Laboratory 6: Design the Sallen-Key Low-Pass Filter

ELEC ENG 2CJ4: Circuits and Systems
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1   Objective

Active filters are widely used in analog circuits, e.g., in power, communication and control systems. In this experiment, we will study a useful second-order Butterworth filter — the Sallen-Key low-pass filter.

2   Euqipment

The following equipments are used in this laboratory:

- DC voltage source with positive and negative output($\pm$9V); Oscilloscope; Function signal generator

- Op-Amp LM358

- Resistors: $10\Omega \times 1$, $10k\Omega \times 3$, $25k\Omega \times 1$, $100k\Omega \times 1$

- Capacitors: $10nF(103) \times 2$

3   Introduction to Sallen-Key Low-pass Filters

The Sallen-Key topology was introduced by R. P. Sallen and E. L. Key of MIT Lincoln Laboratory in 1955[1].

3.1   The Transfer Function

Consider the circuit in Fig. 1. To simplify our analysis, we assume that the Op-Amp is ideal and it works in the linear region. Hence,

$$V_p = V_n. \tag{1}$$

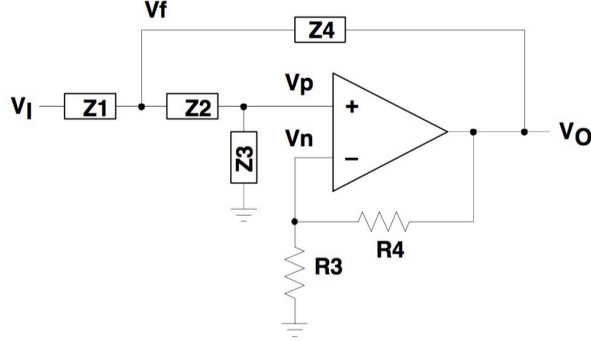[1] https://en.wikipedia.org/wiki/Sallen%E2%80%93Key_topology

Figure 1: A generic Sallen-Key filter topology Karki (1999)

Also note that

$$V_p = V_f \frac{Z_3}{Z_2 + Z_3}, \tag{2a}$$

$$V_n = V_o \frac{R3}{R_3 + R_4}. \tag{2b}$$

It follows by the KCL that

$$\frac{V_i - V_f}{Z_1} = \frac{V_f - V_o}{Z_4} + \frac{V_f}{Z_2 + Z_3}. \tag{3}$$

Combing the above equations yields

$$\frac{V_o}{V_i} = \frac{K}{\frac{Z_1 Z_2}{Z_3 Z_4} + \frac{Z_1}{Z_3} + \frac{Z_2}{Z_3} + \frac{Z_1(1-K)}{Z_4} + 1} \tag{4a}$$

$$= \frac{K Z_3 Z_4}{Z_1 Z_2 + Z_1 Z_4 + Z_2 Z_4 + Z_1 Z_3(1 - K) + Z_3 Z_4}, \tag{4b}$$

where $K = 1 + \frac{R_4}{R_3}$.

3.2   Low-Pass Sallen-Key Circuit

Consider the filter in Fig. 2. We let $Z_1 = R_1$, $Z_2 = R_2$, $Z_3 = \frac{1}{sC_1}$ and $Z_4 = \frac{1}{sC_2}$. The transfer function for this low-pass filter is given by

$$H(s) = \frac{K}{R_1 R_2 C_1 C_2 s^2 + s\left(R_1 C_1 + R_2 C_1 + R_1 C_2(1 - K)\right) + 1}. \tag{5}$$

It can be verified that $f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$ and $Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 C_1 + R_2 C_1 + R_1 C_2(1-K)}$. Setting $R_1 = R_2 = R$, $C_1 = C_2 = C$, and $K \approx 1$ gives $f_c = \frac{1}{2\pi RC}$ and $Q = 1/2$.
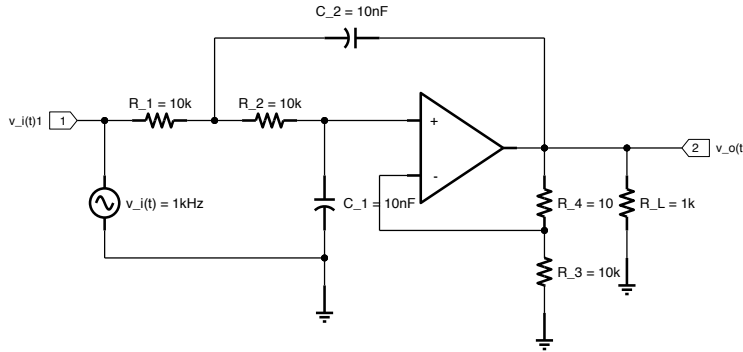
Figure 2: Low-pass Sallen-Key circuit

4 Experiment

4.1 Preparation:

a. Review the connection diagram of LM358.

b. Calculate the cutoff frequency of the filter in Fig. 2. Use Matlab to plot the transfer function. Change the value of R_1 to $25K$, $100K$ and repeat your calculation.

c. Design a low pass filter with cutoff frequency approximately $10kHz$ and a gain of $2(6dB)$ in the passband. Give the values of R_1, R_2, C_1, C_2 and try to maximize Q for the elements you have chosen. Also use Matlab to plot the frequency response.

4.2 Experiment:

a. Build the low-pass filter according to Fig. 2.

b. Sketch the waveform of $v_o(t)$, where the input $v_i(t)$ is chosen to be sinusoid, square, and triangular with frequency 1kHz and 5kHz.

c. Sweep the input signal frequency and try to plot transfer function with $v_i(t)$ chosen to be sinusoid. Hint: You can change the input signal frequency and then measure the input and output waveforms, respectively. To save your time, you can first use a large step size and then use a small step size to add more points to your plot. The input signal should be as large as possible in the permitted range to improve the accuracy. Compare the measured results with the theoretical analysis obtained using Matlab. If they do not agree with each other, can you explain why?

d. *Remove R_L and then sweep the input signal from low frequency (e.g., $10Hz$) to high frequency(e.g, $1MHz$), and observe the output signal.

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5   Results and Conclusions

Your report should include the following things:

    a. Derive the transfer function of the low-pass Sallen-Key circuit. Calculate the cutoff frequency and the Q-factor of the circuit.

    b. Use Matlab (Simulink, PSpice/ORCAD and NI Multisim are also acceptable) to plot the transfer function.

    c. Compare the theoretical analysis with the measured results. Explain the discrepancies (if any).

References

Karki, J. (1999). Analysis of the Sallen Key architecture. Technical report, Texas Instrucments, Dallas, Texas, USA.