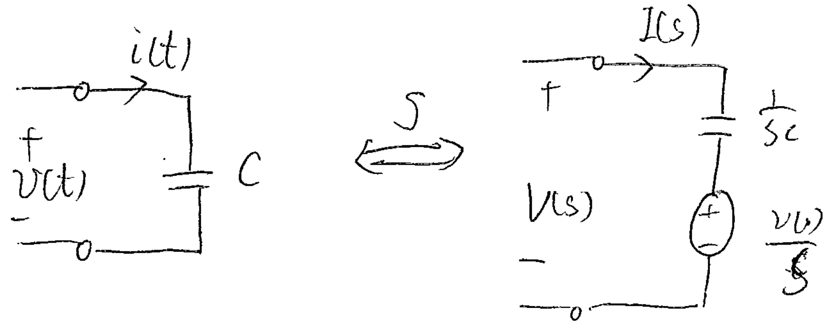
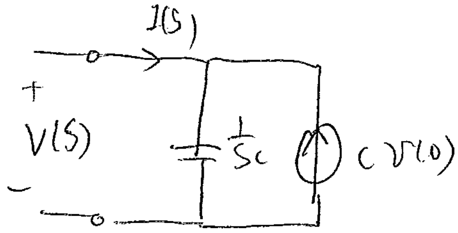
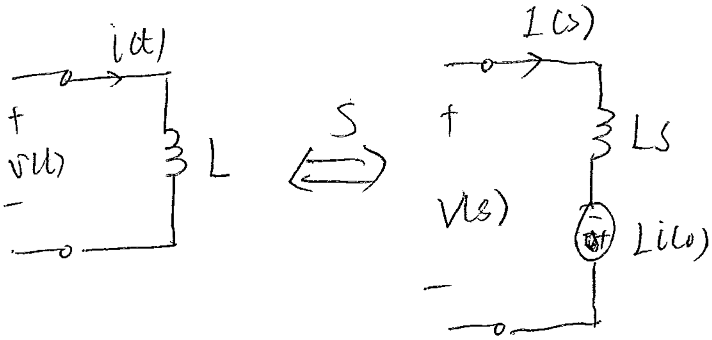


$$i(t) = C\frac{du(t)}{dt}$$

$$\Leftrightarrow I(s) = C\left(sV(s) - v(0)\right)$$

$$V(s) = \frac{1}{sC}I(s) + \frac{v(0)}{s}$$

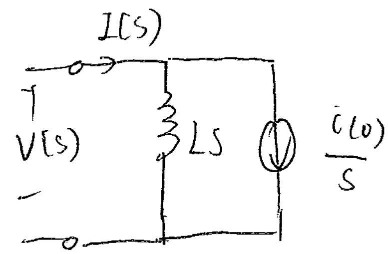




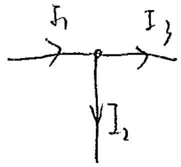
$$v(t) = L\frac{di(t)}{dt}$$

$$\Leftrightarrow V(s) = L\left(sI(s) - i(0)\right)$$

$$= LsI(s) - Li(0)$$



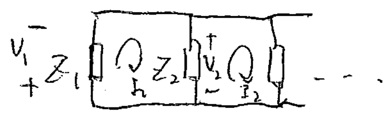
nodal analysis:



$$i_2(t) + i_3(t) = i_1(t)$$

$$I_2(s) + I_3(s) = I_1(s)$$

Loop analysis:

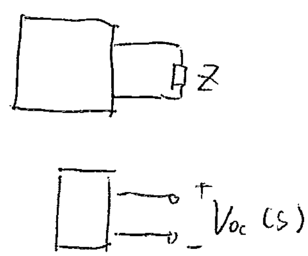


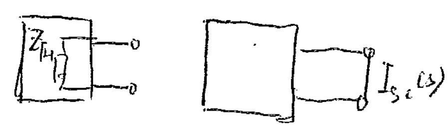
$$V_1 + V_2 = 0$$

$$I_1Z_1 + (I_1 - I_2)V_2 = 0$$

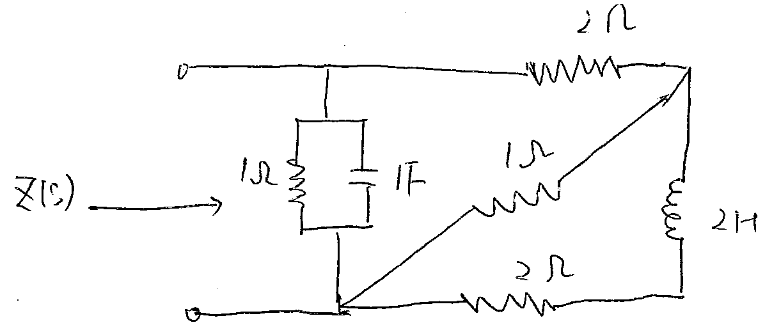
$$I_1(Z_1+Z_2) - I_2V_2 = 0$$

Thevenin's:

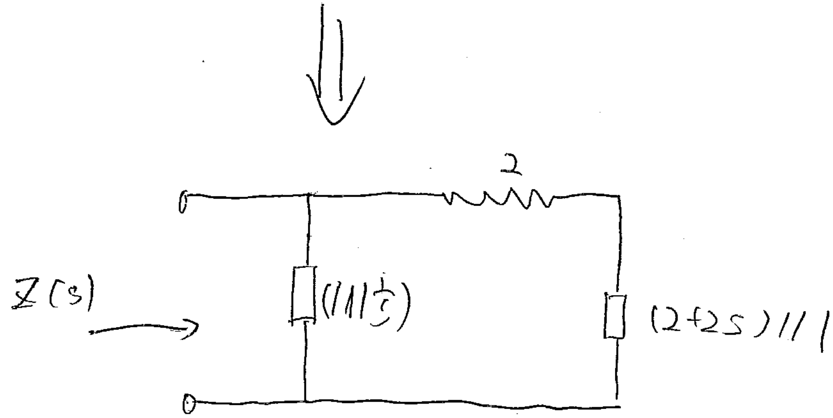




Q1: Find the input impedance $Z(s)$



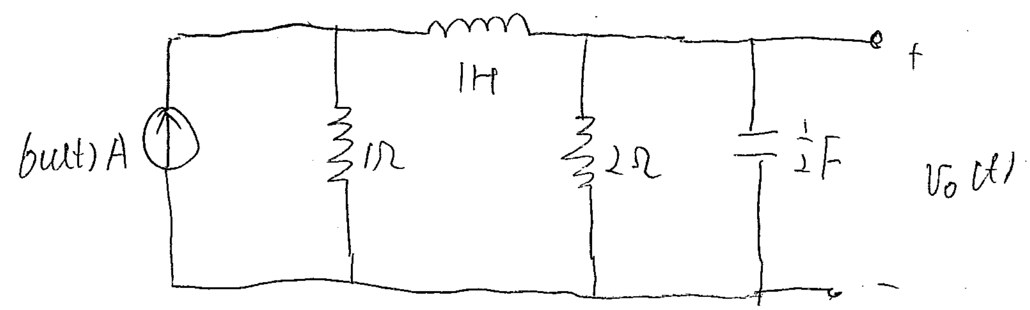
Sol:



$$Z(s) = (1 // \tfrac{1}{s}) \; // \left[2 + (2+2s) // 1 \right]$$

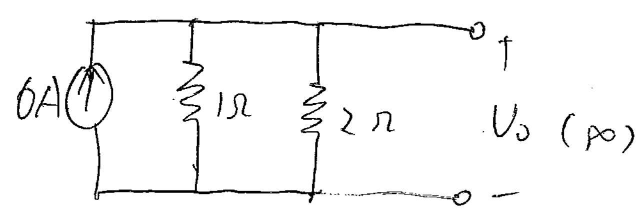
Q_2: Determine the value of the output voltage as $t \to \infty$

~~sol:~~



sol: For $t>0$, the input is DC, all voltages and currents will become DC as well. So. $V_C \to 0$ and $i_L \to 0$ as $t \to \infty$.

When $t \to \infty$,

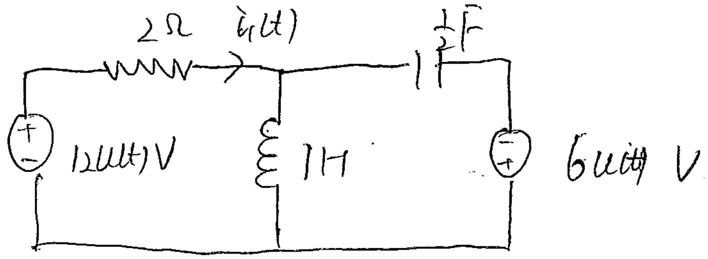


$$V_o(\infty) = 6 \cdot (1 // 2)$$

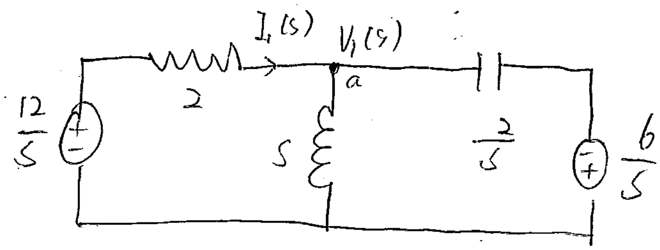
$$= 6 \cdot \frac{2}{1+2}$$

$$= 4V$$

Q3: find $i_1(t)$, $t>0$.



Sol: Because of zero initial conditions, for $t>0$

① 

② KCL at a:

$$\frac{\frac{12}{s} - V_1(s)}{2} = \frac{V_1(s)}{s} + \frac{V_1(s) + \frac{6}{s}}{\frac{2}{s}}$$

$$V_1(s) = \frac{6(2-s)}{s^2+s+2}$$

③ $$I_1(s) = \frac{\frac{12}{s} - V_1(s)}{2} = \frac{3(3s^2+4)}{s(s^2+s+2)}$$

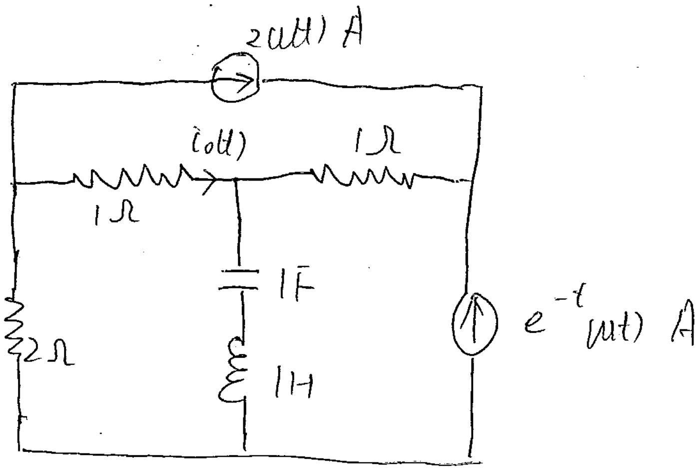
④ $$I_1(s) = \frac{A}{s} + \frac{B}{s+\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{B^*}{s+\frac{1}{2}+j\frac{\sqrt{7}}{2}}$$

⑤ $$A = I_1(s)\,s\Big|_{s=0} = \frac{12}{2} = 6$$

$$B = I_1(s)\left(s+\frac{1}{2}-j\frac{\sqrt{7}}{2}\right)\Big|_{s=-\frac{1}{2}+j\frac{\sqrt{7}}{2}} = 3.21\angle 62.1^\circ$$

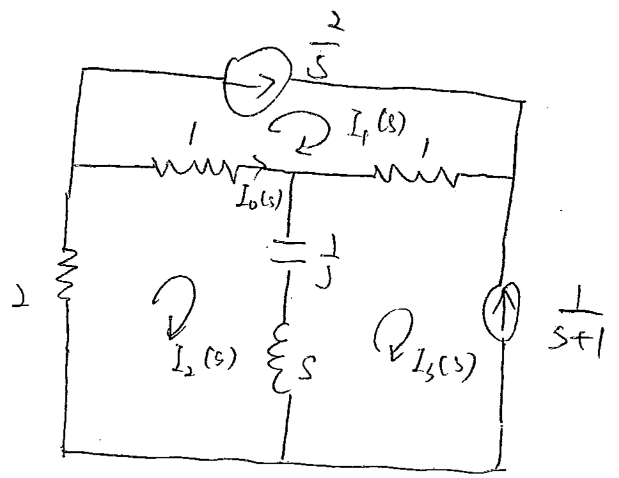
⑥ $$i_1(t) = \left[6 + 6.42\,e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{7}}{2}t + 62.1^\circ\right)\right]u(t)\ \text{A}$$

Q4: find $i_o(t)$, $t>0$.



sol. Because of zero initial conditions, when $t>0$.

①



② Loop equations: $(2+1+\frac{1}{s}+s)I_2(s) = I_1(s) + (\frac{1}{s}+s)\, I_3(s)$

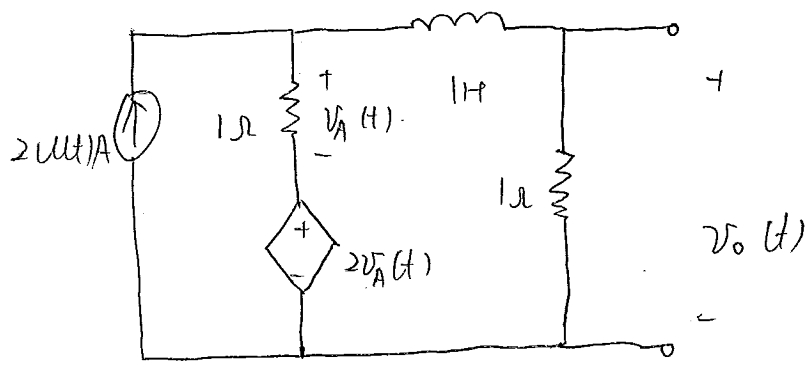
Also: $I_1(s) = \frac{2}{s}$, $I_3(s) = -\frac{1}{s+1}$, $I_o(s) = I_2(s) - I_1(s)$

③ $\Rightarrow I_o(s) = \dfrac{-(3s^3+6s^2+7s+2)}{s(s+1)(s+0.382)(s+2.62)}$

④ $I_o(s) = \dfrac{-2}{s} + \dfrac{2}{s+1} + \dfrac{0.065}{s+0.382} - \dfrac{3.06}{s+2.62}$

⑤ $i_o(t) = [-2+2e^{-t} + 0.065e^{-0.382t} - 3.06e^{-2.62t}]\,u(t)$ A

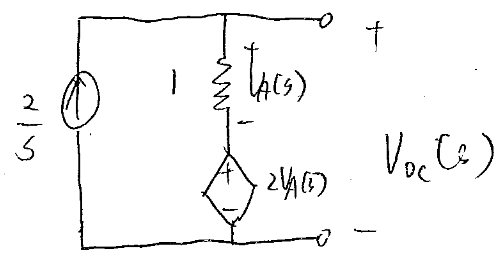
Q5: find $v_o(t)$, $t>0$



Sol: Zero initial conditions.

Using Thevenin's theorem.

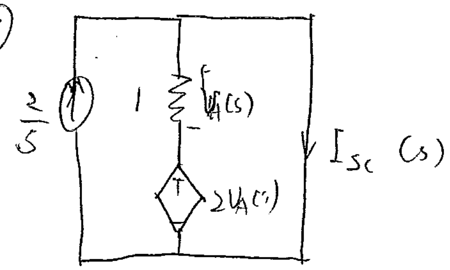
①



$V_A(s) = \frac{2}{s}\cdot 1$

$V_{oc}(s) = 3V_A(s) = \frac{6}{s}$

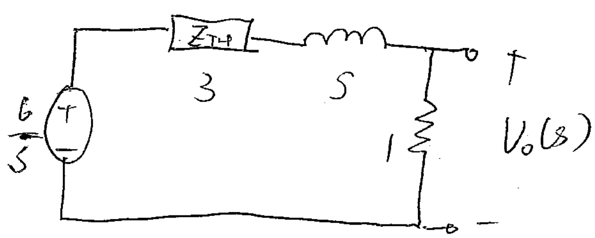
②



$I_{sc}(s) = \frac{2}{s}$

$Z_{TH}(s) = \frac{V_{oc}(s)}{I_{sc}(s)} = 3.$

③

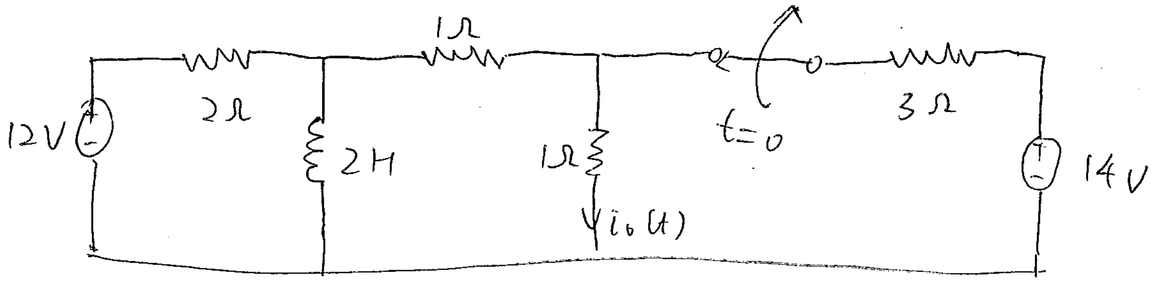


④ $V_o(s) = \frac{1}{4+s} \times \frac{6}{s}$

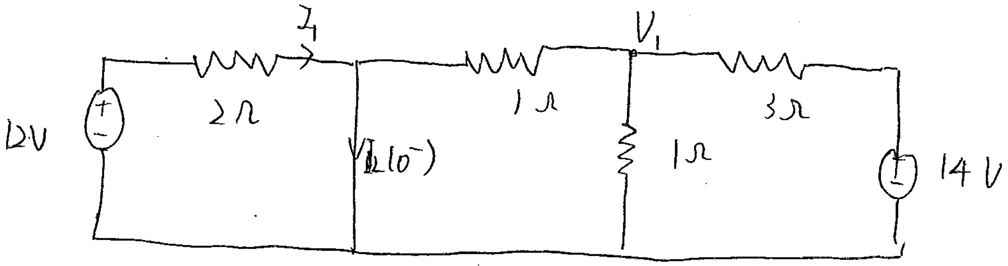
⑤ $V_o(s) = \frac{\frac{3}{2}}{s} - \frac{\frac{3}{2}}{s+4}$

⑥ $v_o(t) = 1.5\,(1-e^{-4t})\,u(t)$ V

Q6: find $i_o(t)$, $t>0$



Sol: ① when $t=0^-$

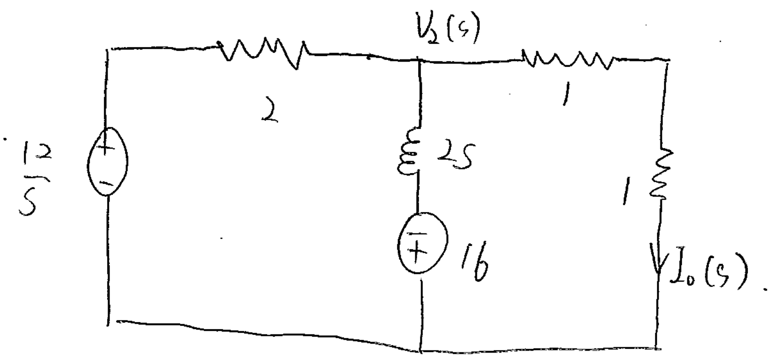


1) $I_1 = \dfrac{12V}{2\Omega} = 6A$

2) $\dfrac{14 - V_1}{3} = \dfrac{V_1}{1} + \dfrac{V_1}{1} \quad \Rightarrow \quad V_1 = 2V,$

3) $I_L(0^-) = I_1 + \dfrac{V_1}{1} = 8A$

② when $t>0$.



1) KCL at V_2: $\dfrac{\frac{12}{s} - V_2(s)}{2} = I_o(s) + \dfrac{V_2(s)+16}{2s}$

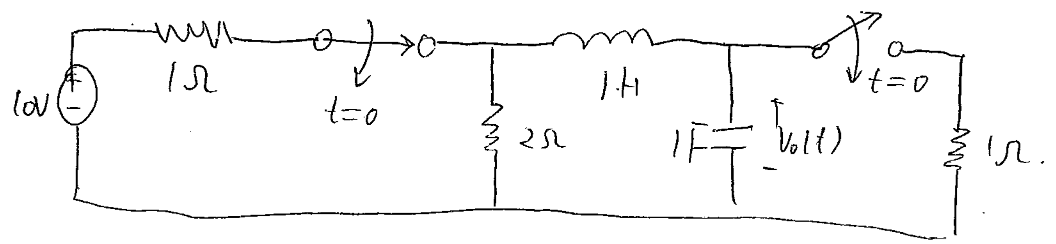
2) Also: $V_2(s) = 2I_o(s)$

Q6:

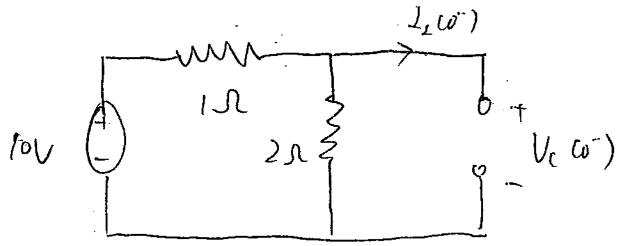
3) $$I_o(s) = \frac{-8s}{s(8s+4)} = \frac{-4}{2s+1} = \frac{-2}{s+\frac{1}{2}}$$

4) $$i_o(t) = -e^{-\frac{t}{2}} u(t) \text{ A}$$

Q7: find $v_o(t)$, $t>0$.



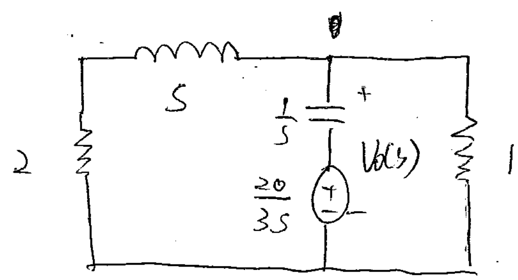
sol. ① When $t=0^-$



1) $I_L(0^-) = 0$ A

2) $V_c(0^-) = \frac{2}{1+2} \cdot 10 = \frac{20}{3}$ V.

② When $t>0$,



1) KCL: $\frac{V_o(s)}{s+2} + \frac{V_o(s)}{1} + \frac{V_o(s) - \frac{20}{3s}}{\frac{1}{s}} = 0$

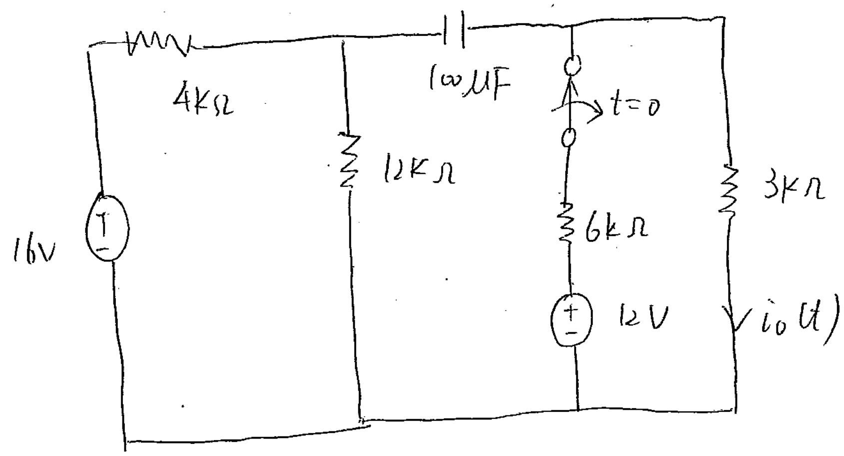
2) $V_o(s) = \frac{\frac{20}{3}(s+2)}{s^2+3s+3}$

3) $V_o(s) = \frac{A}{s+\frac{3}{2}-j\frac{\sqrt{3}}{2}} + \frac{A^*}{s+\frac{3}{2}+j\frac{\sqrt{3}}{2}}$

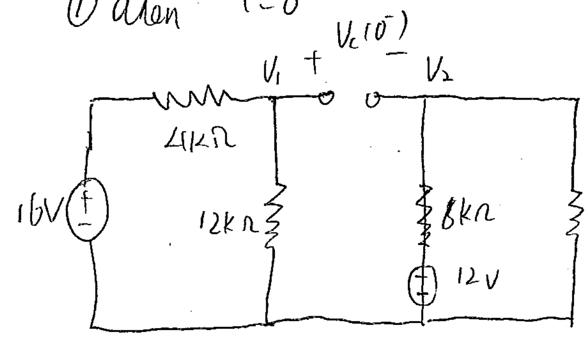
$A = V_o(s)\left(s+\frac{3}{2}-j\frac{\sqrt{3}}{2}\right)\Big|_{s=-\frac{3}{2}+j\frac{\sqrt{3}}{2}} = 3.85\angle -30^\circ$

4) $v_o(t) = \left[7.7e^{-\frac{3}{2}t}\cos\left(\frac{\sqrt{3}}{2}t - 30^\circ\right)\right]u(t)$ V

Q8: find $i_o(t)$, $t>0$.



Sol. ① when $t=0^-$

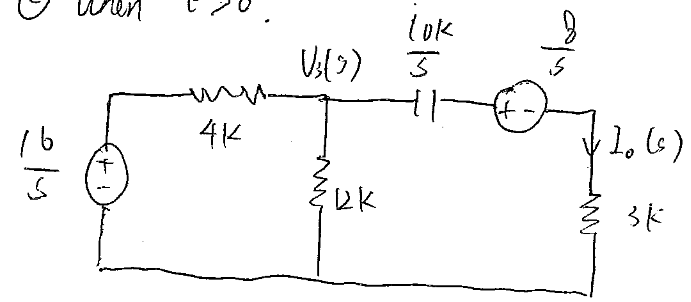


1) $V_1 = \frac{12k}{12k+4k} \cdot 16 = 12V$

$V_2 = \frac{3k}{6k+3k} \cdot 12V = 4V$

2) $V_c(0^-) = V_1 - V_2 = 8V$.

② when $t>0$.



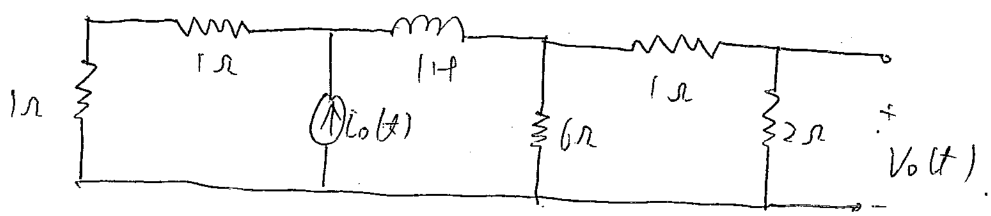
1) KCL: $\frac{\frac{16}{s} - V_3(s)}{4k} = \frac{V_3(s)}{12k} + I_o(s)$

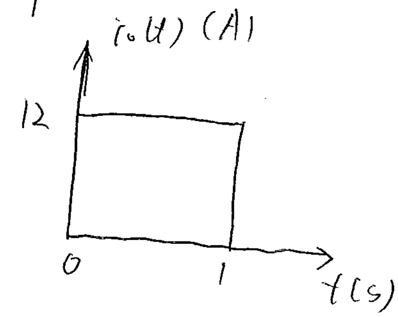
Also: $V_3(s) = \frac{8}{s} + I_o(s) \cdot (3k + \frac{10k}{s})$.

2) $I_o(s) = \frac{2/3}{s + 5/3}$ mA

3) $i_o(t) = \frac{2}{3} e^{-\frac{5}{3}t} u(t)$ mA

Q9: Find $V_o(t)$, $t>0$, if the input is resprensted by the waveform below.

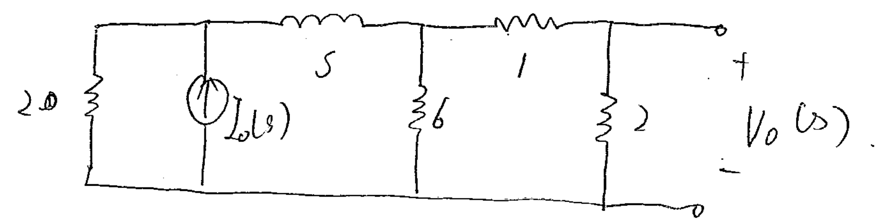




Sol: 1) $i_o(t) = 12\,u(t) - 12\,u(t-1)$ A

$I_o(s) = \frac{12}{s}(1-e^{-s})$ A

2) When $t>0$



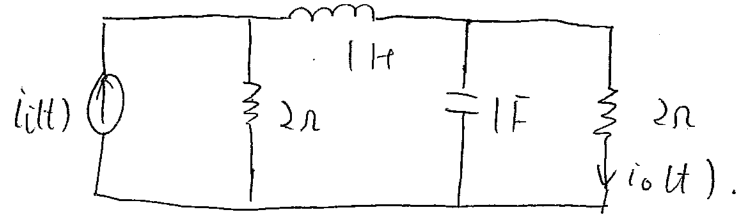
1) $V_o(s) = \frac{2}{(s+6\|3)+2} \times \frac{6}{6+3} I_o(s) \times 2$

2) $V_o(s) = \frac{32}{s(s+4)}(1-e^{-s})$

3) $V_o(s) = \left(\frac{8}{s} - \frac{8}{s+4}\right) - \left(\frac{8}{s} - \frac{8}{s+4}\right)e^{-s}$

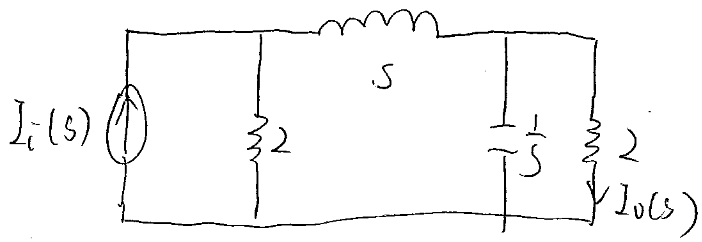
4) $V_o(t) = (8 - 8e^{-4t})u(t) - [8 - 8e^{-4(t-1)}]u(t-1)$ V.

Q10: Determine the transfer function $I_o(s) / I_i(s)$.



Sol:

$$I_o(s) = \frac{2}{2 + (s + \frac{1}{s} // 2)} \times \frac{\frac{1}{s}}{2 + \frac{1}{s}} \times I_i(s)$$



$$\frac{I_o(s)}{I_i(s)} = \frac{1}{s^2 + \frac{5}{2}s + 2}$$

Q11 Transfer function:

$$G(s) = \frac{100s}{s^2+13s+40}$$

Determine the damping ratio, the undamped frenquency, and the type of response that will be exhibited by the network.

Sol: Characteristic equation:

$$s^2 + 2\zeta w_0 s + w_0^2 = 0$$

ζ: damping ratio, w_0: undamped frenquency

$\zeta > 1$: over damped, $\zeta < 1$: underdamped. $\zeta = 1$, critically damped.

In this problem,

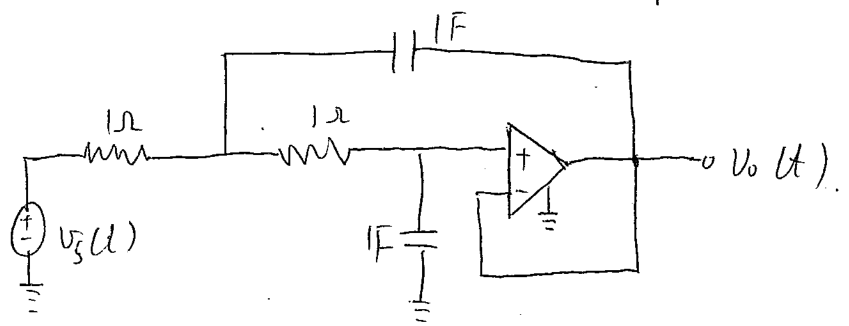
1) $s^2 + 13s + 40 = 0$

2) $w_0 = \sqrt{40}$ rad/s,

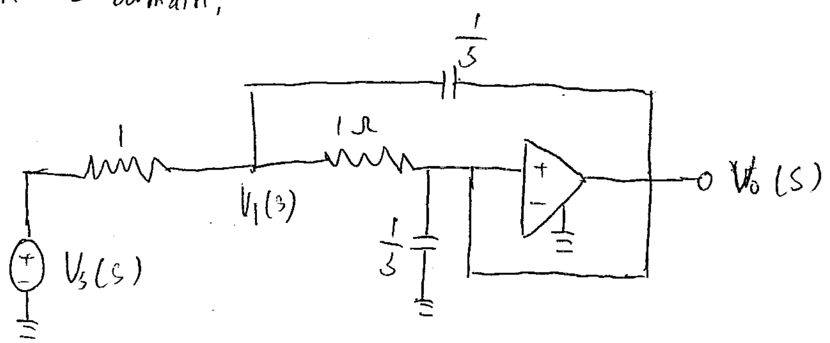
3) $2\zeta w_0 = 13 \Rightarrow \zeta = \frac{13}{2\sqrt{40}} = 1.03$

4) $\zeta > 1$, the system is over damped.

Q2. Find the transfer function. If the step function is applied to the network, will the response be over damped, under damped, or critically damped?



Sol: In s-domain,



1)

$$\begin{cases} \dfrac{V_S(s) - V_1(s)}{1} = \dfrac{V_1(s) - V_o(s)}{\frac{1}{s}} + \dfrac{V_1(s)}{1+\frac{1}{s}} \\[2ex] \dfrac{V_1(s)}{1+\frac{1}{s}} = \dfrac{V_o(s)}{\frac{1}{s}} \end{cases}$$

2)

$$\frac{V_o(s)}{V_S(s)} = \frac{1}{(s+1)^2}$$

3)

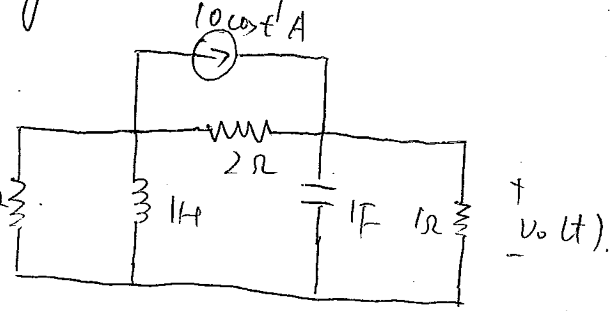
$$s^2+2s+1=0$$

$W_0 = 1$ rad/s

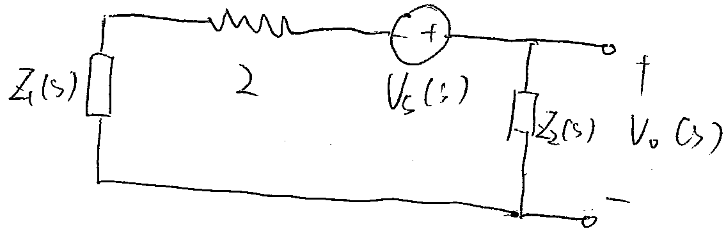
$2\zeta W_0 = 2 \Rightarrow \zeta = 1$

The system is critically damped.

Q13: Find the steady-state response $v_o(t)$.



Sol: Source change:



1) $Z_1(s) = 1 \| s = \frac{s}{1+s}$, $\quad Z_2(s) = 1 \| \frac{1}{s} = \frac{s}{s+1}$

2) $V_s(s) = 20\angle 0^\circ$ V.

3) $V_o(s) = \frac{Z_2(s)}{Z_1(s)+2+Z_2(s)} V_s(s)$

4) $V_o(s) = \frac{1/3}{1+s} V_s(s)$

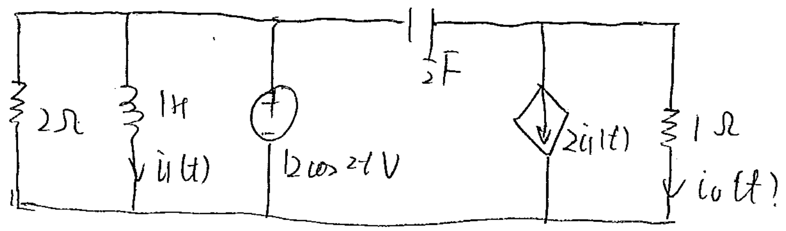
5) In steady-state, let $s = j\omega$

$$V_o(s) = \frac{1/3}{1+j\omega} \cdot 20\angle 0^\circ$$

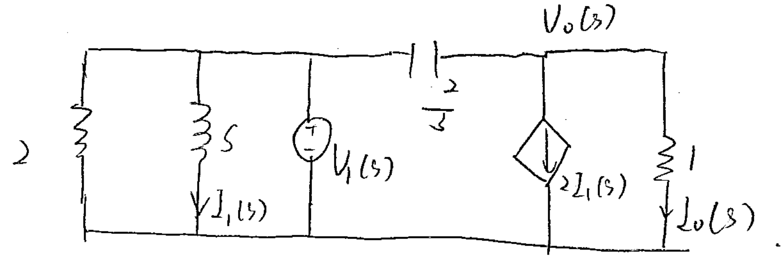
$$= 4.71\angle -45^\circ \text{ V}$$

6) $v_{ss}(t) = 4.71 \cos(t - 45^\circ)$ V.

Q14: Determine the steady-state response $i_o(t)$.



Sol: In s-domain:



$V_1(s) = 12 \angle 0° \ V.$

1) $I_1(s) = \frac{V_1(s)}{s}$, $I_o(s) = \frac{V_o(s)}{1}$

KCL: $\frac{V_o(s) - V_1(s)}{\frac{2}{s}} + 2I_1(s) + \frac{V_o(s)}{1}$

2) $I_o(s) = \frac{s-2}{s} V_1(s)$

3) In steady state, let $s = j2$.

$I_o(s) = \frac{j2-2}{j2} \cdot 12\angle 0°$

$= 16.97 \angle 45° \ A$

4) $i_{oss}(t) = 16.97 \cos(2t + 45°) \ A$