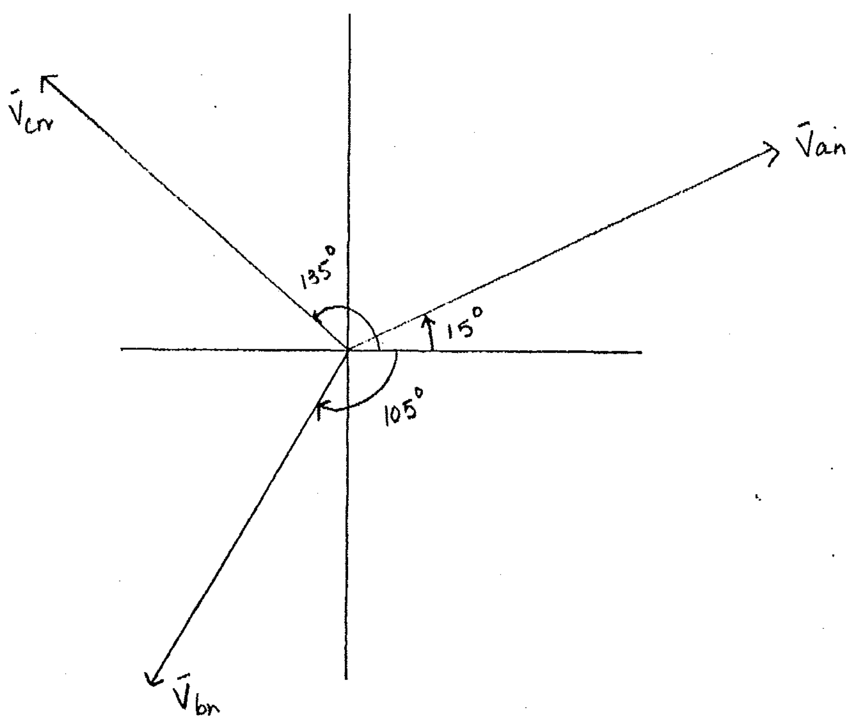
1. Sketch a phasor representation of an abc-sequence balanced three-phase Y-connected source, including V_{an}, V_{bn}, and V_{cn} if $V_{an} = 120 \angle 15°$ Vrms

Solution: $\bar{V}_{an} = 120 \angle 15°$ Vrms

$\bar{V}_{bn} = 120 \angle -105°$ Vrms

$\bar{V}_{cn} = 120 \angle 135°$ Vrms



2. Sketch a phasor representation of a balanced three-phase system containing both phase voltages and line voltages if $V_{ab} = 208 \angle 60° V_{rms}$. Label all phasors and assume an abc-phase sequene.

Solution: $\bar{V}_{ab} = 208 \angle 60° \ V_{rms}$

$$V_{an} = \frac{208}{\sqrt{3}} \angle 60° - 30° = 120 \angle 30° \quad V_{rms}$$

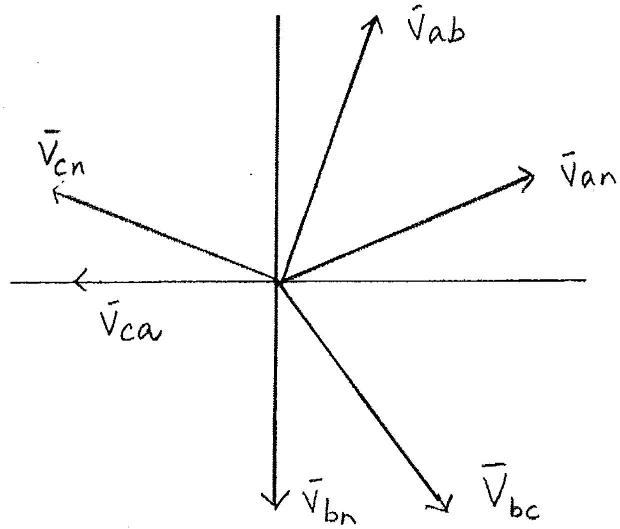
$$\bar{V}_{ba} = \bar{V}_{an} - \bar{V}_{ab} = 120 \angle 30° - 208 \angle 60°$$

$$\bar{V}_{bn} = 120 \angle -90° \quad V_{rms}$$

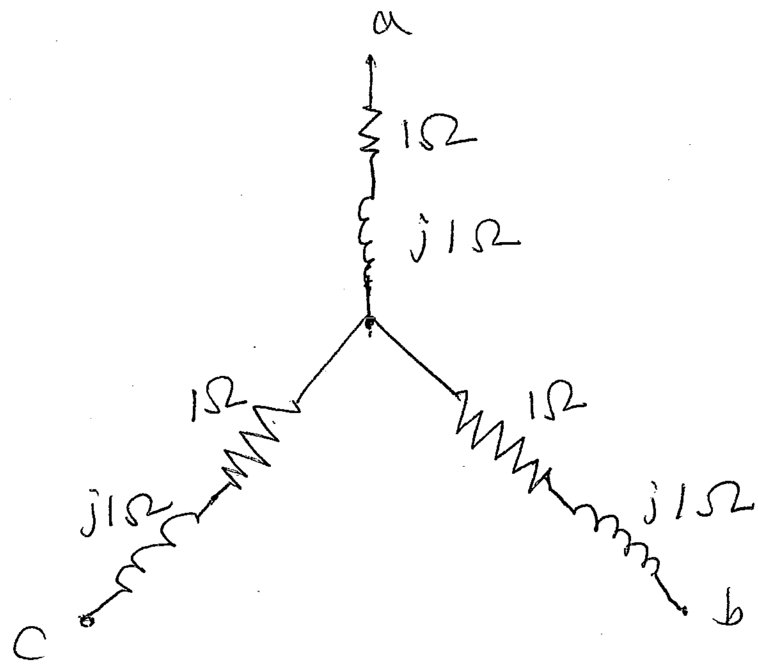
$$V_{cn} = 120 \angle 150° \quad V_{rms}$$

$$V_{bc} = 208 \angle -60° \quad V_{rms}$$

$$V_{ca} = 208 \angle 180 \quad V_{rms}$$



3. Find the equivalent impedances Z_{ab}, Z_{bc}, Z_{ca}.



Solution:

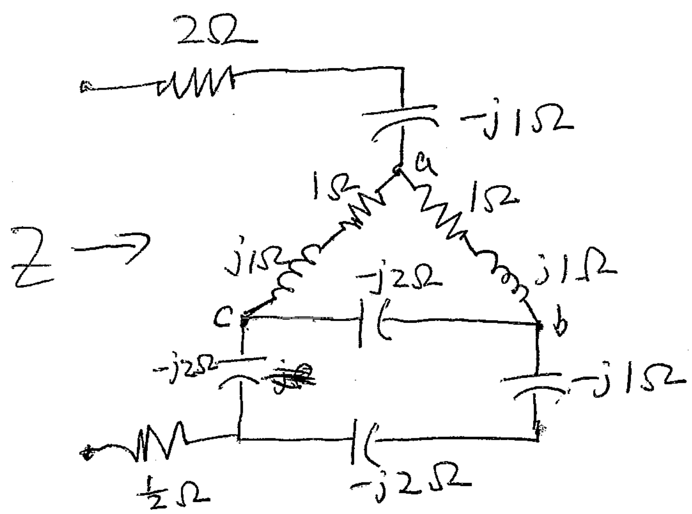
$$Z_{an} = Z_{bn} = Z_{cn} = 1 + j \cdot 1\Omega = Z_Y$$

$$Z_\Delta = 3 Z_Y = 3(1 + j1)$$

$$\bar{Z}_\Delta = 3 + j3\Omega$$

$$\overline{Z_{ab}} = \overline{Z_{bc}} = \overline{Z_{ca}} = 3 + j3\Omega.$$

4. Find the equivalent Z of the network.

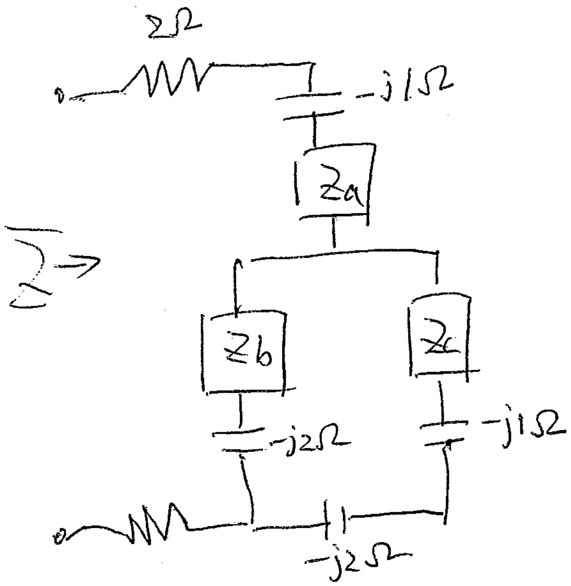


Solution: $Z_1 = 1 + j1\Omega$, $Z_2 = 1 + j1\Omega$, $Z_3 = -j2\Omega$.

$$Z_a = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2 + Z_3} = j1\Omega$$

$$Z_b = \frac{Z_1 \cdot Z_3}{Z_1 + Z_2 + Z_3} = 1 - j1\Omega$$

$$Z_c = \frac{Z_2 \cdot Z_3}{Z_1 + Z_2 + Z_3} = 1 - j1\Omega$$



$$Z = \{(Z_c - j3) // (Z_b - j2)\} + Z_a + \frac{1}{2} - j1 + 2$$

$$= \frac{(1 - j1 - j3)(1 - j1 - j2)}{1 - j1 - j3 + 1 - j1 - j2} + j1 + \frac{1}{2} - j1 + 2$$

$$= 3 - j1.717$$

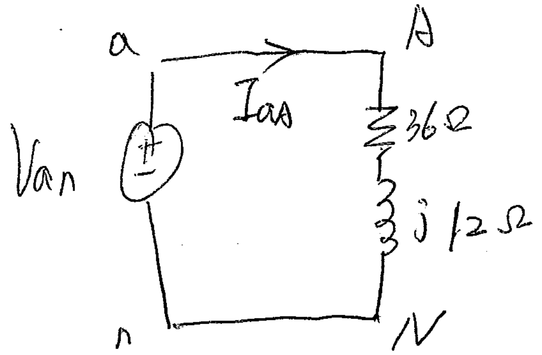
$$= 3.46\angle -29.7^\circ$$

5. A Positive-sequence balanced three-phase wye-Connected Source supplies power to a balanced wye-connected Lode. The magnitude of the line Voltages is 208 Vms. If the load impedance per phase is $36 + j12\,\Omega$, determine the line currents if $\angle V_{an} = 0°$.

Solution: $|V_{Line}| = 208\,V\,rms$,

$$|V_{an}| = \frac{|V_{Line}|}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120\ V_{rms}$$

$$I_{aA} = \frac{V_{an}}{Z_{load}} = \frac{120\angle 0°}{36 + j12} = 3.16\angle -18.43°\,A$$



$$I_{bB} = 3.16\angle -138.43°\ A$$

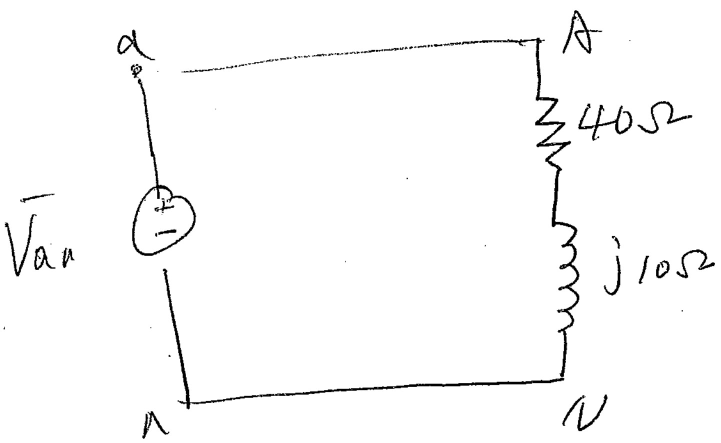
$$I_{cC} = 3.16\angle 101.57°\ A$$

6. A positive-sequence balanced three-phase wye-connected source with a phase voltage of 120 V rms supplies power to a balanced wye-connected load. The per phase load impedance is $40 + j10\,\Omega$, Determine the line currents in the circuit if $\angle V_{an} = 0°$.

Solution. $|\bar{V}_{an}| = 120\,V$ rms

$Z_{load} = 40 + j\,10\,\Omega$

$\angle \bar{V}_{an} = 0°$



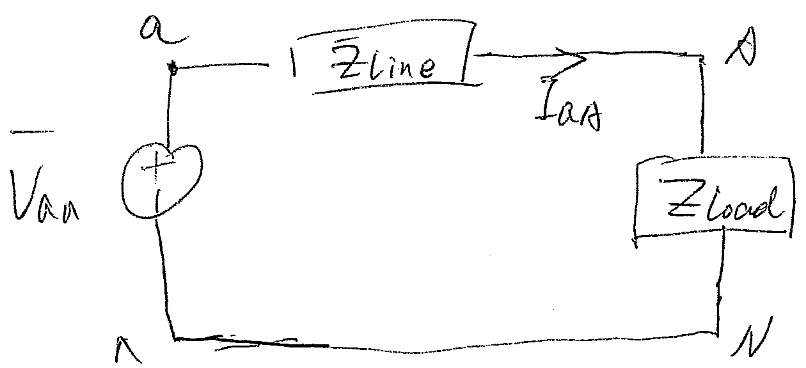
$$I_{aA} = \frac{120\angle 0°}{40 + j10} = 2.91\ \angle -14°\ A$$

$$I_{bB} = 2.91\ \angle -134°\ A$$

$$I_{cC} = 2.91\ \angle 106°\ A$$

7. An abc-sequence balanced three-phase wye-connected source supplies power to a balanced wye-connected load. The line impedance per phase is $1+j5\,\Omega$, and the load impedance per phase is $25+j25\,\Omega$. If the source line voltage is $208\angle 0°$ Vrms. fine the line currents.

Solution: $V_{an} = \frac{208}{\sqrt{3}}\angle -30° = 120\angle -30°$ Vrms.



$$I_{aA} = \frac{V_{an}}{Z_{line} + Z_{load}} = \frac{120\angle -30°}{1+j5+25+j25} = 3.02\angle -79.1° \text{ A}$$

$$I_{bB} = 3.02\angle -199.1° \text{ A}$$

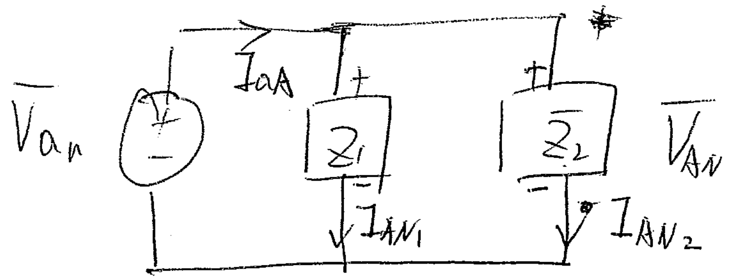
$$I_{cC} = 3.02\angle 40.9° \text{ A}$$

8. In a balanced three-phase system, the abc-phase sequence source is delta connected and $V_{ab} = 120\angle 30^\circ$ V rms. The load consists of two balanced wyes with phase impedances of $10+j1\,\Omega$ and $20+j5\,\Omega$. If the line impedance is 0, find the line currents and the load phase voltage.

Solution: $\bar{V}_{ab} = 120\angle 30^\circ$ Vrms

$Z_1 = 10+j1\,\Omega$, $Z_2 = 20+j5\,\Omega$. $Z_{line} = 0\,\Omega$

$V_{an} = \frac{120}{\sqrt{3}}\angle 30^\circ - 30^\circ = 69.28\angle 0^\circ$ Vrms $= V_{AN}$



$I_{bB} = 10.23\angle -128.44^\circ$ Arms

$I_{cC} = 10.23\angle 111.56^\circ$ Arms

$I_{AN_1} = \frac{V_{an}}{Z_1} = \frac{69.28\angle 0^\circ}{10+j1} = 6.89\angle -5.71^\circ$ A

$I_{AN_2} = \frac{V_{an}}{Z_2} = \frac{69.28\angle 0^\circ}{20+j5} = 3.36\angle -14.04^\circ$ A cont

$I_{aA} = I_{AN_1} + I_{AN_2} = 6.89\angle -5.71^\circ + 3.36\angle -14.04^\circ$ A $= 10.23\angle -8.44^\circ$ Arms

9. The magnitude of the complex power (apparent power) supplied by a three-phase balance wye-wye system is 3600 VA. The line voltage is 208 V rms. If the line impedance is negligible and the power factor angle of the load is 25°, determine the load impedance.

Solution $|\bar{S}_{3-\phi}| = 3600\ VA$

$$|\bar{V}_L| = 208\ V rms$$

$$PF = \cos 25^\circ = 0.906$$

$$S_{3-\phi} = 3600 \angle 25^\circ\ VA$$

$$|\bar{V}_{an}| = \frac{208}{\sqrt{3}} = 120\ V rms$$

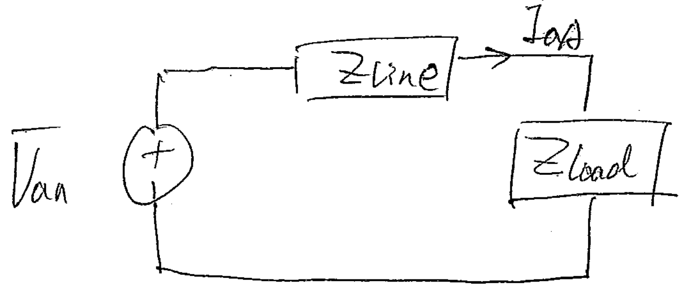
$$\bar{S}_{3-\phi} = 3|V_{an}||I_{aA}| \angle \theta_{V_{an}} - \theta_{I_{aA}}$$

$$I_{aA} = \frac{S_{3-\phi}}{3 V_{an} \angle \theta_{Z_L}} = \frac{3600 \angle 25^\circ}{3(120) \angle 25^\circ} = 10\ A\ rms$$

$$Z_{load} = \frac{120 \angle 25^\circ}{10} = 12 \angle 25^\circ\ \Omega.$$

10. An abc-sequence wye-connected source having a phase-a voltage of $120\angle 0°$ V rms is attached to a wye-connected load having a per-phase impedance of $100\angle 70°\ \Omega$. If the line impedance is $1\angle 20°\ \Omega$, determine the total complex power produced by the voltage sources and the real and reactive power dissipated by the load.

Solution: $\bar{V}_{an} = 120\angle 0°$ Vrms, $\bar{Z}_{load} = 100\angle 70°\ \Omega$, $Z_{line} = 1\angle 20°\ \Omega$.



$$\bar{I}_{aA} = \frac{120\angle 0°}{1\angle 20° + 100\angle 70°} = 1.19\angle -69.6°\ A$$

$$\bar{S}_{3\text{-}\phi} = 3\,V_{an} \cdot I_{aA}^{*} = 3(120\angle 0°)(1.19\angle 69.6°)$$
$$= 428.4\angle 69.6°\ VA$$

$$S_{L3\text{-}\phi} = 3\,I_{aA}^{2} \cdot Z_{load} = 3(1.19)^2(100\angle 70°)$$
$$= 145.3 + j\,399.21\ VA$$

$$P_{L3\text{-}\phi} = 145.3\ W.$$

10