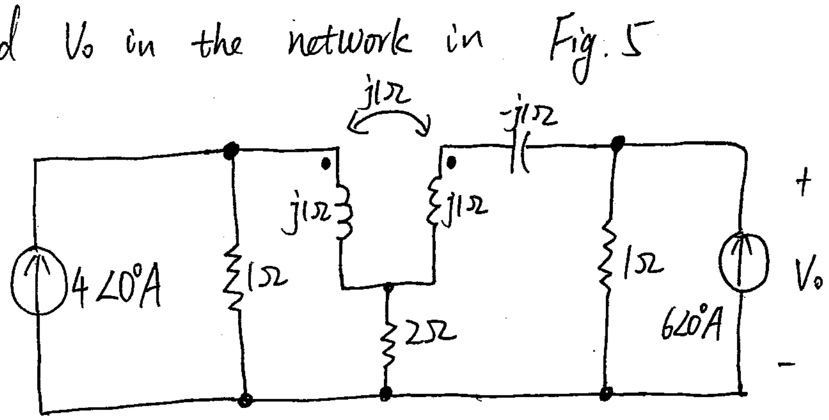
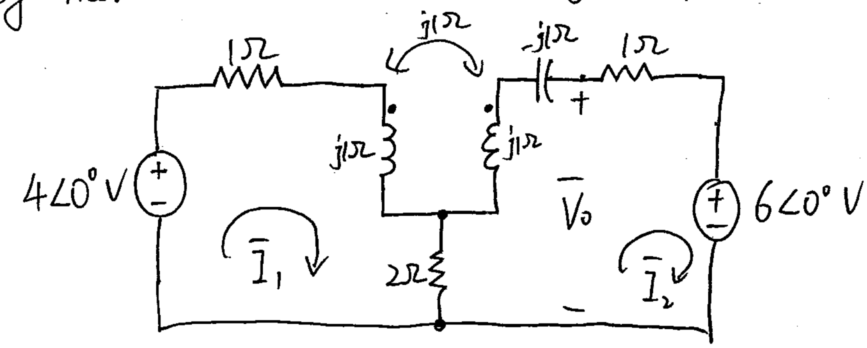


Fig. 5

Solution :

By Thevenin's theorem , we get equivalent ~~ckt~~ circuit .



KVL : $(3+j1)\bar{I}_1 - (2+j1)\bar{I}_2 = 4\angle 0^\circ$

KVL : $-(2+j1)\bar{I}_1 + 3\bar{I}_2 = -6\angle 0^\circ$

$\Rightarrow \bar{I}_1 = 0.986\angle -80.54^\circ A$

$\bar{I}_2 = 1.68\angle -159.23^\circ A$

$\bar{V}_o = 1(\bar{I}_2) + 6\angle 0^\circ$

$= 1.68\angle -159.23^\circ + 6\angle 0^\circ$

$= 4.47\angle -7.66^\circ V$ ✓

6. Determine the input impedance Z_{in} of the circuit in Fig. 6

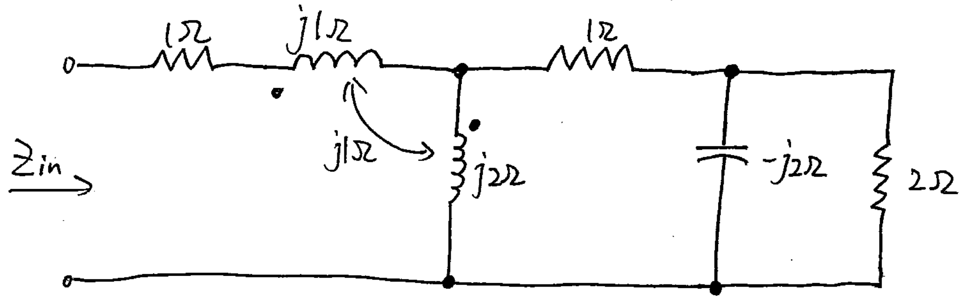
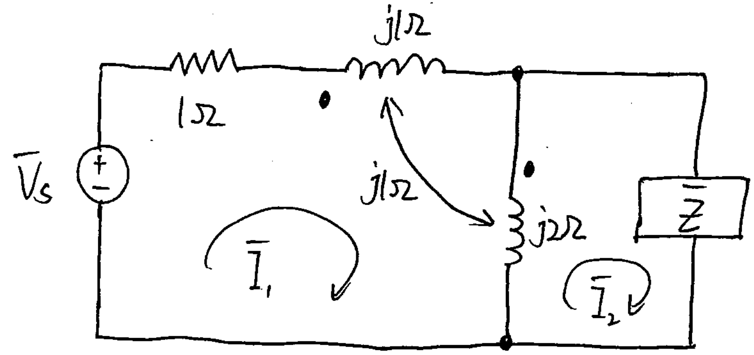


Fig. 6

Solution :



Let $\bar{V}_s = 1\angle 0°\ V$

$$\bar{Z}_{in} = \frac{\bar{V}_s}{\bar{I}_1}$$

$$\bar{Z} = 1 + \frac{2(-j2)}{2-j2} = 1 - j1\ \Omega$$

KVL: $(1+j3)\bar{I}_1 - j2\bar{I}_2 + j1\bar{I}_1 + j1(\bar{I}_1 - \bar{I}_2) = \bar{V}_s$

$$(1+j5)\bar{I}_1 - j3\bar{I}_2 = 1\angle 0°$$

KVL: $-j2\bar{I}_1 + (\bar{Z} + j2)\bar{I}_2 - j1\bar{I}_1 = 0$

$$-j3\bar{I}_1 + (2+j1)\bar{I}_2 = 0$$

$$\Rightarrow \quad \bar{I}_1 = 0.178\angle -34.82°A$$

$$\bar{I}_2 = 0.24\angle 28.61°A$$

$$\bar{Z}_{in} = \frac{\bar{V}_s}{\bar{I}_1}$$

$$= \frac{1\angle 0^\circ}{0.178\angle -34.82^\circ}$$

$$= 4.61 + j3.21\ \Omega$$

7. The currents in the network in Fig. 7 are known to be $i_1(t) = 10\cos(377t - 30°)$ mA and $i_2(t) = 20\cos(377t - 45°)$ mA. The inductances are $L_1 = 2H$, $L_2 = 2H$, and $k = 0.8$. Determine $V_1(t)$ and $V_2(t)$.

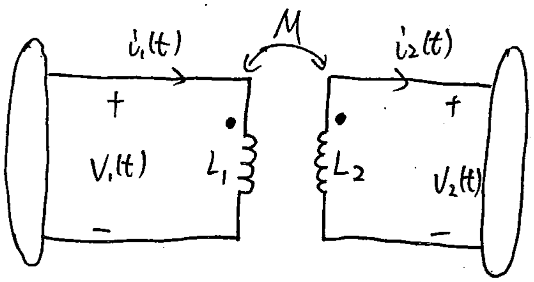
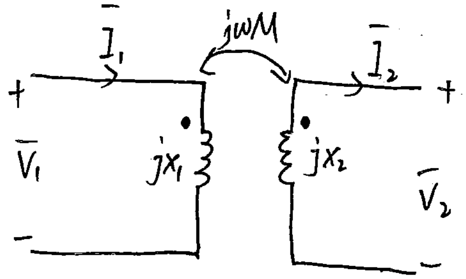


Fig. 7

Solution:



KVL: $jX_1\bar{I}_1 - j\omega M\bar{I}_2 = \bar{V}_1$

KVL: $-j\omega M\bar{I}_1 + jX_2\bar{I}_2 = -\bar{V}_2$

$$X_1 = \omega L_1 = 377 \times 2$$
$$= 754\ \Omega$$

$$X_2 = \omega L_2 = 377 \times 2 = 754\ \Omega$$

$$M = k\sqrt{L_1 L_2}$$
$$= 0.8\sqrt{2 \times 2} = 1.6\ H$$

$$\omega M = 377 \times 1.6 = 603.2\ \Omega$$

$$\bar{I}_1 = 10\angle -30° \, mA$$

$$\bar{I}_2 = 20\angle -45° \, mA$$

$$\bar{V}_1 = j(754)\cdot(10m\angle -30°) - j(603.2)(20m\angle -45°)$$

$$= 5.16\angle -157.2° \, V$$

$$\bar{V}_2 = -[-j(603.2)(10m\angle -30°) + j754(20m\angle -45°)]$$

$$= 9.38\angle -144.58° \, V$$

$$V_1(t) = 5.16\cos(377t - 157.2°) V$$

$$V_2(t) = 9.38\cos(377t - 144.58°) V$$

8. Determine the energy stored in the coupled inductors in the ~~ckt~~ circuit in Fig. 7 at $t = 1ms$

Solution:

$$w(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) - M i_1(t) i_2(t)$$

$$i_1(t) = 10 \cos(377t - 30^\circ) \, mA$$

$$i_2(t) = 20 \cos(377t - 45^\circ) \, mA$$

$L_1 = 2H$, $L_2 = 2H$, $k = 0.8$, $M = 1.6H$

at $t = 1ms$

$$i_1(1ms) = 9.89 \, mA$$

$$i_2(1ms) = 18.4 \, mA$$

$$w(1ms) = \frac{1}{2} \times 2 \times (9.89m)^2 + \frac{1}{2} \times 2 \times (18.4m)^2 - 1.6 \times (9.89m) \times (18.4m)$$

$$= 145.21 \, \mu J$$

9. Determine $\bar{I}_1, \bar{I}_2, \bar{V}_1$ and $\bar{V}_2$ in the network in Fig. 9

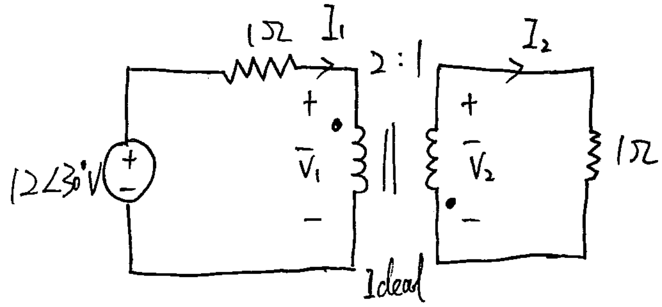


Fig. 9

Solution :

$$1\cdot \bar{I}_1 + \bar{V}_1 = 12\angle 30^\circ$$

$$\bar{V}_2 = 1\cdot \bar{I}_1$$

$$-\bar{V}_1 = 2\bar{V}_2$$

$$2\bar{I}_1 = -\bar{I}_2$$

$$\bar{I}_1 - 2\bar{V}_2 = 12\angle 30^\circ$$

$$\bar{I}_1 - 2\bar{I}_2 = 12\angle 30^\circ$$

$$\bar{I}_1 - 2(-2\bar{I}_1) = 12\angle 30^\circ$$

$$5\bar{I}_1 = 12\angle 30^\circ$$

$$\bar{I}_1 = 2.4\angle 30^\circ \text{ A}$$

$$\bar{I}_2 = -2\times(2.4\angle 30^\circ) = 4.8\angle -150^\circ \text{A}$$

$$\bar{V}_2 = 1\times 4.8\angle -150^\circ = 4.8\angle -150^\circ \text{A}$$

$$\bar{V}_1 = -2(4.8\angle -150^\circ) = 9.6\angle 30^\circ \text{A}$$

10. Determine the input impedance seen by the source in the circuit in Fig. 10

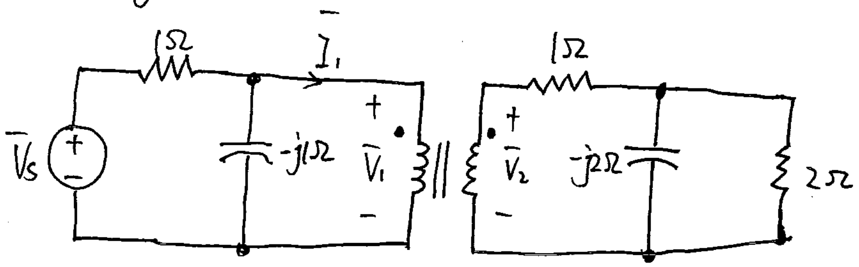
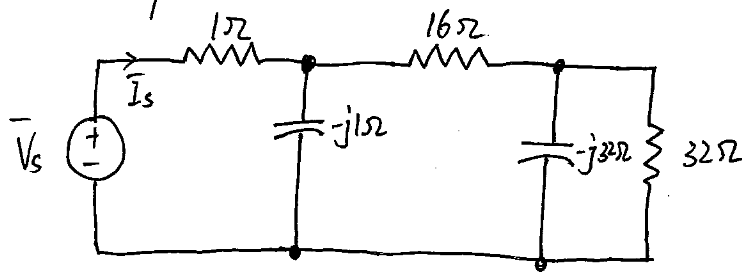


Fig. 10

Solution:

Equivalent circuit:



$$\bar{Z}_{in} = \frac{\bar{V}_s}{\bar{I}_s}$$

$$\bar{Z}_{in} = \{[(32 \parallel -j32) + 16] \parallel -j1\} + 1$$

$$= \{[\frac{32 \cdot (-j32)}{32 - j32} + 16] \parallel -j1\} + 1$$

$$= [(32 - j16) \parallel -j1] + 1$$

$$= \frac{(32 - j16)(-j1)}{32 - j16 - j1} + 1$$

$$= 1.42 \angle -43.94^\circ \ \Omega$$

11. Determine $\bar{I}_s$ in the circuit in Fig. 11

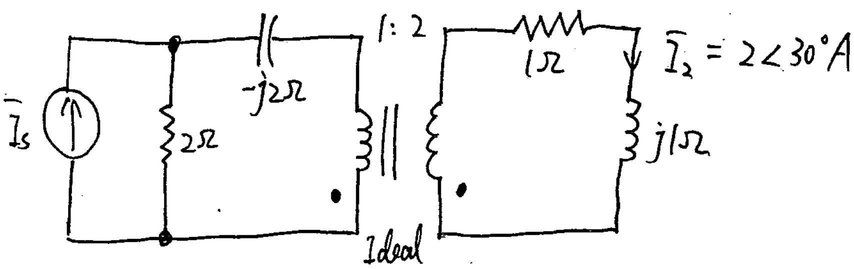
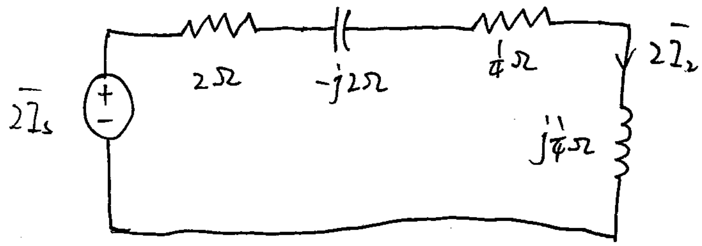


Fig. 11

Solution:

Using impedance reflection and source transformation



$$\text{KVL: } \left(2 - j2 + \frac{1}{4} + j\frac{1}{4}\right) 2\bar{I}_2 = 2\bar{I}_s$$

$$\bar{I}_s = \left(2 - j2 + \frac{1}{4} + j\frac{1}{4}\right)(2\angle 30°)$$

$$= 5.7\angle -7.87°A$$