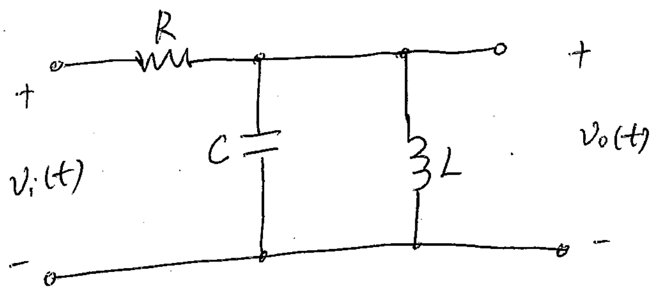
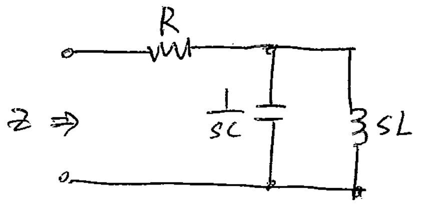
Q1. Determine the driving point impedance at the input terminals of the following network as a function of s.

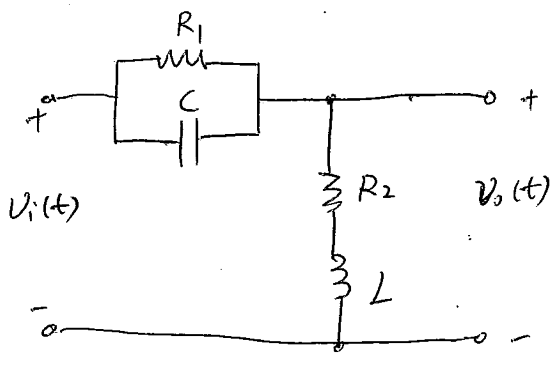


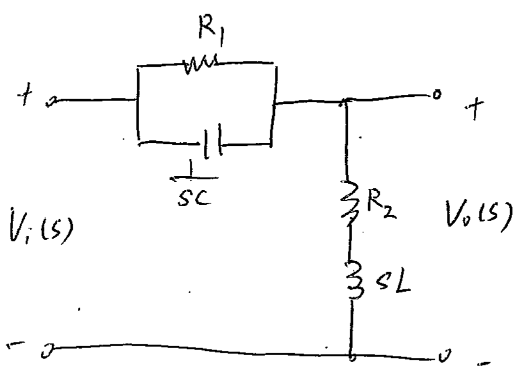


$$Z = R + \frac{1}{sC} \,//\, sL = R + \frac{sL \cdot \frac{1}{sC}}{sL + \frac{1}{sC}} = R + \frac{sL}{s^2LC + 1}$$

$$= \frac{s^2LCR + sL + R}{s^2LC + 1}$$

Q2. Determine the voltage transfor function $V_o(s)/V_i(s)$ as function of s for the following network.



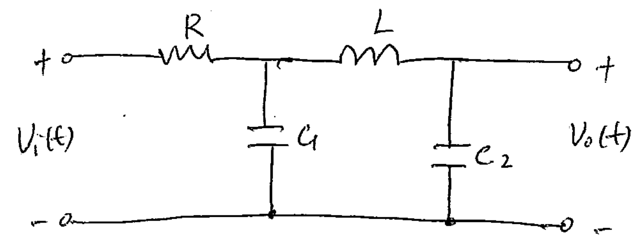


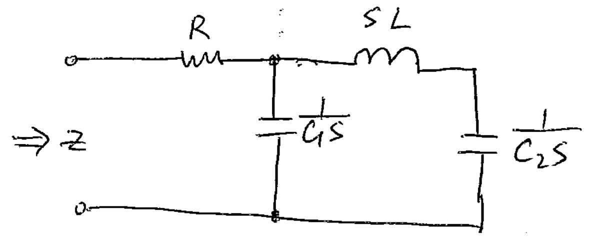
$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 + sL}{R_2 + sL + R_1 \,//\, \frac{1}{sC}} = \frac{R_2 + sL}{R_2 + sL + \frac{R_1 \cdot \frac{1}{sC}}{R_1 + \frac{1}{sC}}} = \frac{R_2 + sL}{R_2 + sL + \frac{R_1}{R_1 C s + 1}}$$

$$= \frac{(R_2 + sL)(sR_1C + 1)}{(R_2 + sL)(sR_1C + 1) + R_1} = \frac{s^2 R_1 CL + sR_1R_2C + sL + R_2}{s^2 R_1 LC + sR_1R_2C + sL + R_2 + R_1}$$

$$= \frac{s^2 + s\left[\frac{R_2}{L} + \frac{1}{R_1 C}\right] + \frac{R_2}{R_1 LC}}{s^2 + s\left[\frac{R_2}{L} + \frac{1}{R_1 C}\right] + \frac{R_1 + R_2}{R_1 LC}}$$

Q3. Determine the driving point impedance at the input terminals of the following network as a function of s.





$$Z = R + \frac{1}{C_1 s} \,//\, \left(sL + \frac{1}{C_2 s}\right) = R + \frac{\frac{1}{sC_1}\cdot\left(sL + \frac{1}{sC_2}\right)}{\frac{1}{sC_1} + sL + \frac{1}{sC_2}}$$

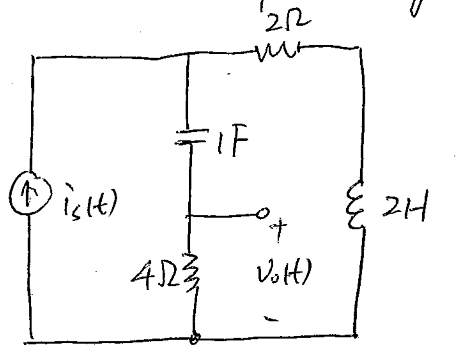
$$= R + \frac{sL + \frac{1}{sC_2}}{1 + s^2 LC_1 + \frac{C_1}{C_2}} = R + \frac{s^2 + \frac{1}{LC_2}}{s\left[s^2 C_1 + \frac{1}{L} + \frac{C_1}{C_2 L}\right]} = R + \frac{s^2 + \frac{1}{LC_2}}{sC_1\left[s^2 + \frac{1}{LC_1} + \frac{1}{LC_2}\right]}$$

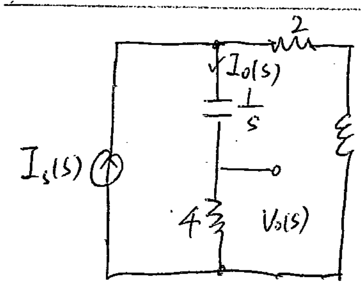
$$= R + \frac{s^2 + \frac{1}{LC_2}}{sC_1\left[s^2 + \frac{C_1 + C_2}{LC_1C_2}\right]} = \frac{RsC_1\left[s^2 + \frac{C_1+C_2}{LC_1C_2}\right] + s^2 + \frac{1}{LC_2}}{sC_1\left[s^2 + \frac{C_1+C_2}{LC_1C_2}\right]}$$

$$= \frac{R\left[s^3 + s\cdot\frac{C_1+C_2}{LC_1C_2} + s^2\cdot\frac{1}{RC_1} + \frac{1}{LC_2RC_1}\right]}{s\left(s^2 + \frac{C_1+C_2}{LC_1C_2}\right)}$$

$$= \frac{R\left(s^3 + \frac{1}{RC_1}s^2 + \frac{C_1+C_2}{LC_1C_2}s + \frac{1}{RLC_1C_2}\right)}{s\left(s^2 + \frac{C_1+C_2}{LC_1C_2}\right)}$$

Q4. Find the transfer impedance $V_o(s) / I_s(s)$ for the network shown in the following figure.





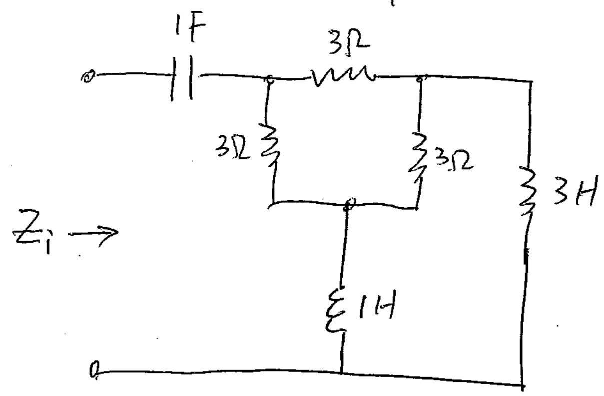
Let $Z_1 = 4 + \frac{1}{s} = \frac{4s+1}{s}\ \Omega$

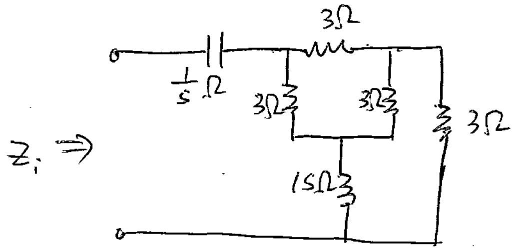
$Z_2 = 2 + 2s$

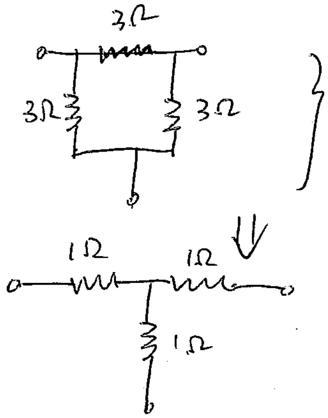
$$\frac{I_o}{I_s} = \frac{Z_2}{Z_1 + Z_2} = \frac{2+2s}{2+2s+\frac{4s+1}{s}} = \frac{2s(s+1)}{2s^2+6s+1}$$

$$\frac{V_o(s)}{I_s(s)} = \frac{I_o(s)\cdot 4}{I_s(s)} = \frac{8s(s+1)}{2s^2+6s+1}$$

Q5. Find the driving point impedance at the input terminals of the circuit as a function of s.

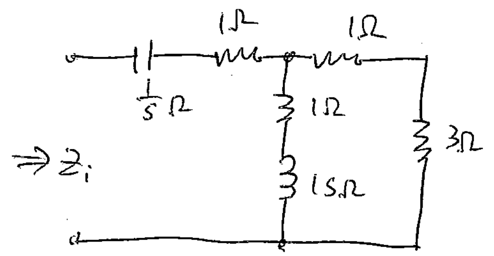






Δ connection

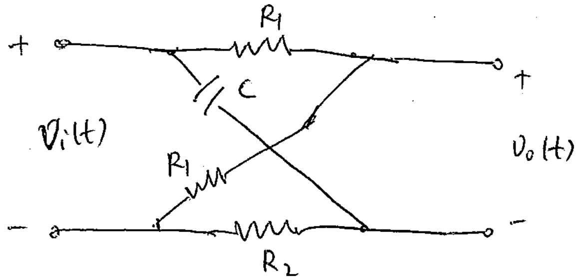
Y connection

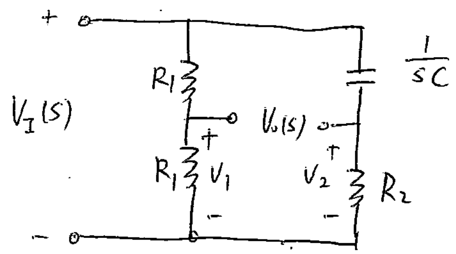


$$Z_i = \frac{1}{s} + 1 + (1+1s) \,//\, (1+3) = 1 + \frac{1}{s} + \frac{4\times(1+s)}{4+1+s} = 1 + \frac{1}{s} + \frac{4(s+1)}{s+5}$$

$$= \frac{s^2+5s+s+5+4s^2+4s}{s(s+5)} = \frac{5s^2+10s+5}{s(s+5)} = \frac{5(s^2+2s+1)}{s(s+5)}\ \Omega$$

Q6. Determine the voltage transfer function $\frac{V_o(s)}{V_I(s)}$





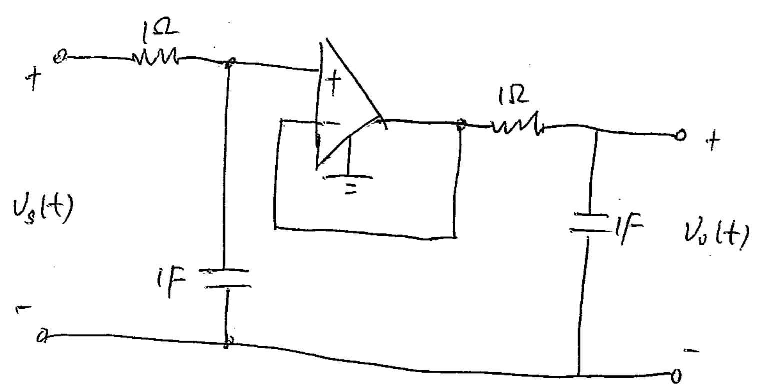
$$V_1 = V_I \cdot \frac{R_1}{R_1 + R_1} = \frac{V_I}{2}$$

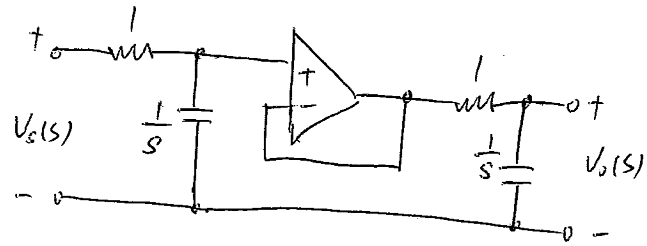
$$V_2 = V_I \cdot \frac{R_2}{R_2 + \frac{1}{sC}} = \frac{sR_2C}{sR_2C+1} \cdot V_I$$

$$V_o = V_1 - V_2 = \frac{1}{2}V_I - \frac{sR_2C}{sR_2C+1} V_I = V_I \frac{sR_2C + 1 - 2sR_2C}{2(sR_2C+1)}$$

$$\frac{V_o(s)}{V_I(s)} = \frac{1}{2} \cdot \frac{1 - sR_2C}{1 + sR_2C}$$

Q7. Find $\frac{V_o(s)}{V_s(s)}$





$$\frac{V_+(s)}{V_s(s)} = \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{1}{s+1}$$

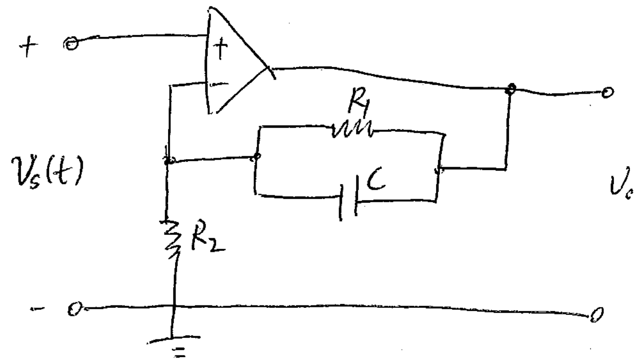
$$V_- = V_+ = \frac{1}{s+1} \cdot V_s$$

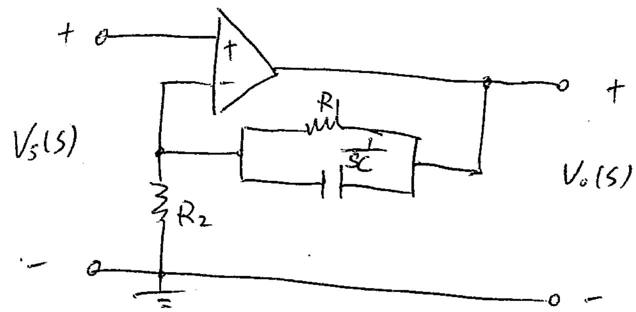
$$\frac{V_o}{V_-} = \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{1}{s+1}$$

$$V_o = \frac{1}{s+1} V_- = \frac{1}{(s+1)^2} V_s$$

$$\therefore \frac{V_o}{V_s} = \frac{1}{(s+1)^2}$$

Q8. Find the transfer function $\frac{V_o(s)}{V_s(s)}$.



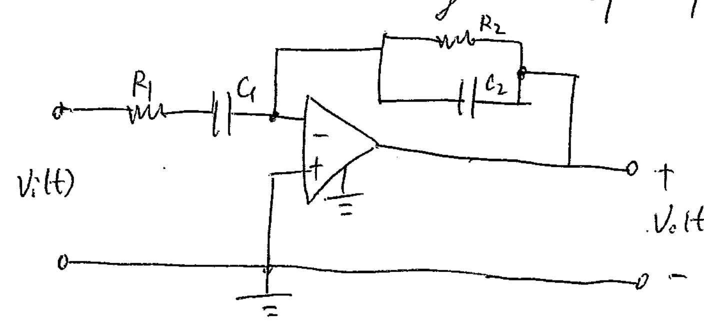


$$V_- = V_+ = V_s(s)$$

$$\frac{V_o}{V_s} = \frac{R_2 + R_1 \parallel \frac{1}{sC}}{R_2} = 1 + \frac{R_1 \cdot \frac{1}{sC}}{R_2 \cdot (R_1 + \frac{1}{sC})} = 1 + \frac{R_1}{R_2(R_1CS+1)}$$

$$= \frac{sR_1R_2C + R_2 + R_1}{sR_1R_2C + R_2} = \frac{s + \frac{R_1+R_2}{R_1R_2C}}{s + \frac{1}{R_1C}}$$

Q9. Determine the voltage transfer function $\frac{V_o(s)}{V_i(s)}$



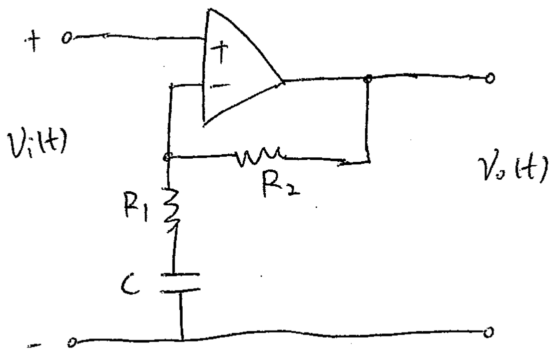
Let $Z_1 = R_1 + \frac{1}{sC_1} = \frac{sR_1C_1+1}{sC_1}$

$$Z_2 = R_2 \,//\, \frac{1}{sC_2} = \frac{R_2 \cdot \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{sC_2R_2+1}$$

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2}{Z_1} = -\frac{R_2}{sC_2R_2+1} \cdot \frac{sC_1}{sR_1C_1+1}$$

$$= -\frac{sR_2C_1}{(sC_2R_2+1)(sR_1C_1+1)}$$

Q10. Find $\dfrac{V_o(s)}{V_i(s)}$



$$V_- = V_+ = V_I(s)$$

$$\frac{V_o(s)}{V_I(s)} = \frac{R_2 + R_1 + \frac{1}{sC}}{R_1 + \frac{1}{sC}} = \frac{s(R_1+R_2)C + 1}{sR_1C + 1}$$

Q11, Skech the magnitude characteristic of Bode Plot for the transfer function $G(j\omega) = \dfrac{10(j\omega+2)(j\omega+100)}{j\omega(-\omega^2+4j\omega+100)}$.

$$G(j\omega) = \frac{10(j\omega+2)(j\omega+100)}{j\omega(-\omega^2+4j\omega+100)}$$

$$= 10\,\frac{2(\frac{j\omega}{2}+1)\cdot 100(\frac{j\omega}{100}+1)}{j\omega\cdot 100\cdot(-\frac{\omega^2}{100}+\frac{4j\omega}{100}+1)}$$

$$= \frac{20(\frac{j\omega}{2}+1)(\frac{j\omega}{100}+1)}{j\omega(-\frac{\omega^2}{100}+\frac{4j\omega}{100}+1)}$$

$K_0 = 20$

First order pole at origin.

two simple zeros : $\omega_1 = 2$ rad/s , $\omega_2 = 100$ rad/s

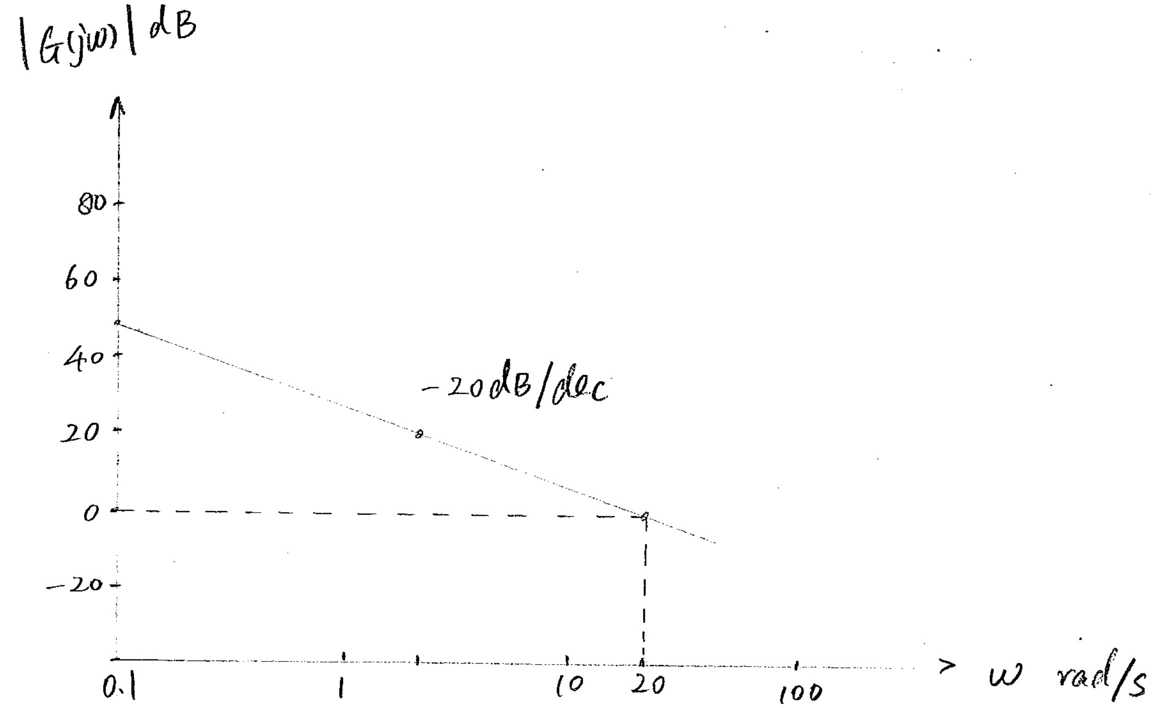
quadratic poles at : $\omega_3 = 10$ rad/s, $\zeta_3 = 0.2$

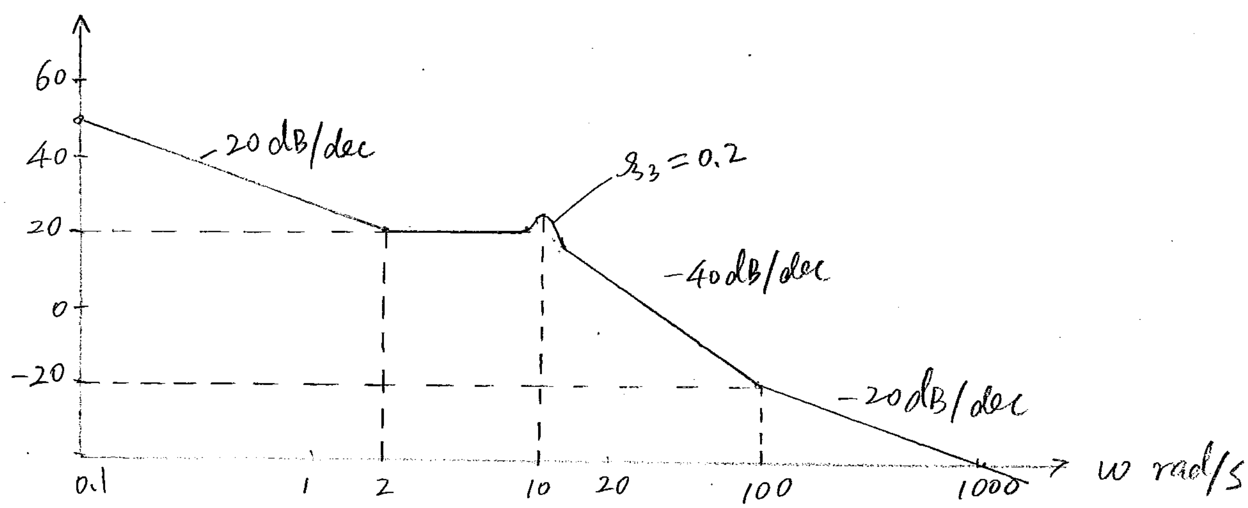
$$-\frac{\omega^2}{100} + \frac{4j\omega}{100} + 1 = \left(\frac{j\omega}{100}\right)^2 + \frac{4j\omega}{100} + 1$$

Standard form : $(j\omega\tau_3)^2 + 2\zeta_3 j\omega\tau_3 + 1$

$$\tau_3^2 = 100 \Rightarrow \tau_3 = \frac{1}{10} \qquad \omega_3 = \frac{1}{\tau_3} = 10 \text{ rad/s}$$

$$2\zeta_3\frac{1}{10} = \frac{4}{100} \Rightarrow \zeta_3 = 0.2$$


|G(jω)| dB
80
60
40
20
0
-20
-20dB/dec
0.1
1
10
20
100
ω rad/s


60
40
20
0
-20
-20dB/dec
ξ3=0.2
-40dB/dec
-20dB/dec
0.1
1
2
10
20
100
1000
ω rad/s

Q12. Find $H(j\omega)$ if its amplitude characteristic is shown in Fig. 2.

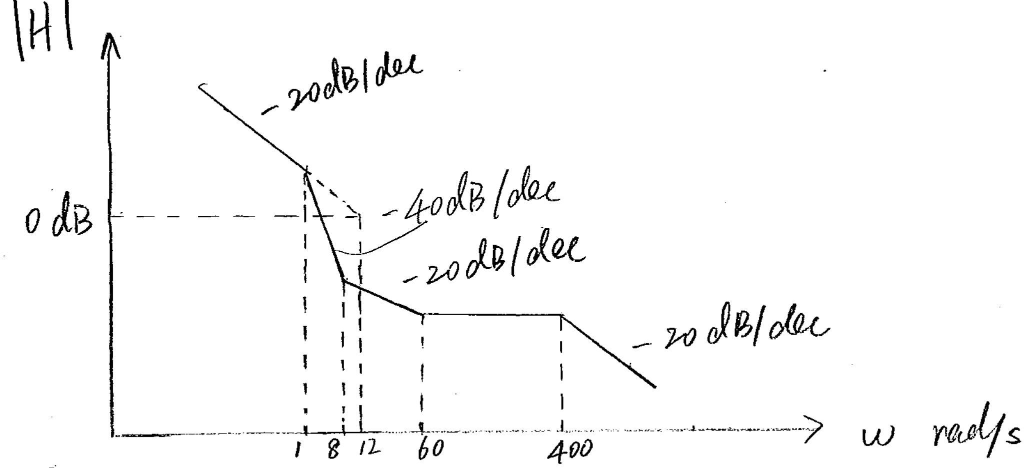


Figure. 2.

First order pole at origin

Simple poles : $\omega_1 = 1$ rad/s , $\omega_2 = 400$ rads/s

Simple zeros : $\omega_3 = 8$ rad/s $\omega_4 = 60$ rad/s

$$H(j\omega) = \frac{12}{j\omega}\,\frac{1}{j\omega+1}\cdot\left(\frac{j\omega}{8}+1\right)\cdot\left(\frac{j\omega}{60}+1\right)\cdot\frac{1}{\left(\frac{j\omega}{400}+1\right)}$$

$$= 12 \times \frac{1}{8} \times \frac{1}{60} \times \frac{1}{\frac{1}{400}} \cdot \frac{(j\omega+8)(j\omega+6)}{j\omega(j\omega+1)(j\omega+400)}$$

$$= 10 \cdot \frac{(j\omega+8)(j\omega+6)}{j\omega(j\omega+1)(j\omega+400)}$$

Q13. Given the network in Fig 3., sketch the magnitude characteristic of the transfer function

$$G_v(j\omega) = \frac{V_o}{V_i}(j\omega)$$

Indentify the type of filter

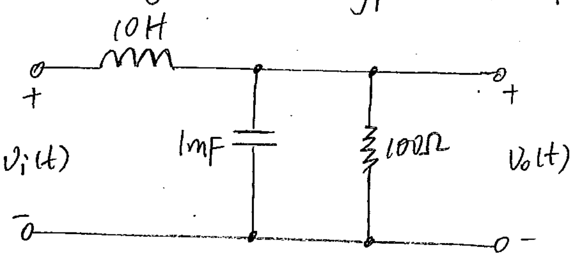
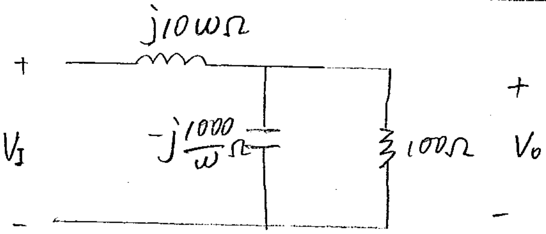


Figure 3



$$j\omega L = j10\omega\ \Omega$$

$$\frac{1}{j\omega C} = \frac{1}{j10^{-3}\omega} = -j\frac{1000}{\omega}\ \Omega$$

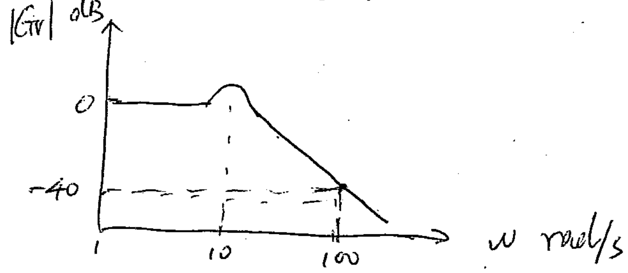
$$Z_{11} = -j\frac{1000}{\omega} \parallel 100 = \frac{-j\frac{1000}{\omega} \times 100}{-j\frac{1000}{\omega} + 100} = \frac{-j\frac{1000}{\omega}}{-j\frac{10}{\omega} + 1} = \frac{-j1000}{-j10 + \omega}$$

$$= \frac{1000}{10 + j\omega}$$

$$\frac{V_o}{V_I} = \frac{Z_{11}}{Z_{11} + j10\omega} = \frac{\frac{1000}{10+j\omega}}{\frac{1000}{10+j\omega} + j10\omega} = \frac{10^3}{10^3 + j10\omega(10+j\omega)} = \frac{100}{100 - \omega^2 + j10\omega}$$

$$\frac{V_o}{V_i} = \frac{100}{(j\omega)^2 + j\omega 10 + 100} = \frac{1}{\frac{(j\omega)^2}{100} + \frac{j\omega}{10} + 1}$$

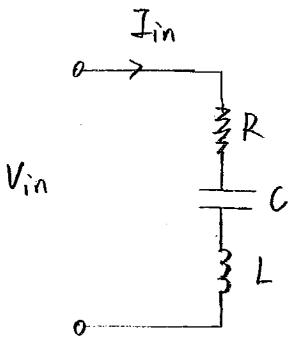
Complex conjugate poles : $\tau = \frac{1}{10}$ $\quad \frac{10}{100} = 2\xi\tau \Rightarrow \xi = 0.5$



Lowpass filter

Resonant Circuit

Series Resonance



$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$

Resonance frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$

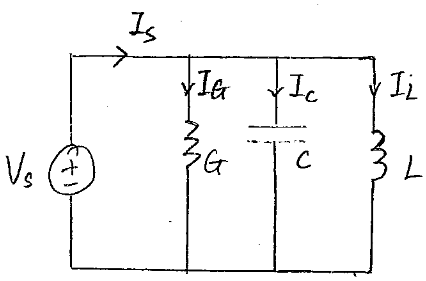
$$\min |Z(j\omega)| = Z(j\omega_0) = R$$

Quality factor:

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}} = 2\pi \cdot \frac{E \text{ stored in } C, L}{E \text{ dissipated on } R}$$

Bandwidth: $BW = \omega_{HI} - \omega_{LO} = \frac{\omega_0}{Q}$

Parallel Resonance



$$Y(j\omega) = G + j\omega C + \frac{1}{j\omega L}$$

Resonance frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Q = \frac{R}{\omega_0 L} = \frac{1}{G\,\omega_0 L} = R\omega_0 C = \frac{\omega_0 C}{G}$$

$$Y(j\omega_0) = G$$

$$BW = \frac{\omega_0}{Q}$$

Q 14. A series RLC circuit resonates at 1000 rad/s. If $C = 20\mu F$, and it is known that the impedance at resonance is 2.4Ω, compute the value of L, the Q of the circuit, and the bandwidth.

$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$

$$\omega_0 = 1000 \text{ rad/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

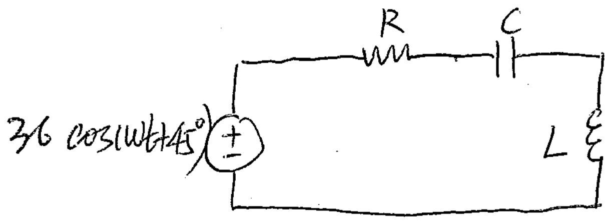
$$L = \frac{1}{\omega_0^2 C} = \frac{1}{1000^2 \times 20 \times 10^{-6}} = \frac{1}{20} = 50 mH$$

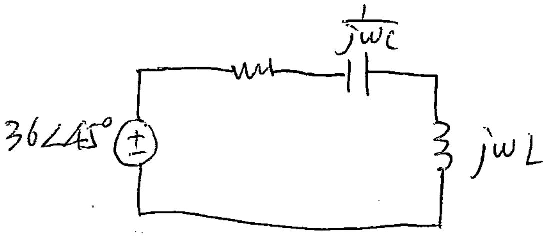
$$Z(j\omega_0) = R = 2.4\Omega$$

$$Q = \frac{\omega_0 L}{R} = \frac{1000 \times 50 \times 10^{3}}{2.4} \approx 20.83$$

$$BW = \frac{\omega_0}{Q} = \frac{1000}{20.83} \approx 48.01 \text{ rad/s.}$$

Q15. In the network, the inductor value is 10 mH, and the circuit is driven by a variable-frequency source. If the magnitude of the current at resonance is 12 A, $\omega_0 = 1000$ rad/s, and $L = 10$ mH, find C, Q and the bandwidth of the circuit.





$$Z = R + \frac{1}{j\omega C} + j\omega L = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

at resonance $\quad Z = R = \frac{36}{12} = 3\,\Omega$

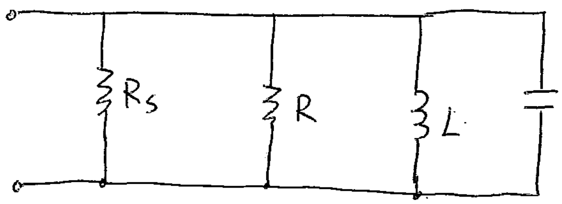
$$Q = \frac{\omega_0 L}{R} = \frac{10^3 \times 10 \times 10^{-3}}{3} = \frac{10}{3} \approx 3.33$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(10^3)^2 \cdot 10 \times 10^{-3}} = \frac{1}{10^4} = 100\,\mu F$$

$$BW = \frac{\omega_0}{Q} = \frac{10^3}{\frac{10}{3}} = 300 \text{ rad/s}$$

Q16. Consider the network, if $R = 1k\Omega$, $L = 20mH$, $C = 50\mu F$, and $R_s = \infty$, determine the resonance frequency ω_o and Q of the network, and the ~~bad~~ bandwidth of the network. What impact does an R_s of $10 k\Omega$ have on the quantities determined?



$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\times10^{-3}\times50\times10^{-6}}} = 10^3 \text{ rad/s}$$

If $R_s = \infty$,

$$Q = \omega_o RC = 10^3 \times 10^3 \times 50\times10^{-6} = 50$$

$$BW = \frac{\omega_o}{Q} = \frac{1000}{50} = 20 \text{ rad/s}$$

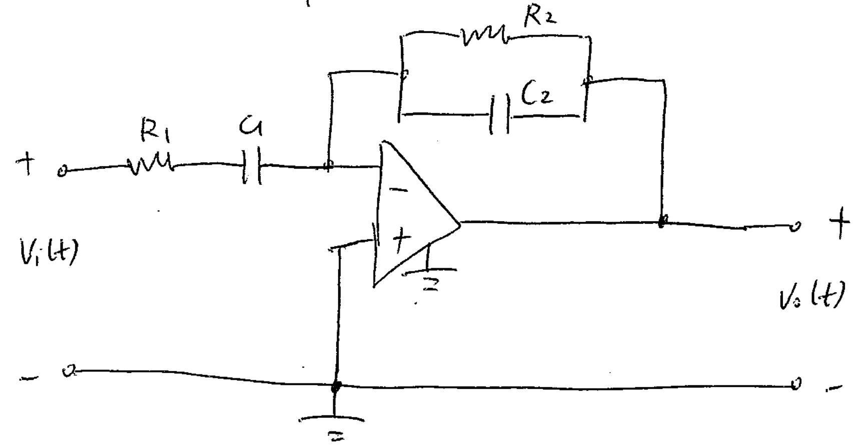
When $R_s = 10k\Omega$

$$R_{eq} = R_s \parallel R = 10k\Omega \parallel 1k\Omega = \frac{10\times1}{10+1} = \frac{10}{11} k\Omega \approx 909\Omega$$

$$Q = \omega_o R_{eq} C = 10^3 \times 909 \times 50\times10^{-6} = 45.45$$

$$BW = \frac{\omega_o}{Q} = \frac{1000}{45.45} = 22 \text{ rad/s}$$

Q17. Determine the voltage transfer function and its magnitude characteristic for the network and identify the filter properties.



(see Q9)

$$Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1+j\omega C_1 R_1}{j\omega C_1} \qquad Z_2 = R_2 \,//\, \frac{1}{j\omega C_2} = \frac{R_2 \cdot \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1+j\omega C_2 R_2}$$

$$G_v(j\omega) = \frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{R_2}{(1+j\omega C_2 R_2)} \cdot \frac{j\omega C_1}{(1+j\omega C_1 R_1)}$$

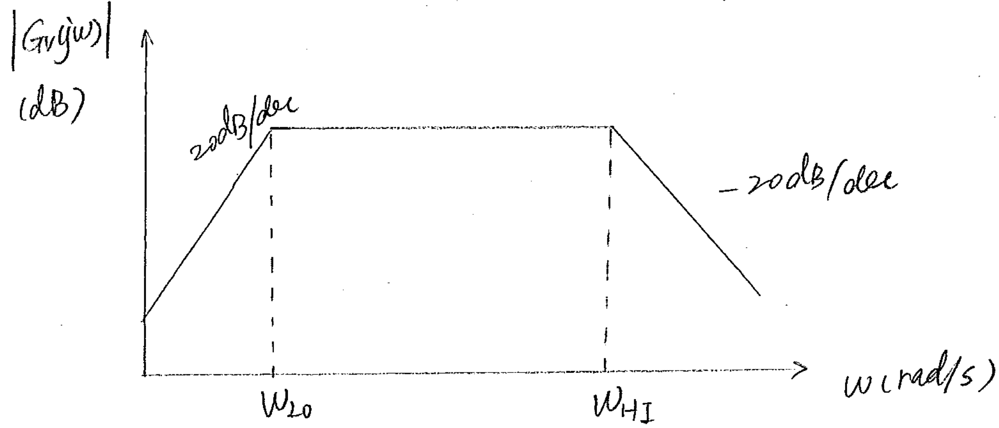
$$= -\frac{j\omega C_1 R_2}{(j\omega C_1 R_1 + 1)(j\omega C_2 R_2 + 1)}$$

zero at origin

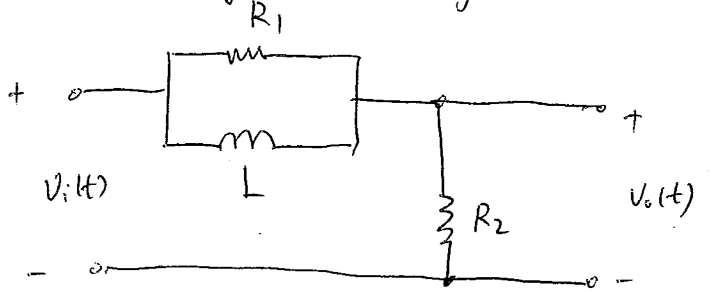
poles : $\omega_1 = \frac{1}{R_1 C_1}$ $\quad \omega_2 = \frac{1}{R_2 C_2}$

$\left|G_v(j\omega)\right|_{\omega\to 0} \to 0$ $\qquad \left|G_v(j\omega)\right|_{\omega\to\infty} \to 0$, $\qquad \left|G_v(j\omega)\right|_{0<\omega<\infty} > 0$

The filter is a bandpass filter.



Q18. Determine what type of filter the network represents by determining the voltage transfor funition.



$$G_v(j\omega) = \frac{V_o(j\omega)}{V_I(j\omega)} = \frac{R_2}{R_2 + R_1 \,//\, j\omega L} = \frac{R_2}{R_2 + \dfrac{R_1 \cdot j\omega L}{R_1 + j\omega L}}$$

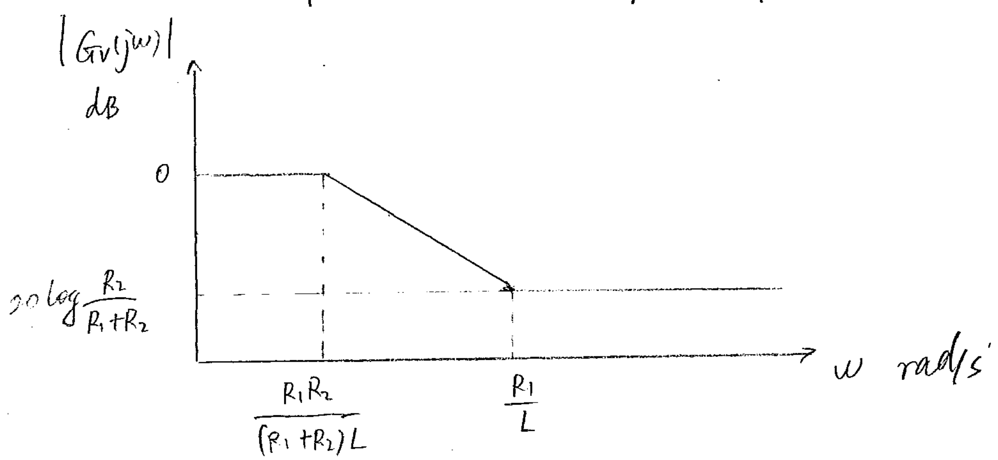
$$= \frac{R_1 R_2 + j\omega R_2 L}{R_1 R_2 + j\omega R_2 L + j\omega R_1 L}$$

$$= \frac{R_2 (R_1 + j\omega L)}{R_1 R_2 + j\omega L (R_1 + R_2)}$$

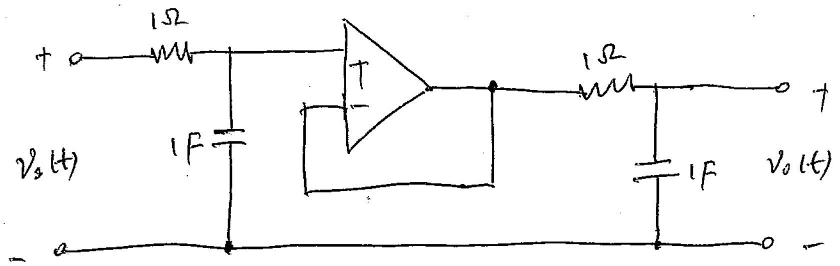
$$= \frac{R_2}{R_1 + R_2} \cdot \frac{j\omega + \frac{R_1}{L}}{j\omega + \frac{R_1 R_2}{(R_1 + R_2) L}}$$

$$G_v(j0) = 1 \qquad G_v(j\infty) = \frac{R_2}{R_1 + R_2} < 1$$

The filter is a lowpass filter



Q19. Given the network in the following figure, and employing the voltage follower analyzed in Chapter 4, determine the voltage transfer function and its magnitude characteristic. What type of filter does the network represent?

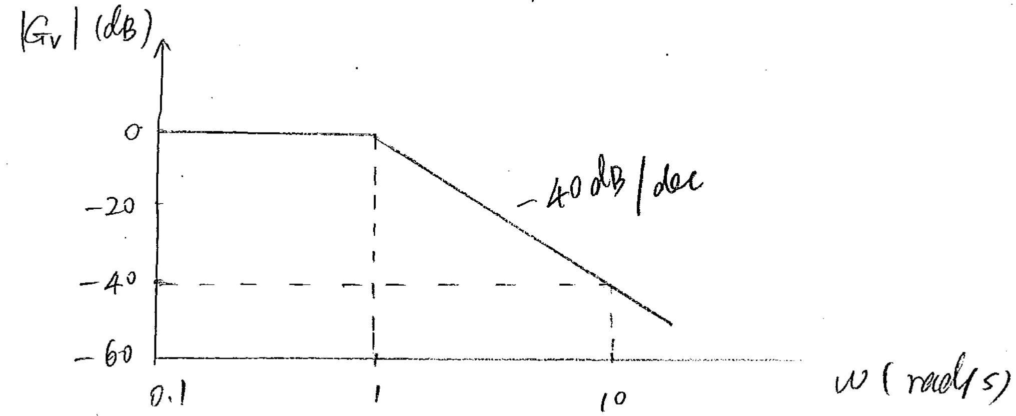


(see Q7)

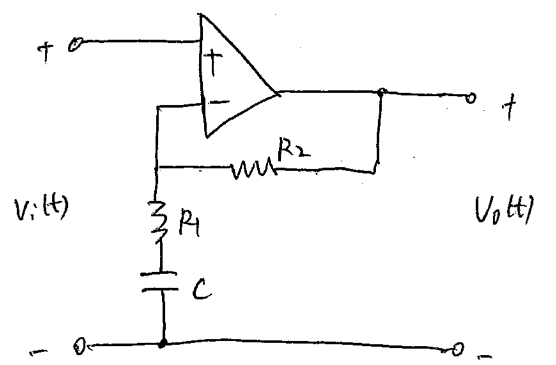
$$G_v(j\omega) = \frac{V_o(j\omega)}{V_s(j\omega)} = \frac{1}{(1+j\omega)^2}$$

second order pole at 1 rad/s

The filter is a lowpass filter.



Q20. Given the network, find the transfer function $\frac{V_o}{V_i}(j\omega)$ and determine what type of filter the network represents.



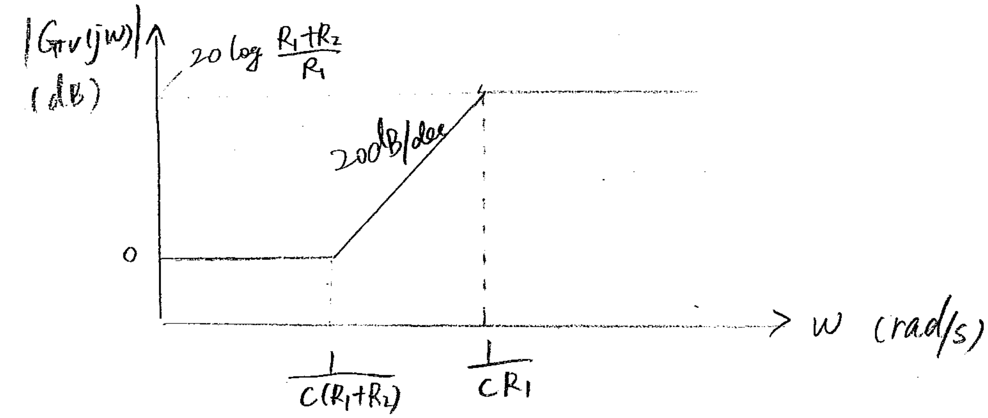
(see Q10)

$$G_V(j\omega) = \frac{V_o}{V_I}(j\omega) = \frac{1 + j\omega C(R_1 + R_2)}{1 + j\omega C R_1}$$

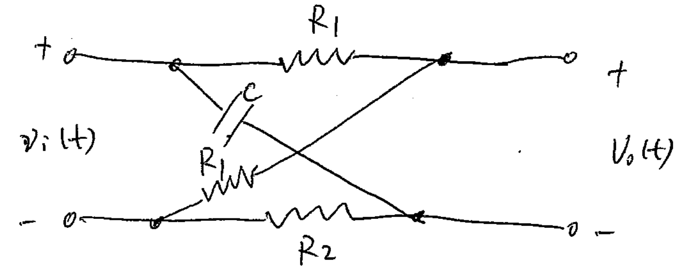
First order simple zero : $\omega_1 = \frac{1}{C(R_1 + R_2)}$

First order simple pole : $\omega_2 = \frac{1}{CR_1}$

$|G_V(j0)| = 1$ $\qquad |G_V(j\infty)| = \frac{R_1 + R_2}{R_1} > 1$ $\qquad$ High pass filter



Q21. Determine what type of filter this network represents by determining the voltage transfer function.



(see Q6)

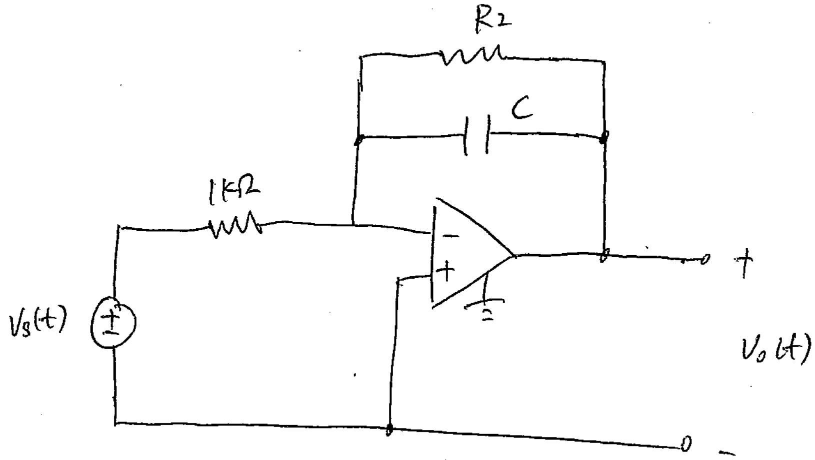
$$G_v(j\omega) = \frac{1}{2}\left[\frac{1-j\omega R_2 C}{1+j\omega R_2 C}\right]$$

$$|G_v(j\omega)| = \frac{1}{2}\,\frac{\sqrt{1+\omega^2 R_2^2 C^2}}{\sqrt{1+\omega^2 R_2^2 C^2}} = \frac{1}{2}$$

$|G_v(j\omega)|$ is independent of ω.

The filter is an allpass filter.

Q22. For the low-pass active filter in the following figure, choose R_2 and C such that $H_0 = -7$ and $f_c = 10\ kHz$



Let $Z_1 = 1\,k\Omega$

$$Z_2 = R_2 \,//\, \frac{1}{j\omega C} = \frac{R_2 \cdot \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{j\omega C R_2 + 1}$$

$$H(j\omega) = -\frac{Z_2}{Z_1} = -\frac{R_2}{(j\omega C R_2 + 1)\cdot 10^3}$$

$$= -\frac{1}{10^3 C} \cdot \frac{1}{j\omega + \frac{1}{R_2 C}}$$

$$H_0 = -\frac{R_2 C}{10^3 C} = -7 \Rightarrow R_2 = 7\,k\Omega$$

$$\omega_0 = \frac{1}{R_2 C} = 2\pi f_c = 2\pi \times 10^3 \times 10$$

$$C = \frac{1}{R_2 \cdot 2\pi \times 10^3 \times 10} = \frac{1}{2\pi \times 10^3 \times 7 \times 10^3 \times 10} \approx 2.27\,nF$$