On the Optimality of the Greedy Policy for Battery Limited Energy Harvesting Communications

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***Abstract*— Consider a battery limited energy harvesting communication system with online power control. Assuming independent and identically distributed (i.i.d.) energy arrivals and the harvest-store-use architecture, it is shown that the greedy policy achieves the maximum throughput if and only if the battery capacity is below a certain positive threshold that admits a precise characterization. Simple lower and upper bounds on this threshold are established. The asymptotic relationship between the threshold and the mean of the energy arrival process is analyzed for several examples.**

***Index Terms*— Bellman equation, energy harvesting, greedy policy, power control, throughput.**

I. Introduction

DESIGNING power control policies is a central problem in energy harvesting communications [2]. The body of literature on this problem is already quite rich (see, e.g., [3]–[22]) and continues to grow with the introduction of new mathematical techniques and the impetus from other emerging fields (see, e.g., some of the recent additions [23], [24]). Though the exact problem formulation varies depending on the system model and the performance metric, the essential challenge remains the same, which is, roughly speaking, to deal with random energy availability. In this paper we consider online power control for a battery limited energy harvesting communication system with the goal of maximizing the long-term average throughput. The aforementioned challenge is arguably most pronounced in this setting. Indeed, it is known that the impact of random energy arrivals can be smoothed out if the system is equipped with a battery of unlimited capacity [9], [21], and offline power control can achieve the same effect to a certain extent. The standard approach to the problem under consideration is based on the theory of Markov decision processes [25]. Although in principle the maximum throughput and the associated optimal online power control policy can be found by solving the relevant Bellman equation, it is often very difficult to accomplish this task analytically. Indeed, even for a simple pair of energy arrival process and power control policy, their interplay can lead to exceedingly complex dynamic behaviors. As such, there is no shortage of technical challenges in characterizing the performance of a specific policy, let alone finding the optimal one. To the best of our knowledge, no exact chacterization of the throughput-maximizing power control policy is known except for Bernoulli energy arrivals [18], [19], and typically one can only resort to policies that are asymptotically optimal [12] or approximately optimal [19], [20]. In this work, we shall show that it is possible to make definite progress by tackling the problem from a different angle. Specifically, instead of directly solving the Bellman equation to get the optimal power control policy, we use it to verify whether a given power control policy is optimal. This strategy effectively turns a hard optimization problem into a simple decision problem for which more conclusive results can be obtained (see [26], [27] for similar strategies applied in other information-theoretic contexts). In particular, it enables us to derive a threshold on the battery capacity below which the simple strategy that depletes the battery in every time slot, known as the greedy policy, is optimal. We further show that this threshold is tight, and consequently establish a sufficient and necessary condition for the optimality of the greedy policy. As a byproduct, we also obtain an exact characterization of the maximum throughput in the low-battery-capacity regime.

A significant portion of this paper is devoted to the technical aspect of the proposed strategy, namely, verifying the optimality of a given policy based on the Bellman equation. However, the importance of the preparational step, which is to identify a potential candidate, should not be overlooked. In particular, it is instructive to understand the rationale behind our special attention to the greedy policy as detailed below. Firstly, by examining the numerical plots of the optimal policy in the low-battery-capacity regime, one can get clear evidences

Manuscript received September 27, 2020; revised March 12, 2021; accepted June 21, 2021. Date of publication June 24, 2021; date of current version September 15, 2021. This work was supported in part by the Natural Sciences and Engineering Research Council (NSERC) of Canada through the Discovery Grant. This article was presented in part at the 2019 IEEE International Workshop on Signal Processing Advances in Wireless Communications [1]. *(Corresponding authors: Ali Zibaeenejad; Jun Chen.)*

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Communicated by T. Koch, Associate Editor for Communications.

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TIT.2021.3092232.

Digital Object Identifier 10.1109/TIT.2021.3092232

indicating that the greedy policy is exactly optimal[1]. Secondly, the greedy policy has the desirable and, in some sense, rare property that its induced throughput admits a simple expression under any i.i.d. energy arrivals, which greatly facilitates the optimality verification process. Last but not least, our work is also partly motivated by the observation that the optimal policy for Bernoulli energy arrivals [18], [19] degenerates to the greedy policy when the battery capacity is low enough.

The rest of the paper is organized as follows. We state the main results in Section II and present the proofs in Section III. Section IV contains the asymptotic analysis for several illustrative examples. We conclude the paper in Section V. Throughout this paper, little-o notation $f(x) = o_{x\downarrow 0}(\psi(x))$ ($f(x) = o_{x\uparrow\infty}(\psi(x))$) means $\lim_{x\downarrow 0} \frac{f(x)}{\psi(x)} = 0$ ($\lim_{x\uparrow\infty} \frac{f(x)}{\psi(x)} = 0$), and the base of the logarithm function is e.

II. Main Results

Consider a discrete-time energy harvesting communication system equipped with a battery of capacity c. Let X_t denote the amount of energy harvested at time t, $t = 1, 2, \cdots$, where $\{X_t\}_{t=1}^{\infty}$ are assumed to be i.i.d. copies of a non-negative random variable X. The following quantities will be used frequently in our analysis:

$$\rho(x) \triangleq \mathbb{P}(X < x),$$
$$\underline{x} \triangleq \max\{x \geq 0 : \rho(x) = 0\},$$
$$\overline{x} \triangleq \inf\{x \geq 0 : \rho(x) = 1\},$$
$$\mu \triangleq \mathbb{E}[X].$$

An online power control policy is a sequence of mappings $\{f_t\}_{t=1}^{\infty}$ specifying the level of energy consumption G_t in time slot t based on $X^t \triangleq (X_1, \cdots, X_t)$ for all t:

$$G_t = f_t(X^t), \quad t = 1, 2, \cdots.$$

Let B_t denote the amount of energy stored in the battery at the beginning of time slot t. We have[2]

$$B_t = \min\{B_{t-1} - G_{t-1} + X_t, c\}, \quad t = 1, 2, \cdots,$$

where $B_0 \triangleq 0$ and $G_0 \triangleq 0$. An online power control policy is said to be admissible if

$$G_t \leq B_t, \quad t = 1, 2, \cdots.$$

The throughput induced by policy $\{f_t\}_{t=1}^{\infty}$ is defined as

$$\gamma(c) \triangleq \liminf_{n\uparrow\infty} \frac{1}{n} \mathbb{E}\left[\sum_{t=1}^{n} r(f_t(X^t))\right],$$

where $r : [0, \infty) \to [0, \infty)$ is a reward function that specifies the instantaneous rate achievable with the given level of energy consumption. The maximum throughput is defined as

$$\gamma^*(c) \triangleq \sup \gamma(c),$$

where the supremum is taken over all admissible online power control policies.

In this paper, we assume that r is a non-decreasing concave function with continuous first-order derivative r' **[Assumption 1]**. Special attention is paid to the case

$$r(x) = \frac{1}{2}\log(1 + x), \quad x \geq 0, \tag{1}$$

which is relevant to the scenario where the underlying communication system is capacity-achieving for additive Gaussian noise channels.

An online power control policy $\{f_t\}_{t=1}^{\infty}$ is said to be stationary if the resulting $\{G_t\}_{t=1}^{\infty}$ and $\{B_t\}_{t=1}^{\infty}$ satisfy $G_t = f(B_t)$, $t = 1, 2, \cdots$, for some time-invariant function f. The following Bellman equation [19, Prop. 1] provides an implicit characterization of the maximum throughput and the associated optimal power control policy.

Proposition 1 (Bellman Equation): If there exist a non-negative scalar γ and a bounded function $h : [0, c] \to [0, \infty)$ that satisfy

$$\gamma + h(b) = \sup_{g\in[0,b]} \{r(g) + \mathbb{E}[h(\min\{b - g + X, c\})]\} \tag{2}$$

for all $b \in [0, c]$, then $\gamma^*(c) = \gamma$; moreover, every stationary policy f such that $f(b)$ attains the supremum in (2) for all $b \in [0, c]$ is throughput-optimal.

The greedy policy is a simple stationary policy of the form

$$G_t = B_t, \quad t = 1, 2, \cdots. \tag{3}$$

The throughput induced by the greedy policy can serve as a lower bound on $\gamma^*(c)$:

$$\gamma^*(c) \geq \underline{\gamma}(c) \triangleq \mathbb{E}[r(\min\{X, c\})].$$

On the other hand, the concavity of the reward function implies the following upper bound on $\gamma^*(c)$ [19, Prop. 2]:

$$\gamma^*(c) \leq \overline{\gamma}(c) \triangleq r(\mathbb{E}[\min\{X, c\}]).$$

We shall assume[3] $r'(\underline{x}) > r'(\overline{x})$ **[Assumption 2]** since otherwise $\underline{\gamma}(c) = \overline{\gamma}(c)$ for all $c \geq 0$. It is clear that

$$\lim_{c\downarrow 0} \frac{\underline{\gamma}(c)}{\overline{\gamma}(c)} = 1.$$

In other words, the greedy policy is asymptotically optimal when $c \downarrow 0$. To gain a better understanding, we plot[4] $\gamma^*(c)$, $\underline{\gamma}(c)$, and $\overline{\gamma}(c)$ associated with the reward function defined in (1) for various distributions[5] of X. It can be seen from the examples in Fig. 1 that, somewhat surprisingly, $\underline{\gamma}(c)$ coincides with $\gamma^*(c)$ when c is below a certain

[1]Unfortunately, such numerical evidences are not widely known. A possible explanation is as follows. Although solving the Bellman equation via value iteration yields numerical data on both the optimal policy and the maximum throughput, often only the latter is presented, as done almost exclusively in the existing literature. However, the maximum throughput by itself does not give much clue regarding the optimality of the greedy policy unless it is compared against the throughput induced by the greedy policy. On the other hand, without the right hypothesis in mind, one hardly has any motivation to perform this comparison. In contrast, it is widely known that the greedy policy is asymptotically optimal in the low-battery-capacity regime. But these two notions of optimality (exact vs. asymptotic) are of completely different nature and should not be confused.

[2]Here we adopt the popular harvest-store-use architecture, which should be contrasted with the harvest-use-store architecture in [15].

[3]We let $r'(\infty) \triangleq \lim_{x\uparrow\infty} r'(x)$, which is well-defined since r' is a non-increasing function.

[4]Here $\gamma^*(c)$ is obtained by numerically solving the Bellman equation (i.e., (2)).

[5]The definition of these distributions can be found in Section IV.

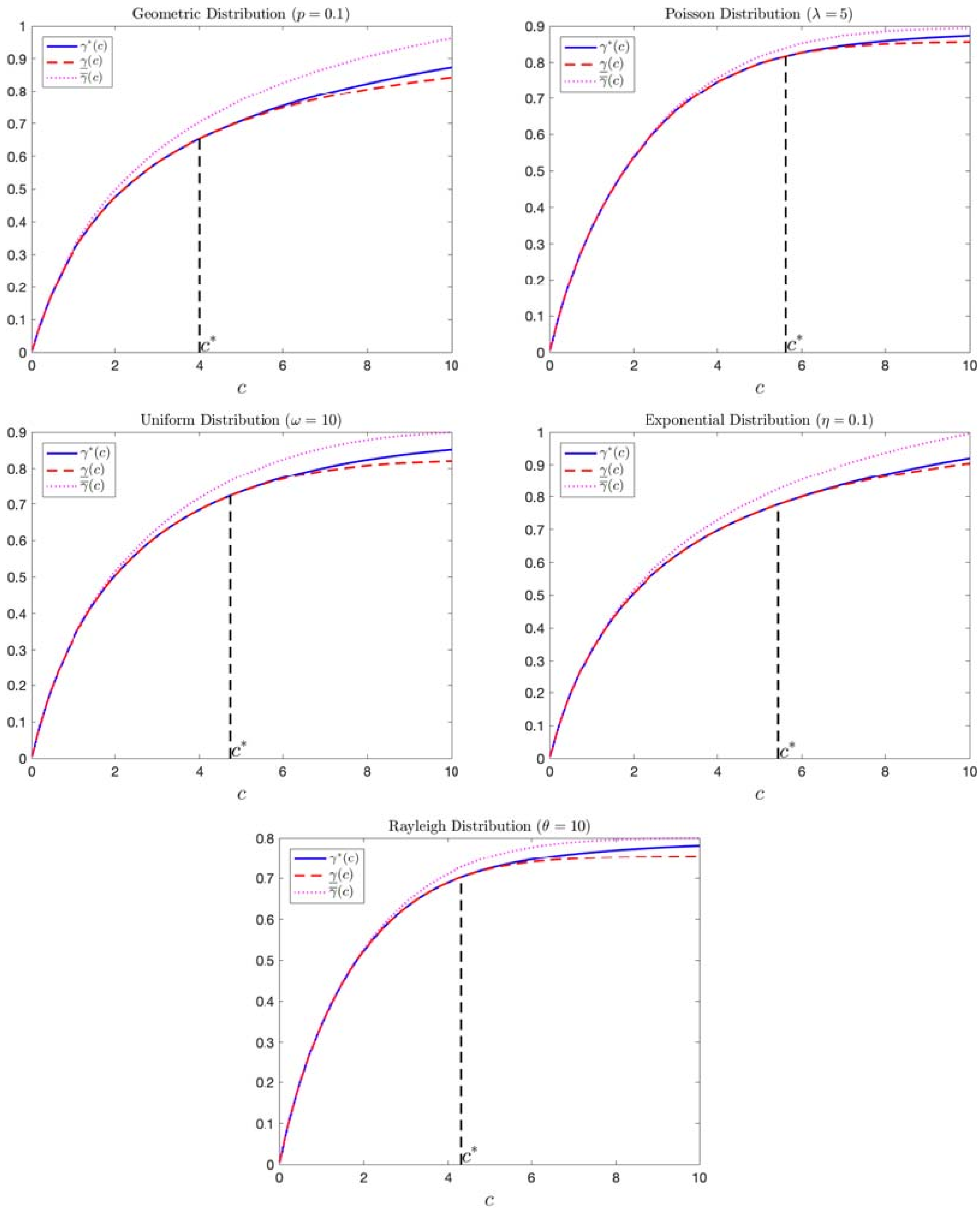


Fig. 1. Illustrations of $\gamma^*(c)$, $\underline{\gamma}(c)$, and $\overline{\gamma}(c)$ for several different distributions.

positive threshold c^* (as a consequence, the greedy policy is in fact exactly optimal in that regime). This turns out to be a general phenomenon, as shown by the following result, which also provides an analytical characterization of c^*.

Theorem 1 (Threshold c^*): Under Assumptions 1 and 2, the greedy policy is optimal, i.e., $\gamma^*(c) = \underline{\gamma}(c)$, if and only if $c \leq c^*$, where

$$c^* \triangleq \max\{c \geq 0 : r'(c) \geq \rho(c)\mathbb{E}[r'(X)|X < c]\}.$$

In particular, for the reward function defined in (1),

$$c^* = \max\left\{c \geq 0 : \frac{1}{1+c} \geq \rho(c)\mathbb{E}\left[\frac{1}{1+X}\middle| X < c\right]\right\}. \quad (4)$$

Remark 1: It is easy to see that $r'(c)$ is a non-increasing continuous function of c, and $\rho(c)\mathbb{E}[r'(X)|X < c]$ is a non-decreasing left-continuous function of c; moreover,

$$r'(\underline{x}) > \mathbb{P}(X = \underline{x})r'(\underline{x}) = \lim_{c \downarrow \underline{x}} \rho(c)\mathbb{E}[r'(X)|X < c],$$
$$r'(\overline{x}) < \mathbb{E}[r'(X)] = \lim_{c \downarrow \overline{x}} \rho(c)\mathbb{E}[r'(X)|X < c], \quad \underline{x} < \infty,$$
$$r'(\overline{x}) < \mathbb{E}[r'(X)] = \lim_{c \uparrow \overline{x}} \rho(c)\mathbb{E}[r'(X)|X < c], \quad \underline{x} = \infty.$$

These facts imply that c^* is well-defined and more generally

$$\{c \geq 0 : r'(c) \geq \rho(c)\mathbb{E}[r'(X)|X < c]\} = [0, c^*]$$

with $\underline{x} < c^* \leq \overline{x}$ (the second inequality is strict if $\overline{x} = \infty$).

Remark 2: To gain a deeper understanding of (4), it is instructive to consider the following two cases separately (see also Fig. 2).

1) Let X be a discrete random variable with probability mass function p_X. For simplicity, we assume the support of p_X is a countable set $\{\xi_1, \xi_2, \cdots\}$ with $0 \leq \xi_1 < \xi_2 < \cdots$. In this case, c^* is the unique positive number satisfying one of the following two conditions.

i) $c^* \in (\xi_j, \xi_{j+1})$ for some j and

$$\frac{1}{1+c^*} = \sum_{i=1}^{j} \frac{1}{1+\xi_i} p_X(\xi_i).$$

ii) $c^* = \xi_{j+1}$ for some j and

$$\sum_{i=1}^{j} \frac{1}{1+\xi_i} p_X(\xi_i) \leq \frac{1}{1+c^*} \leq \sum_{i=1}^{j+1} \frac{1}{1+\xi_i} p_X(\xi_i).$$

2) Let X be a continuous random variable with probability density function f_X. In this case, c^* is the unique positive number satisfying

$$\frac{1}{1+c^*} = \int_0^{c^*} \frac{1}{1+x} f_X(x) \mathrm{d}x. \tag{5}$$

Remark 3: The proof of the "if" part can be slightly modified to show that as long as r' is continuous and positive (or constantly zero) over $[0, \nu]$ for some $\nu > 0$, the greedy policy is optimal when c is sufficiently close to 0. Characterizing the sufficient and necessary condition for the optimality of the greedy policy under relaxed assumptions on the reward function is left for future work.

Remark 4: Intuitively, it makes sense to save energy only when the expected future return exceeds the current loss; with a small battery, one has no impetus to keep some energy for later because there is a good chance that the next energy arrival by itself will get the battery fully charged, rendering the saved energy wasted. This is exactly the reason why the greedy policy is optimal in the low-battery-capacity regime. On the other hand, it also suggests that the optimality of the greedy policy is specific to online power control. Indeed, for offline power control or, more generally, power control with the knowledge of future energy arrivals in a look-ahead window [28], [29], one can effectively avoid the situation that the saved energy gets wasted due to battery overflow and consequently the greedy policy is in general strictly suboptimal (even in the low-battery-capacity regime).

Proof: See Section III-A. Note that for the reward function defined in (1),

$$r'(x) = \frac{1}{2(1+x)}, \quad x \geq 0,$$

from which (4) follows immediately. ∎

Next we establish bounds on c^* that are in general easier to evaluate than c^* itself. For $c > \underline{x}$, let $\overline{r'}_{[\underline{x},c]}$ ($\underline{r'}_{[\underline{x},c]}$) denote the upper concave envelope (the lower convex envelope) of r' over $[\underline{x}, c]$.

Proposition 2 (Lower Bound on c^*):

$$c^* \geq \underline{c} \triangleq \sup\{c \in (\underline{x}, \overline{x}) : r'(c) \geq \rho(c)\overline{r'}_{[\underline{x},c]}(\underline{\xi})\}, \tag{6}$$

where

$$\underline{\xi} \triangleq \max\left\{\frac{\mu - (1-\rho(c))\overline{x}}{\rho(c)}, \underline{x}\right\}.$$

In particular, for the reward function defined in (1),

$$\underline{c} = \sup\{c \in (\underline{x}, \overline{x}) : c \leq \overline{\zeta}(c)\}, \tag{7}$$

where

$$\overline{\zeta}(c) \triangleq \frac{(1-\rho(c))(1+\underline{x}) + \rho(c)\underline{\xi}}{\rho(c)}.$$

Remark 5: It is clear that

$$r'(\underline{x}) > \mathbb{P}(X = \underline{x})r'(\underline{x}) = \lim_{c \downarrow \underline{x}} \rho(c)\overline{r'}_{[\underline{x},c]}(\underline{\xi}),$$
$$r'(\overline{x}) < r'(\underline{x}) = \lim_{c \uparrow \overline{x}} \rho(c)\overline{r'}_{[\underline{x},c]}(\underline{\xi}), \quad \overline{x} = \infty.$$

Therefore, we must have $\underline{x} < \underline{c} \leq \overline{x}$ (the second inequality is strict if $\overline{x} = \infty$).

Proof: See Section III-B. Note that for the reward function defined in (1),

$$\overline{r'}_{[\underline{x},c]}(x) = \frac{1 + \underline{x} + c - x}{2(1+\underline{x})(1+c)}, \quad x \in [\underline{x}, c],$$

from which (7) follows immediately. ∎

Proposition 3 (Upper Bound on c^*):

$$c^* \leq \overline{c} \triangleq \sup\{c \in (\underline{x}, \overline{x}) : r'(c) \geq \rho(c)\underline{r'}_{[\underline{x},c]}(\overline{\xi})\}, \tag{8}$$

where

$$\overline{\xi} \triangleq \min\left\{\frac{\mu - (1-\rho(c))c}{\rho(c)}, c\right\}.$$

In particular, for the reward function defined in (1),

$$\overline{c} = \sup\left\{c \in (\underline{x}, \overline{x}) : c \leq \underline{\zeta}(c)\right\}, \tag{9}$$

where

$$\underline{\zeta}(c) \triangleq \frac{1 - \rho(c) + \overline{\xi}}{\rho(c)}.$$

Remark 6: It is clear that

$$r'(\mu) > \mathbb{P}(X \leq \mu)r'(\mu) = \lim_{c \downarrow \mu} \rho(c)\underline{r'}_{[\underline{x},c]}(\overline{\xi}).$$

Therefore, we must have $\mu < \overline{c} \leq \overline{x}$. This implies that "$c \in (\underline{x}, \overline{x})$" in (8) and (9) can be replaced by "$c \in (\mu, \overline{x})$".

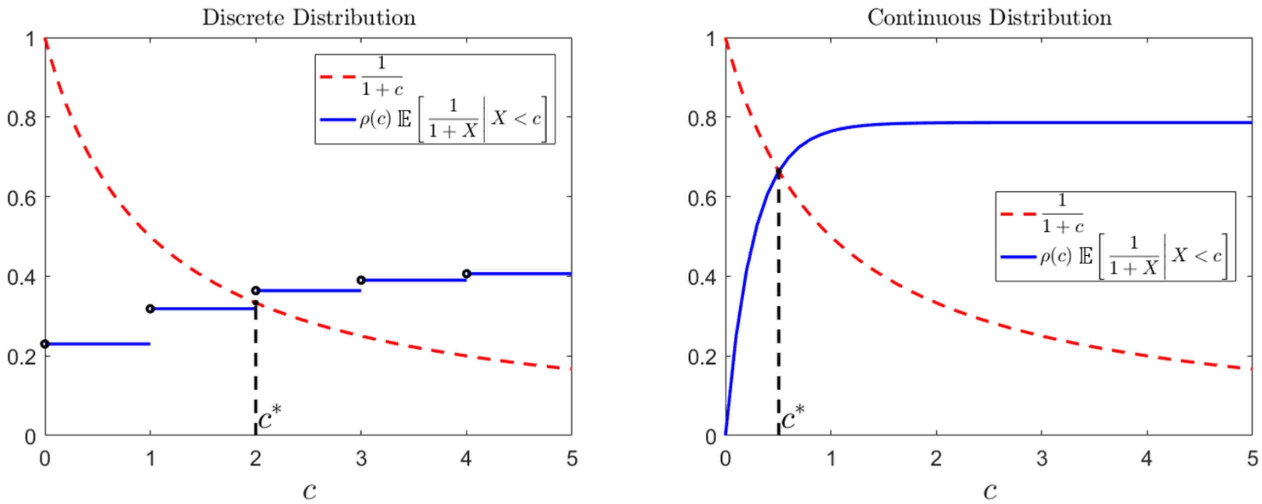


Fig. 2. Characterization of c^* for the case where X has a discrete distribution and the case where X has a continuous distribution.

In particular, through some algebraic manipulations, we can write (9) equivalently as

$$\overline{c} = \sup\left\{c \in (\mu, \overline{x}) : c \le \frac{\mu + \rho(c) - \rho^2(c)}{1 - \rho(c) + \rho^2(c)}\right\}.$$

Note that $\overline{c} = \overline{x}$ may hold even if $\overline{x} = \infty$. As shown in Appendix A, $\overline{c} = \infty$ if $\overline{x} = \infty$ and $r'(\mu - \epsilon) = r'(\overline{x})$ for some $\epsilon > 0$; on the other hand, if $\overline{x} = \infty$ and $r'(\mu) > r'(\overline{x})$, then $\overline{c} < \infty$.

Proof: See Section III-C. Note that for the reward function defined in (1),

$$\underline{r'}_{[\underline{x},c]}(x) = \frac{1}{2(1+x)}, \quad x \in [\underline{x}, c],$$

from which (9) follows immediately. ■

We further establish semi-universal bounds on c^* that depend only on $\underline{x}$, $\overline{x}$, and μ.

Proposition 4 (Semi-Universal Lower Bound on c^*):

$$c^* \ge \underline{\underline{c}} \triangleq \sup\{c \in (\underline{x}, \overline{x}) : r'(c) \ge \overline{\chi}(c)\}, \tag{10}$$

where

$$\overline{\chi}(c) \triangleq \begin{cases} \sup\limits_{\rho(c) \in \left(0, \frac{\overline{x}-\mu}{\overline{x}-c}\right)} \rho(c)\overline{r'}_{[\underline{x},c]}(\underline{\xi}), & c \in (\underline{x}, \mu], \\ \sup\limits_{\rho(c) \in \left(\frac{c-\mu}{c-\underline{x}}, 1\right)} \rho(c)\overline{r'}_{[\underline{x},c]}(\underline{\xi}), & c \in (\mu, \overline{x}). \end{cases}$$

In particular, for the reward function defined in (1),

$$\underline{\underline{c}} = \begin{cases} \frac{(1+\underline{x})(\overline{x}-\underline{x})}{\overline{x}-\mu} - 1, & \mu \le \overline{x} - \underline{x} - 1, \\ \mu, & \mu > \overline{x} - \underline{x} - 1. \end{cases} \tag{11}$$

Remark 7: We let $\underline{\underline{c}} \triangleq \underline{x}$ if $\{c \in (\underline{x}, \overline{x}) : r'(c) \ge \overline{\chi}(c)\} = \emptyset$.

Proof: See Section III-D. ■

Proposition 5 (Semi-Universal Upper Bound on c^*):

$$c^* \le \overline{\overline{c}} \triangleq \sup\{c \in (\underline{x}, \overline{x}) : r'(c) \ge \underline{\chi}(c)\}, \tag{12}$$

where

$$\underline{\chi}(c) \triangleq \begin{cases} \inf\limits_{\rho(c) \in \left(0, \frac{\overline{x}-\mu}{\overline{x}-c}\right)} \rho(c)\underline{r'}_{[\underline{x},c]}(\overline{\xi}), & c \in (\underline{x}, \mu], \\ \inf\limits_{\rho(c) \in \left(\frac{c-\mu}{c-\underline{x}}, 1\right)} \rho(c)\underline{r'}_{[\underline{x},c]}(\overline{\xi}), & c \in (\mu, \overline{x}). \end{cases}$$

In particular, for the reward function defined in (1),

$$\overline{\overline{c}} = \begin{cases} \min\{c_1, \overline{x}\}, & \mu \le \frac{3}{2}\underline{x} + \frac{1}{2}, \\ \min\{c_2, \overline{x}\}, & \mu > \frac{3}{2}\underline{x} + \frac{1}{2}, \end{cases} \tag{13}$$

where

$$c_1 \triangleq \frac{\mu + \underline{x} + \sqrt{(\mu + \underline{x})^2 - 4(\underline{x}^2 + \underline{x} - \mu)}}{2},$$
$$c_2 \triangleq \frac{4}{3}\mu + \frac{1}{3}.$$

Proof: See Section III-E. ■

Consider the reward function defined in (1) and assume that X is a Bernoulli random variable with $\mathbb{P}(X = \underline{x}) = 1 - p$ and $\mathbb{P}(X = \overline{x}) = p$, where $p \in (0, 1)$. For this special example, a simple calculation shows that

$$c^* = \underline{c} = \begin{cases} \frac{\underline{x}+p}{1-p}, & \frac{\underline{x}+p}{1-p} \le \overline{x}, \\ \overline{x}, & \frac{\underline{x}+p}{1-p} > \overline{x}, \end{cases}$$

$$\overline{c} = \begin{cases} \frac{(1-p)(\underline{x}+p)+p\overline{x}}{1-p+p^2}, & \frac{\underline{x}+p}{1-p} \le \overline{x}, \\ \overline{x}, & \frac{\underline{x}+p}{1-p} > \overline{x}, \end{cases}$$

$$\underline{\underline{c}} = \begin{cases} \frac{\underline{x}+p}{1-p}, & \frac{(2-p)\underline{x}+1}{1-p} \le \overline{x}, \\ (1-p)\underline{x} + p\overline{x}, & \frac{(2-p)\underline{x}+1}{1-p} > \overline{x}, \end{cases}$$

$$\overline{\overline{c}} = \begin{cases} \min\{c_1, \overline{x}\}, & \frac{(1+2p)\underline{x}+1}{2p} \ge \overline{x}, \\ \min\{c_2, \overline{x}\}, & \frac{(1+2p)\underline{x}+1}{2p} < \overline{x}, \end{cases}$$

where

$$c_1 = \frac{(2-p)\underline{x} + p\overline{x}}{2} + \frac{\sqrt{((2-p)\underline{x} + p\overline{x})^2 - 4(\underline{x}^2 + p(\underline{x} - \overline{x}))}}{2},$$
$$c_2 = \frac{4}{3}((1-p)\underline{x} + p\overline{x}) + \frac{1}{3}.$$

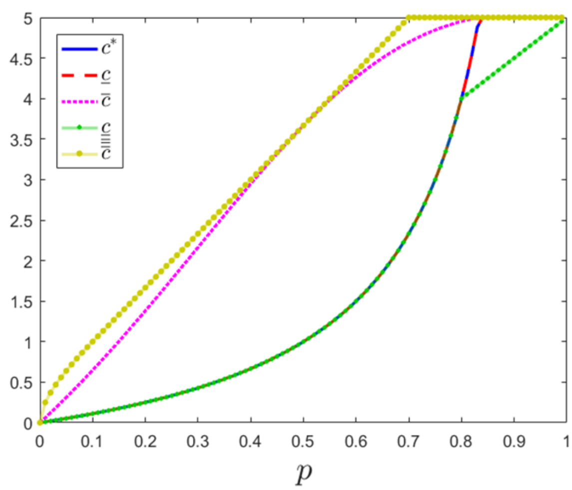


Fig. 3. Plots of c^*, $\underline{c}$, $\overline{c}$, $\underline{\underline{c}}$, and $\overline{\overline{c}}$ against p with $\underline{x} = 0$ and $\overline{x} = 5$.

Moreover, it can be verified that $\overline{c} = c^*$ if $\frac{\underline{x}+p}{1-p} \geq \overline{x}$, $\underline{c} = c^*$ if $\frac{(2-p)\underline{x}+1}{1-p} \leq \overline{x}$, and $\overline{\overline{c}} = c^*$ if $\overline{x} \leq \min\{\frac{(1+2p)\underline{x}+1}{2p}, c_1\}$ or $\frac{(3-4p)\overline{x}-1}{4(1-p)} \leq \underline{x} \leq \frac{2p\overline{x}-1}{1+2p}$. Therefore, the bounds in Propositions 2, 3, 4, and 5 are tight for non-trivial cases. Plots of c^*, $\underline{c}$, $\overline{c}$, $\underline{\underline{c}}$, and $\overline{\overline{c}}$ against p with $\underline{x} = 0$ and $\overline{x} = 5$ can be found in Fig. 3.

It is also interesting to compare the greedy policy with the well-known fixed fraction policy [19]. Both polices are linear functions of the current batter energy level, one with slope 1 and the other with slope $p \triangleq \frac{\mathbb{E}[\min\{X,c\}]}{c}$. It is known the fixed fraction policy is universally near optimal in the sense that its induced throughput is at least $\frac{1}{2-p}$ of the maximum throughput for the reward function defined in (1) (see the proofs of [19, Prop. 4 and Thm. 2]). Interestingly, this worst-case multiplicative gap is attained when X is a Bernoulli random variable with $\underline{x} = 0$ as $c \downarrow 0$ (see the proof of [30, Thm. 7]). Note that $\frac{1}{2-p}$ approaches $\frac{1}{2}$ as $p \downarrow 0$. So in the low-battery-capacity regime, the throughput induced by the greedy policy can be almost twice as large as that induced by the fixed fraction policy.

III. Proofs

A. Proof of Theorem 1

The proof is divided into two parts. We first use the Bellman equation (i.e., (2)) to verify that the greedy policy is optimal when $c \leq c^*$. Then we show that a slightly modified version of the greedy policy achieves strictly higher throughput when $c > c^*$.

The main difficulty in solving the Bellman equation is that the function h associated with the optimal power control policy is in general unknown. However, since we only aim to check the optimality of the greedy policy, it is easy to construct a candidate function h. Specifically, in view of Proposition 1, the greedy policy is optimal if

$$\begin{aligned}&\sup_{g\in[0,b]} \{r(g) + \mathbb{E}[h(\min\{b-g+X, c\})]\}\\&= r(b) + \mathbb{E}[h(\min\{X, c\})]\\&= \underline{\gamma}(c) + h(b)\end{aligned}$$

for all $b \in [0, c]$, and the second equality naturally suggests that $h(x) = r(x)$ for $x \in [0, c]$. Therefore, it suffices to check whether the supremum of $\phi(g) \triangleq r(g) + \mathbb{E}[r(\min\{b - g + X, c\})]$ over $[0, b]$ is attained at $g = b$ for all $b \in [0, c]$. We show in Appendix B that for $g \in (0, b]$,

$$\begin{aligned}&\lim_{\epsilon\downarrow 0} \frac{1}{\epsilon}(\phi(g) - \phi(g-\epsilon))\\&= r'(g) - \rho(c-b+g)\mathbb{E}[r'(b-g+X)|X < c-b+g],\end{aligned} \tag{14}$$

and for $g \in [0, b)$,

$$\begin{aligned}&\lim_{\epsilon\downarrow 0} \frac{1}{\epsilon}(\phi(g+\epsilon) - \phi(g))\\&= r'(g) - \rho(c-b+g)\mathbb{E}[r'(b-g+X)|X < c-b+g]\\&\quad - \mathbb{P}(X = c-b+g)r'(c).\end{aligned} \tag{15}$$

Therefore, ϕ is semi-differentiable and consequently is continuous over $[0, b]$. Note that for $c \in [0, c^*]$, $b \in [0, c]$, and $g \in [0, b]$,

$$\begin{aligned}&r'(g) - \rho(c-b+g)\mathbb{E}[r'(b-g+X)|X < c-b+g]\\&\geq r'(c) - \rho(c)\mathbb{E}[r'(X)|X < c]\\&\geq 0.\end{aligned}$$

So ϕ is a non-decreasing function[6] over $[0, b]$ for all $b \in [0, c]$ when $c \leq c^*$. This proves the "if" part of Theorem 1.

To prove the "only if" part of Theorem 1, we shall construct an online power control policy that outperforms the greedy policy when $c > c^*$. To this end, we modify the greedy policy, which is defined in (3), as follows: for $t = 1, 2, \cdots$,

$$\begin{aligned}G_{2t-1} &= \begin{cases} B_{2t-1} - \epsilon, & X_{2t-1} \geq \min\{\overline{x}, c\} - \epsilon,\\ B_{2t-1}, & \text{otherwise},\end{cases}\\ G_{2t} &= B_{2t},\end{aligned}$$

where $\epsilon \in (0, \frac{1}{2}\min\{\overline{x}, c\}]$. As compared to the greedy policy, the modified policy incurs a rate loss $\mathbb{E}[r(\min\{X_{2t-1}, c\})] - \mathbb{E}[r(G_{2t-1})]$ in time slot $2t-1$, but gains $\mathbb{E}[r(G_{2t})] - \mathbb{E}[r(\min\{X_{2t}, c\})]$ in time slot $2t$. It can be verified that

$$\begin{aligned}&\mathbb{E}[r(\min\{X_{2t-1}, c\})] - \mathbb{E}[r(G_{2t-1})]\\&= \mathbb{P}(X_{2t-1} \geq \min\{\overline{x}, c\} - \epsilon)\mathbb{E}[r(\min\{X_{2t-1}, c\})\\&\quad - r(\min\{X_{2t-1}, c\} - \epsilon)|X_{2t-1} \geq \min\{\overline{x}, c\} - \epsilon]\\&= \mathbb{P}(\min\{\overline{x}, c\} - \epsilon \leq X_{2t-1} < c)\mathbb{E}[r(X_{2t-1})\\&\quad - r(X_{2t-1} - \epsilon)| \min\{\overline{x}, c\} - \epsilon \leq X_{2t-1} < c]\\&\quad + \mathbb{P}(X_{2t-1} \geq c)\mathbb{E}[r(c) - r(c-\epsilon)|X_{2t-1} \geq c]\\&\leq \mathbb{P}(\min\{\overline{x}, c\} - \epsilon \leq X_{2t-1} < c)\mathbb{E}[r'(\min\{\overline{x}, c\} - 2\epsilon)\\&\quad \times \epsilon| \min\{\overline{x}, c\} - \epsilon \leq X_{2t-1} < c]\\&\quad + \mathbb{P}(X_{2t-1} \geq c)\mathbb{E}[r'(\min\{\overline{x}, c\} - 2\epsilon)\epsilon|X_{2t-1} \geq c]\\&= \mathbb{P}(X \geq \min\{\overline{x}, c\} - \epsilon)r'(\min\{\overline{x}, c\} - 2\epsilon)\epsilon,\end{aligned}$$

[6]Here we have invoked the fact that a continuous function f with non-negative left derivative must be non-decreasing. This fact can be proved as follows. Assume there exist $\alpha < \beta$ such that $f(\alpha) > f(\beta)$. Let $\kappa \triangleq \frac{f(\alpha)-f(\beta)}{2(\beta-\alpha)}$ and $\tau \triangleq \max\{x \in [\alpha, \beta] : f(x') - f(\beta) > \kappa(\beta - x')$ for all $x' \in [\alpha, x)\}$. It follows by the continuity of f that $\tau \in (\alpha, \beta]$ and $f(\tau) - f(\beta) = \kappa(\beta - \tau)$. Since the left derivative of f is non-negative at τ, there exists $\tau' \in [\alpha, \tau)$ such that $f(\tau') - f(\tau) \leq \kappa(\tau - \tau')$. Therefore, we have $f(\tau') - f(\beta) = f(\tau') - f(\tau) + f(\tau) - f(\beta) \leq \kappa(\beta - \tau')$, which is contradictory to the definition of τ.

and

$$\begin{aligned}
&\mathbb{E}[r(G_{2t})] - \mathbb{E}[r(\min\{X_{2t}, c\})] \\
&= \mathbb{P}(X_{2t-1} \geq \min\{\overline{x}, c\} - \epsilon)\mathbb{E}[r(\min\{X_{2t} + \epsilon, c\}) \\
&\quad - r(\min\{X_{2t}, c\})] \\
&= \mathbb{P}(X_{2t-1} \geq \min\{\overline{x}, c\} - \epsilon)(\mathbb{P}(X_{2t} < c - \epsilon) \\
&\quad \times \mathbb{E}[r(X_{2t} + \epsilon) - r(X_{2t}) | X_{2t} < c - \epsilon] \\
&\quad + \mathbb{P}(c - \epsilon \leq X_{2t} < c)\mathbb{E}[r(c) - r(X_{2t}) \\
&\qquad\qquad |c - \epsilon \leq X_{2t} < c]) \\
&\geq \mathbb{P}(X_{2t-1} \geq \min\{\overline{x}, c\} - \epsilon)\mathbb{P}(X_{2t} < c - \epsilon) \\
&\quad \times \mathbb{E}[r(X_{2t} + \epsilon) - r(X_{2t}) | X_{2t} < c - \epsilon] \\
&\geq \mathbb{P}(X_{2t-1} \geq \min\{\overline{x}, c\} - \epsilon)\mathbb{P}(X_{2t} < c - \epsilon) \\
&\quad \times \mathbb{E}[r'(X_{2t} + \epsilon)\epsilon | X_{2t} < c - \epsilon] \\
&= \mathbb{P}(X \geq \min\{\overline{x}, c\} - \epsilon)\rho(c - \epsilon) \\
&\quad \times \mathbb{E}[r'(X + \epsilon) | X < c - \epsilon]\epsilon.
\end{aligned}$$

Clearly, we have

$$\mathbb{P}(X \geq \min\{\overline{x}, c\} - \epsilon) > 0, \quad \epsilon > 0.$$

Moreover,

$$\lim_{\epsilon \downarrow 0} r'(\min\{\overline{x}, c\} - 2\epsilon) = r'(\min\{\overline{x}, c\}),$$

and it follows by the monotone convergence theorem that

$$\begin{aligned}
&\lim_{\epsilon \downarrow 0} \rho(c - \epsilon)\mathbb{E}[r'(X + \epsilon) | X < c - \epsilon] \\
&= \rho(c)\mathbb{E}[r'(X) | X < c].
\end{aligned}$$

If $c \leq \overline{x}$,

$$\begin{aligned}
r'(\min\{\overline{x}, c\}) &= r'(c) \\
&< \rho(c)\mathbb{E}[r'(X) | X < c], \qquad (16)
\end{aligned}$$

where (16) is due to the assumption that $c > c^*$. If $c > \overline{x}$,

$$\begin{aligned}
r'(\min\{\overline{x}, c\}) &= r'(\overline{x}) \\
&< \mathbb{E}[r'(X)] \qquad (17) \\
&= \rho(c)\mathbb{E}[r'(X) | X < c],
\end{aligned}$$

where (17) is due to the assumption that $r'(\underline{x}) > r'(\overline{x})$. Therefore, when ϵ is sufficiently close to 0,

$$\begin{aligned}
&\mathbb{E}[r(\min\{X_{2t-1}, c\})] - \mathbb{E}[r(G_{2t-1})] \\
&< \mathbb{E}[r(G_{2t})] - \mathbb{E}[r(\min\{X_{2t}, c\})]
\end{aligned}$$

and the overall throughput is improved. This proves the "only if" part of Theorem 1.

B. Proof of Proposition 2

For $c \in (\underline{x}, \overline{x})$,

$$\begin{aligned}
\mathbb{E}[r'(X) | X < c] &\leq \mathbb{E}[\overline{r'}_{[\underline{x}, c]}(X) | X < c] \\
&\leq \overline{r'}_{[\underline{x}, c]}(\mathbb{E}[X | X < c]), \qquad (18)
\end{aligned}$$

where (18) is due to Jensen's inequality. Note that

$$\begin{aligned}
\mu &= \rho(c)\mathbb{E}[X | X < c] + (1 - \rho(c))\mathbb{E}[X | X \geq c] \\
&\leq \rho(c)\mathbb{E}[X | X < c] + (1 - \rho(c))\overline{x},
\end{aligned}$$

which implies

$$\mathbb{E}[X | X < c] \geq \frac{\mu - (1 - \rho(c))\overline{x}}{\rho(c)}.$$

Moreover, we have $\mathbb{E}[X | X < c] \geq \underline{x}$. Since $\overline{r'}_{[\underline{x}, c]}(x)$ is a non-increasing[7] function of x over $[\underline{x}, c]$, it follows that

$$\overline{r'}_{[\underline{x}, c]}(\mathbb{E}[X | X < c]) \leq \overline{r'}_{[\underline{x}, c]}(\underline{\xi}). \qquad (19)$$

Combining (18) and (19) gives

$$\mathbb{E}[r'(X) | X < c] \leq \overline{r'}_{[\underline{x}, c]}(\underline{\xi}).$$

Therefore,

$$\begin{aligned}
&\left\{c \in (\underline{x}, \overline{x}) : r'(c) \geq \rho(c)\overline{r'}_{[\underline{x}, c]}(\underline{\xi})\right\} \\
&\subseteq \left\{c \in (\underline{x}, \overline{x}) : r'(c) \geq \rho(c)\mathbb{E}[r'(X) | X < c]\right\},
\end{aligned}$$

which, together with the fact (see Remark 1) that

$$\sup\left\{c \in (\underline{x}, \overline{x}) : r'(c) \geq \rho(c)\mathbb{E}[r'(X) | X < c]\right\} = c^*,$$

proves (6).

C. Proof of Proposition 3

For $c \in (\underline{x}, \overline{x})$,

$$\begin{aligned}
\mathbb{E}[r'(X) | X < c] &\geq \mathbb{E}[\underline{r'}_{[\underline{x}, c]}(X) | X < c] \\
&\geq \underline{r'}_{[\underline{x}, c]}(\mathbb{E}[X | X < c]), \qquad (20)
\end{aligned}$$

where (20) is due to Jensen's inequality. Note that

$$\begin{aligned}
\mu &= \rho(c)\mathbb{E}[X | X < c] + (1 - \rho(c))\mathbb{E}[X | X \geq c] \\
&\geq \rho(c)\mathbb{E}[X | X < c] + (1 - \rho(c))c,
\end{aligned}$$

which implies

$$\mathbb{E}[X | X < c] \leq \frac{\mu - (1 - \rho(c))c}{\rho(c)}.$$

Moreover, we have $\mathbb{E}[X | X < c] \leq c$. Since $\underline{r'}_{[\underline{x}, c]}(x)$ is a non-increasing[8] function of x over $[\underline{x}, c]$, it follows that

$$\underline{r'}_{[\underline{x}, c]}(\mathbb{E}[X | X < c]) \geq \underline{r'}_{[\underline{x}, c]}(\overline{\xi}). \qquad (21)$$

Combining (20) and (21) gives

$$\mathbb{E}[r'(X) | X < c] \geq \underline{r'}_{[\underline{x}, c]}(\overline{\xi}).$$

Therefore,

$$\begin{aligned}
&\left\{c \in (\underline{x}, \overline{x}) : r'(c) \geq \rho(c)\overline{r'}_{[\underline{x}, c]}(\underline{\xi})\right\} \\
&\supseteq \left\{c \in (\underline{x}, \overline{x}) : r'(c) \geq \rho(c)\mathbb{E}[r'(X) | X < c]\right\},
\end{aligned}$$

which, together with the fact (see Remark 1) that

$$\sup\left\{c \in (\underline{x}, \overline{x}) : r'(c) \geq \rho(c)\mathbb{E}[r'(X) | X < c]\right\} = c^*,$$

proves (8).

[7]This is because $\overline{r'}_{[\underline{x}, c]}$ is the upper concave envelope of a non-increasing function r' over $[\underline{x}, c]$.

[8]This is because $\underline{r'}_{[\underline{x}, c]}$ is the lower convex envelope of a non-increasing function r' over $[\underline{x}, c]$.

D. Proof of Proposition 4

For $c \in (\underline{x}, \overline{x})$, we have $\rho(c) \in (0,1)$ and

$$\rho(c)\underline{x} + (1-\rho(c))c < \mu < \rho(c)c + (1-\rho(c))\overline{x},$$

which can be written equivalently as

$$\frac{c-\mu}{c-\underline{x}} < \rho(c) < \frac{\overline{x}-\mu}{\overline{x}-c}.$$

Therefore, we have

$$\begin{aligned}&\{c \in (\underline{x}, \overline{x}) : r'(c) \geq \overline{\chi}(c)\}\\ &\subseteq \{c \in (\underline{x}, \overline{x}) : r'(c) \geq \rho(c)\overline{r'}_{[\underline{x},c]}(\underline{\xi})\}\end{aligned}$$

and consequently $\underline{\underline{c}} \leq \underline{c}$. Invoking Proposition 2 proves (10).

Now we proceed to prove (11). It suffices to consider the case $\overline{x} < \infty$ since otherwise $\underline{\underline{c}} = \underline{x}$ and (11) is obviously true. Clearly, $r'(c) \geq \overline{\chi}(c)$ if and only if

$$c \leq \begin{cases} \inf\limits_{\rho(c)\in\left(0,\frac{\overline{x}-\mu}{\overline{x}-c}\right)} \overline{\zeta}(c), & c \in (\underline{x}, \mu],\\ \inf\limits_{\rho(c)\in\left(\frac{c-\mu}{c-\underline{x}},1\right)} \overline{\zeta}(c), & c \in (\mu, \overline{x}),\end{cases}$$

where

$$\overline{\zeta}(c) = \begin{cases} \frac{1+\underline{x}}{\rho(c)} - 1, & \rho(c) \in \left(0, \frac{\overline{x}-\mu}{\overline{x}-\underline{x}}\right],\\ \frac{\mu-\overline{x}+\underline{x}+1}{\rho(c)} + \overline{x} - \underline{x} - 1, & \rho(c) \in \left(\frac{\overline{x}-\mu}{\overline{x}-\underline{x}}, 1\right).\end{cases}$$

For $c \in (\underline{x}, \mu]$,

$$\begin{aligned}&\inf_{\rho(c)\in\left(0,\frac{\overline{x}-\mu}{\overline{x}-c}\right)} \overline{\zeta}(c)\\ &= \min\Bigg\{\inf_{\rho(c)\in\left(0,\frac{\overline{x}-\mu}{\overline{x}-\underline{x}}\right]} \frac{1+\underline{x}}{\rho(c)} - 1,\\ &\qquad \inf_{\rho(c)\in\left(\frac{\overline{x}-\mu}{\overline{x}-\underline{x}},\frac{\overline{x}-\mu}{\overline{x}-c}\right)} \frac{\mu-\overline{x}+\underline{x}+1}{\rho(c)} + \overline{x} - \underline{x} - 1\Bigg\}\\ &= \inf_{\rho(c)\in\left(\frac{\overline{x}-\mu}{\overline{x}-\underline{x}},\frac{\overline{x}-\mu}{\overline{x}-c}\right)} \frac{\mu-\overline{x}+\underline{x}+1}{\rho(c)} + \overline{x} - \underline{x} - 1 \qquad (22)\\ &= \begin{cases} \frac{(1+\underline{x})(\overline{x}-\underline{x})}{\overline{x}-\mu} - 1, & \mu \leq \overline{x} - \underline{x} - 1,\\ \frac{(\mu-\overline{x}+\underline{x}+1)(\overline{x}-c)}{\overline{x}-\mu} + \overline{x} - \underline{x} - 1, & \mu > \overline{x} - \underline{x} - 1,\end{cases} \qquad (23)\end{aligned}$$

where (22) is due to the fact that

$$\begin{aligned}&\inf_{\rho(c)\in\left(0,\frac{\overline{x}-\mu}{\overline{x}-\underline{x}}\right]} \frac{1+\underline{x}}{\rho(c)} - 1\\ &= \frac{\mu-\overline{x}+\underline{x}+1}{\rho(c)} + \overline{x} - \underline{x} - 1\Bigg|_{\rho(c)=\frac{\overline{x}-\mu}{\overline{x}-\underline{x}}}. \qquad (24)\end{aligned}$$

For $c \in (\mu, \overline{x})$,

$$\begin{aligned}&\inf_{\rho(c)\in\left(\frac{c-\mu}{c-\underline{x}},1\right)} \overline{\zeta}(c)\\ &= \min\Bigg\{\inf_{\rho(c)\in\left(\frac{c-\mu}{c-\underline{x}},\frac{\overline{x}-\mu}{\overline{x}-\underline{x}}\right]} \frac{1+\underline{x}}{\rho(c)} - 1,\\ &\qquad \inf_{\rho(c)\in\left(\frac{\overline{x}-\mu}{\overline{x}-\underline{x}},1\right)} \frac{\mu-\overline{x}+\underline{x}+1}{\rho(c)} + \overline{x} - \underline{x} - 1\Bigg\}\\ &= \inf_{\rho(c)\in\left(\frac{\overline{x}-\mu}{\overline{x}-\underline{x}},1\right)} \frac{\mu-\overline{x}+\underline{x}+1}{\rho(c)} + \overline{x} - \underline{x} - 1 \qquad (25)\\ &= \begin{cases} \frac{(1+\underline{x})(\overline{x}-\underline{x})}{\overline{x}-\mu} - 1, & \mu \leq \overline{x} - \underline{x} - 1,\\ \mu, & \mu > \overline{x} - \underline{x} - 1,\end{cases} \qquad (26)\end{aligned}$$

where (25) is due to (24). One can readily prove (11) given (23) and (26).

E. Proof of Proposition 5

We shall only prove (13) since the proof of (12) is similar to that of (10). Clearly, $r'(c) \geq \underline{\chi}(c)$ if and only if

$$c \leq \begin{cases} \sup\limits_{\rho(c)\in\left(0,\frac{\overline{x}-\mu}{\overline{x}-c}\right)} \underline{\zeta}(c), & c \in (\underline{x}, \mu],\\ \sup\limits_{\rho(c)\in\left(\frac{c-\mu}{c-\underline{x}},1\right)} \underline{\zeta}(c), & c \in (\mu, \overline{x}),\end{cases} \qquad (27)$$

where

$$\underline{\zeta}(c) = \begin{cases} \frac{1+c}{\rho} - 1, & c \in (\underline{x}, \mu],\\ \frac{\mu+(1-\rho(c))(\rho(c)-c)}{\rho^2(c)}, & c \in (\mu, \overline{x}).\end{cases}$$

For $c \in (\underline{x}, \mu]$,

$$\sup_{\rho(c)\in\left(0,\frac{\overline{x}-\mu}{\overline{x}-c}\right)} \underline{\zeta}(c) = \infty$$

and consequently (27) trivially holds. For $c \in (\mu, \overline{x})$, we have

$$\begin{aligned}&\sup_{\rho(c)\in\left(\frac{c-\mu}{c-\underline{x}},1\right)} \underline{\zeta}(c)\\ &= \begin{cases} \frac{(c-\underline{x})(1+\underline{x})}{c-\mu} - 1, & c \leq 2\underline{x}+1,\\ \frac{(1+c)^2}{4(c-\mu)} - 1, & c \in (2\underline{x}+1, 2\mu+1],\\ \mu, & c > 2\mu+1,\end{cases}\end{aligned}$$

which is a non-increasing function of c. For $c > \mu$,

$$c = \frac{(c-\underline{x})(1+\underline{x})}{c-\mu} - 1$$

has a unique solution $c = c_1$, and

$$c = \frac{(1+c)^2}{4(c-\mu)} - 1$$

has a unique solution $c = c_2$. Note that

$$\frac{(c-\underline{x})(1+\underline{x})}{c-\mu} \leq \frac{(1+c)^2}{4(c-\mu)}, \quad c \in (\mu, 2\mu+1].$$

Therefore, $c_2 \leq 2\underline{x}+1$ (i.e., $\mu \leq \frac{3}{2}\underline{x}+\frac{1}{2}$) implies $c_1 \leq 2\underline{x}+1$. Now one can readily complete the proof of (13).

IV. Asymptotic Relationship Between c^* and μ

We shall focus on the reward function defined in (1) and provide a detailed analysis of c^* for a few examples, with a particular interest in understanding how c^* scales with μ as $\mu \downarrow 0$ or $\mu \uparrow \infty$. In the sequel we write $c^* \sim_0 \psi(\mu)$ ($c^* \sim_\infty \psi(\mu)$) to indicate that $\lim_{\mu\downarrow 0} \frac{c^*}{\psi(\mu)} = 1$ ($\lim_{\mu\uparrow\infty} \frac{c^*}{\psi(\mu)} = 1$).

A. Discrete Distribution

1) Geometric Distribution:

$$p_X(k) = (1-p)^k p, \quad k = 0, 1, \cdots,$$

where $p \in (0, 1)$.

Note that $\mu = \frac{1-p}{p}$. Clearly,

$$c^* = \mu, \quad \mu \in (0, 1],$$

which implies $c^* \sim_0 \mu$.

For any $a > 0$,

$$\begin{aligned}
&\lim_{\mu\uparrow\infty}\left(1+\frac{a\mu}{\log\mu}\right)\sum_{k=0}^{\lfloor\frac{a\mu}{\log\mu}\rfloor}\frac{(1-p)^k p}{1+k}\\
&= \lim_{\mu\uparrow\infty}\left(1+\frac{a\mu}{\log\mu}\right)\sum_{k=0}^{\lfloor\frac{a\mu}{\log\mu}\rfloor}\frac{\left(\frac{\mu}{1+\mu}\right)^k}{(1+\mu)(1+k)}\\
&= \lim_{\mu\uparrow\infty}\left(1+\frac{a\mu}{\log\mu}\right)\sum_{k=0}^{\lfloor\frac{a\mu}{\log\mu}\rfloor}\frac{1}{(1+\mu)(1+k)} \qquad (28)\\
&= \lim_{\mu\uparrow\infty}\left(1+\frac{a\mu}{\log\mu}\right)\frac{1}{1+\mu}\log\left(1+\left\lfloor\frac{a\mu}{\log\mu}\right\rfloor\right)\\
&= \lim_{\mu\uparrow\infty}\frac{a}{\log\mu}\log\left(\frac{a\mu}{\log\mu}\right)\\
&= a,
\end{aligned}$$

where (28) is due to the fact that

$$1 \geq \left(\frac{\mu}{1+\mu}\right)^k \geq \left(\frac{\mu}{1+\mu}\right)^{\frac{a\mu}{\log\mu}}, \quad k = 0, 1, \cdots, \left\lfloor\frac{a\mu}{\log\mu}\right\rfloor,$$

and

$$\lim_{\mu\uparrow\infty}\left(\frac{\mu}{1+\mu}\right)^{\frac{a\mu}{\log\mu}} = 1.$$

Therefore, we must have $c^* \sim_\infty \frac{\mu}{\log\mu}$.

2) Poisson Distribution:

$$p_X(k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k = 0, 1, \cdots,$$

where $\lambda > 0$.

Note that $\mu = \mathbb{E}[(X-\mu)^2] = \lambda$. Clearly,

$$c^* = e^\mu - 1, \quad \mu \in (0, \log 2],$$

which implies $c^* \sim_0 \mu$.

It is shown in Appendix C that for any $a > 0$,

$$\lim_{\mu\uparrow\infty}(1+a\mu)\sum_{k=\lceil\mu-\mu^{\frac{2}{3}}\rceil}^{\lfloor\mu+\mu^{\frac{2}{3}}\rfloor}\frac{e^{-\lambda}\lambda^k}{(1+k)(k!)} = a, \qquad (29)$$

$$\lim_{\mu\uparrow\infty}(1+a\mu)\sum_{k=\lfloor\mu+\mu^{\frac{2}{3}}\rfloor+1}^{\infty}\frac{e^{-\lambda}\lambda^k}{(1+k)(k!)} = 0, \qquad (30)$$

$$\lim_{\mu\uparrow\infty}(1+a\mu)\sum_{k=0}^{\lceil\mu-\mu^{\frac{2}{3}}\rceil-1}\frac{e^{-\lambda}\lambda^k}{(1+k)(k!)} = 0. \qquad (31)$$

Therefore, we have

$$\lim_{\mu\uparrow\infty}(1+a\mu)\sum_{k=0}^{\lfloor a\mu\rfloor}\frac{e^{-\lambda}\lambda^k}{(1+k)(k!)} = \begin{cases}0, & a<1,\\ a, & a>1,\end{cases}$$

which implies $c^* \sim_\infty \mu$.

We plot c^* against μ in Fig. 4 for the geometric distribution and the Poisson distribution, which confirms our asymptotic analysis.

B. Continuous Distribution

1) Uniform Distribution:

$$f_X(x) = \begin{cases}\frac{1}{\omega}, & x \in [0, \omega],\\ 0, & x \notin [0, \omega],\end{cases}$$

where $\omega > 0$.

We can write (5) equivalently as

$$\frac{1+c^*}{\omega}\log(1+c^*) = 1.$$

Note that $\mu = \frac{\omega}{2}$. For any $a > 0$,

$$\begin{aligned}
&\lim_{\mu\downarrow 0}\frac{1+a\mu}{\omega}\log(1+a\mu)\\
&= \lim_{\mu\downarrow 0}\frac{1+a\mu}{2\mu}\log(1+a\mu)\\
&= \frac{a}{2}.
\end{aligned}$$

Therefore, we must have $c^* \sim_0 2\mu$.

For any $a > 0$,

$$\begin{aligned}
&\lim_{\mu\uparrow\infty}\frac{1+\frac{a\mu}{\log\mu}}{\omega}\log\left(1+\frac{a\mu}{\log\mu}\right)\\
&= \lim_{\mu\uparrow\infty}\frac{1+\frac{a\mu}{\log\mu}}{2\mu}\log\left(1+\frac{a\mu}{\log\mu}\right)\\
&= \lim_{\mu\uparrow\infty}\frac{a}{2\log\mu}\log\left(\frac{a\mu}{\log\mu}\right)\\
&= \frac{a}{2}.
\end{aligned}$$

Therefore, we must have $c^* \sim_\infty \frac{2\mu}{\log\mu}$.

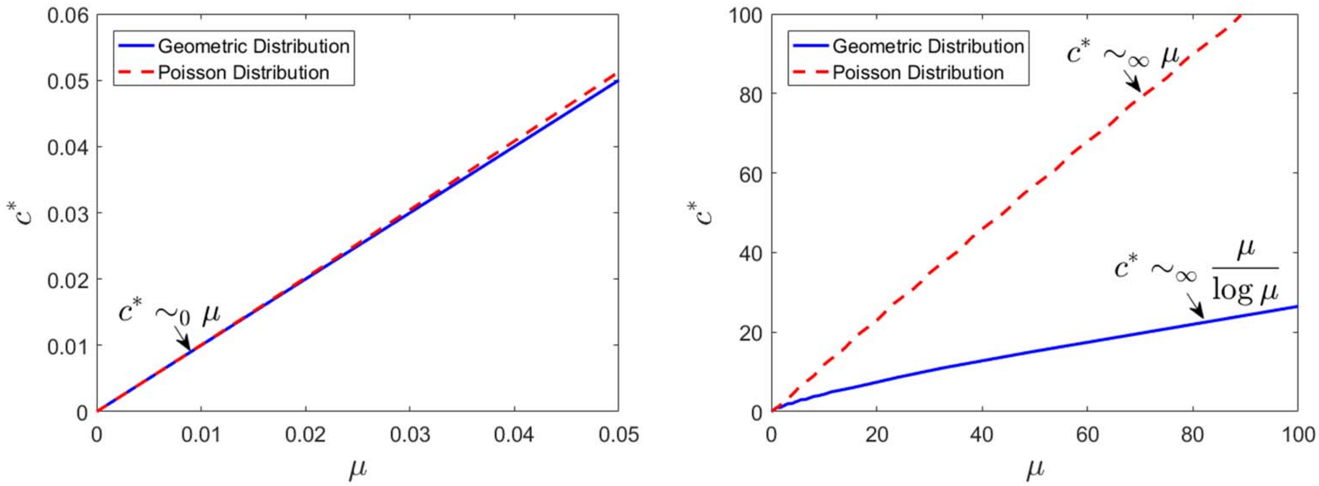


Fig. 4. The relationship between c^* and μ for some discrete distributions.

2) Exponential Distribution:

$$f_X(x) = \begin{cases} \eta e^{-\eta x}, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

where $\eta > 0$.

We can write (5) equivalently as

$$(1+c^*) \int_0^{c^*} \frac{\eta e^{-\eta x}}{1+x} \mathrm{d}x = 1.$$

Note that $\mu = \frac{1}{\eta}$. For any $a > 0$,

$$\begin{aligned} &(1 - a\mu \log \mu) \int_0^{-a\mu \log \mu} \frac{\eta e^{-\eta x}}{1+x} \mathrm{d}x \\ &= (1 - a\mu \log \mu) \int_0^{-a\mu \log \mu} \frac{e^{-\frac{x}{\mu}}}{\mu(1+x)} \mathrm{d}x \\ &= (1 - a\mu \log \mu) \int_0^{-a\mu \log \mu} \frac{e^{-\frac{x}{\mu}}}{\mu} \\ &\qquad \times (1 - x + o_{x\downarrow 0}(x)) \mathrm{d}x, \end{aligned} \tag{32}$$

where $o_{x\downarrow 0}(x) = \frac{x^2}{1+x} \leq x^2$. It can be verified that

$$\int_0^{-a\mu \log \mu} \frac{e^{-\frac{x}{\mu}}}{\mu} \mathrm{d}x = 1 - \mu^a, \tag{33}$$

and

$$\int_0^{-a\mu \log \mu} \frac{x e^{-\frac{x}{\mu}}}{\mu} \mathrm{d}x = a\mu^{a+1} \log \mu - \mu^{a+1} + \mu. \tag{34}$$

Substituting (33) and (34) into (32) gives

$$\begin{aligned} &(1 - a\mu \log \mu) \int_0^{-a\mu \log \mu} \frac{\eta e^{-\eta x}}{1+x} \mathrm{d}x \\ &= 1 - a\mu \log \mu - \mu^a + o_{\mu\downarrow 0}(\mu \log \mu). \end{aligned}$$

When μ is sufficiently close to 0,

$$1 - a\mu \log \mu - \mu^a + o_{\mu\downarrow 0}(\mu \log \mu) \begin{cases} < 1, & a < 1, \\ > 1, & a > 1. \end{cases}$$

Therefore, we must have $c^* \sim_0 -\mu \log \mu$.

For any $a > 0$,

$$\begin{aligned} &\lim_{\mu\uparrow\infty} \left(1 + \frac{a\mu}{\log \mu}\right) \int_0^{\frac{a\mu}{\log \mu}} \frac{\eta e^{-\eta x}}{1+x} \mathrm{d}x \\ &= \lim_{\mu\uparrow\infty} \left(1 + \frac{a\mu}{\log \mu}\right) \int_0^{\frac{a\mu}{\log \mu}} \frac{e^{-\frac{x}{\mu}}}{\mu(1+x)} \mathrm{d}x \\ &= \lim_{\mu\uparrow\infty} \left(1 + \frac{a\mu}{\log \mu}\right) \int_0^{\frac{a\mu}{\log \mu}} \frac{1}{\mu(1+x)} \mathrm{d}x \qquad (35) \\ &= \lim_{\mu\uparrow\infty} \left(1 + \frac{a\mu}{\log \mu}\right) \frac{1}{\mu} \log\left(1 + \frac{a\mu}{\log \mu}\right) \\ &= \lim_{\mu\uparrow\infty} \frac{a}{\log \mu} \log\left(\frac{a\mu}{\log \mu}\right) \\ &= a, \end{aligned}$$

where (35) is due to the fact that

$$1 \geq e^{-\frac{x}{\mu}} \geq e^{-\frac{a}{\log \mu}}, \quad x \in \left[0, \frac{a\mu}{\log \mu}\right],$$

and

$$\lim_{\mu\uparrow\infty} e^{-\frac{a}{\log \mu}} = 1.$$

Therefore, we must have $c^* \sim_\infty \frac{\mu}{\log \mu}$.

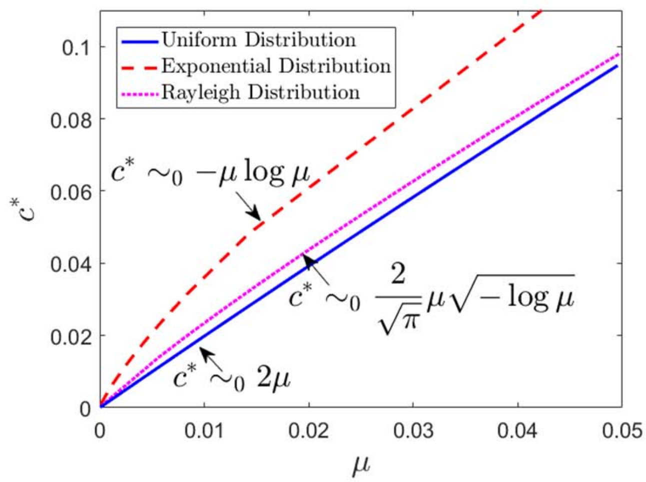
3) Rayleigh Distribution:

$$f_X(x) = \begin{cases} \frac{x}{\theta} e^{-\frac{x^2}{2\theta}}, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

where $\theta > 0$.

We can write (5) equivalently as

$$(1+c^*) \int_0^{c^*} \frac{x e^{-\frac{x^2}{2\theta}}}{\theta(1+x)} \mathrm{d}x = 1.$$



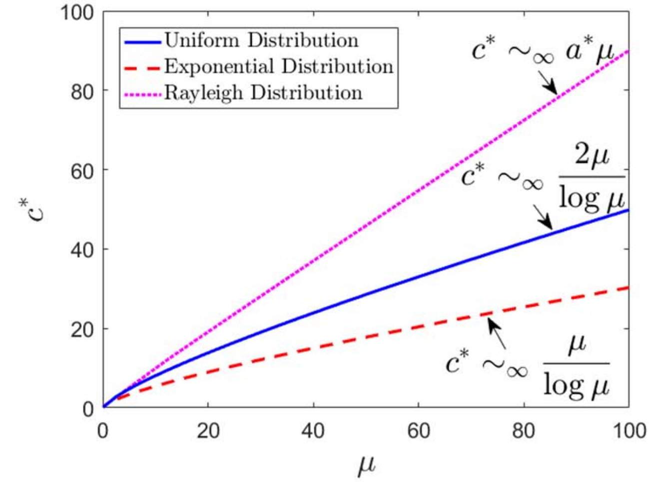


Fig. 5. The relationship between c^* and μ for some continuous distributions.

Note that $\mu = \sqrt{\frac{\pi\theta}{2}}$. For any $a > 0$,

$$\begin{aligned}&(1 + a\mu\sqrt{-\log \mu}) \int_0^{a\mu\sqrt{-\log \mu}} \frac{xe^{-\frac{x^2}{2\theta}}}{\theta(1+x)}\mathrm{d}x \\ &= (1 + a\mu\sqrt{-\log \mu}) \int_0^{a\mu\sqrt{-\log \mu}} \frac{\pi xe^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2(1+x)}\mathrm{d}x \\ &= (1 + a\mu\sqrt{-\log \mu}) \int_0^{a\mu\sqrt{-\log \mu}} \frac{\pi e^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2} \\ &\qquad \times (x - x^2 + o_{x\downarrow 0}(x^2))\mathrm{d}x, \qquad (36)\end{aligned}$$

where $o_{x\downarrow 0}(x^2) = \frac{x^3}{1+x} \le x^3$. It can be verified that

$$\int_0^{a\mu\sqrt{-\log \mu}} \frac{\pi xe^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2}\mathrm{d}x = 1 - \mu^{\frac{\pi a^2}{4}}, \qquad (37)$$

and

$$\begin{aligned}&\int_0^{a\mu\sqrt{-\log \mu}} \frac{\pi x^2 e^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2}\mathrm{d}x \\ &= -a\mu^{\frac{\pi a^2}{4}+1}\sqrt{-\log \mu} + \mu \int_0^{a\sqrt{-\log \mu}} e^{-\frac{\pi y^2}{4}}\mathrm{d}y. \qquad (38)\end{aligned}$$

Substituting (37) and (38) into (36) gives

$$\begin{aligned}&(1 + a\mu\sqrt{-\log \mu}) \int_0^{a\mu\sqrt{-\log \mu}} \frac{xe^{-\frac{x^2}{2\theta}}}{\theta(1+x)}\mathrm{d}x \\ &= 1 + a\mu\sqrt{-\log \mu} - \mu^{\frac{\pi a^2}{4}} + o_{\mu\downarrow 0}(\mu\sqrt{-\log \mu}).\end{aligned}$$

When μ is sufficiently close to 0,

$$\begin{aligned}&1 + a\mu\sqrt{-\log \mu} - \mu^{\frac{\pi a^2}{4}} + o_{\mu\downarrow 0}(\mu\sqrt{-\log \mu}) \\ &\begin{cases} < 1, & a < \frac{2}{\sqrt{\pi}}, \\ > 1, & a > \frac{2}{\sqrt{\pi}}. \end{cases}\end{aligned}$$

Therefore, we must have $c^* \sim_0 \frac{2}{\sqrt{\pi}}\mu\sqrt{-\log \mu}$.

For any $a > 0$,

$$\begin{aligned}&\lim_{\mu\uparrow\infty}(1 + a\mu) \int_0^{a\mu} \frac{xe^{-\frac{x^2}{2\theta}}}{\theta(1+x)}\mathrm{d}x \\ &= \lim_{\mu\uparrow\infty}(1 + a\mu) \int_0^{a\mu} \frac{\pi xe^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2(1+x)}\mathrm{d}x \\ &= \lim_{\mu\uparrow\infty}(1 + a\mu) \int_0^{\log \mu} \frac{\pi xe^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2(1+x)}\mathrm{d}x \\ &\quad + \lim_{\mu\uparrow\infty}(1 + a\mu) \int_{\log \mu}^{a\mu} \frac{\pi xe^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2(1+x)}\mathrm{d}x. \qquad (39)\end{aligned}$$

It can be verified that

$$\begin{aligned}0 &\le \lim_{\mu\uparrow\infty}(1 + a\mu) \int_0^{\log \mu} \frac{\pi xe^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2(1+x)}\mathrm{d}x \\ &\le \lim_{\mu\uparrow\infty}(1 + a\mu) \int_0^{\log \mu} \frac{\pi}{2\mu^2}\mathrm{d}x \\ &= \lim_{\mu\uparrow\infty} \frac{\pi(1 + a\mu)\log \mu}{2\mu^2} \\ &= 0,\end{aligned}$$

which implies

$$\lim_{\mu\uparrow\infty}(1 + a\mu) \int_0^{\log \mu} \frac{\pi xe^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2(1+x)}\mathrm{d}x = 0. \qquad (40)$$

Moreover,

$$\begin{aligned}&\lim_{\mu\uparrow\infty}(1 + a\mu) \int_{\log \mu}^{a\mu} \frac{\pi xe^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2(1+x)}\mathrm{d}x \\ &= \lim_{\mu\uparrow\infty}(1 + a\mu) \int_{\log \mu}^{a\mu} \frac{\pi e^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2}\mathrm{d}x \qquad (41) \\ &= \lim_{\mu\uparrow\infty}(1 + a\mu) \int_{\frac{\log \mu}{\mu}}^{a} \frac{\pi e^{-\frac{\pi y^2}{4}}}{2\mu}\mathrm{d}y \\ &= \frac{\pi a}{2} \int_0^a e^{-\frac{\pi y^2}{4}}\mathrm{d}y, \qquad (42)\end{aligned}$$

where (41) is due to the fact that

$$\frac{\log \mu}{1+\log \mu} \leq \frac{x}{1+x} \leq \frac{a\mu}{1+a\mu}, \quad x \in [\log \mu, a\mu],$$

and

$$\lim_{\mu \uparrow \infty} \frac{\log \mu}{1+\log \mu} = \lim_{\mu \uparrow \infty} \frac{a\mu}{1+a\mu} = 1.$$

Substituting (40) and (42) into (39) gives

$$\lim_{\mu \uparrow \infty} (1+a\mu) \int_0^{a\mu} \frac{x e^{-\frac{x^2}{2\theta}}}{\theta(1+x)} \mathrm{d}x = \frac{\pi a}{2} \int_0^{a} e^{-\frac{\pi y^2}{4}} \mathrm{d}y.$$

Therefore, we must have $c^* \sim_\infty a^* \mu$, where $a^* \approx 0.875$ is the unique positive number satisfying

$$\frac{\pi a^*}{2} \int_0^{a^*} e^{-\frac{\pi y^2}{4}} \mathrm{d}y = 1.$$

We plot c^* against μ in Fig. 5 for the uniform distribution, the exponential distribution, and the Rayleigh distribution, which confirms our asymptotic analysis.

V. Conclusion

We have studied the problem of online power control for battery limited energy harvesting communications. The main finding of this work is that the greedy policy achieves the maximum throughput if and only if the battery capacity is below a certain threshold. It is worth noting that this threshold depends on the distribution of the energy arrival process although the greedy policy itself does not. In fact, there does not exist a positive threshold on the battery capacity below which the greedy policy (or any other universal policy) is throughput-optimal for all energy arrival processes. Nevertheless, as shown in [19], it is possible to define certain weakened notion of universality and optimality, and construct the associated online power control policy. Further progress along this line of research can be found in [30].

The optimality condition for the greedy policy requires that the battery capacity is no greater than the peak of the energy arrival process, which might not be fulfilled in many scenarios where the current energy harvesting technologies are being considered. However, it becomes relevant in certain emerging applications with massive deployment of microscale batteries (e.g., biobatteries and nanobatteries) in energy-dense environments (e.g., living body and radiation-rich outer space). It may even have implications beyond energy harvesting communications, say, explaining the energy consumption patterns of certain biological cells.

It is also worth pointing out that the optimality of the greedy policy in the low-battery-capacity regime is a manifestation of the following more general phenomenon: under mild conditions on the distribution of the energy arrival process, for any battery capacity $c > 0$, the optimal policy f behaves exactly like the greedy policy when the battery energy level is below a certain threshold[9] $\overline{b} \in (0, c]$, i.e., $f(b) = b$ for all $b \in [0, \overline{b}]$. A variant of the proof of Theorem 1 can be used to provide a rigorous explanation of this phenomenon. As such, our work not only gives insight into the greedy policy, but also sheds light on the optimal policy in general.

[9] Here the threshold $\overline{b}$ depends on the battery capacity and the distribution of the energy arrival process. Theorem 1 basically deals with the special case where $\overline{b}$ coincides with c.

Appendix A
Proof of a Statement in Remark 6

We assume $\overline{x} = \infty$ throughout this proof.

First consider the case $r'(\mu - \epsilon) = r'(\overline{x})$ for some $\epsilon > 0$, which implies

$$r'(x) = r'(\overline{x}), \quad x \geq \mu - \epsilon. \tag{43}$$

Note that for $c \geq \mu$,

$$r'(c) = r'(\overline{x}), \tag{44}$$

$$\rho(c) \underline{r'}_{[\underline{x},c]}(\overline{\xi}) \leq r'\left(\frac{\mu - (1-\rho(c))c}{\rho(c)}\right). \tag{45}$$

Moreover, in view of (43) and the fact that

$$\lim_{c \uparrow \overline{x}} \frac{\mu - (1-\rho(c))c}{\rho(c)} = \mu,$$

we have

$$r'\left(\frac{\mu - (1-\rho(c))c}{\rho(c)}\right) = r'(\overline{x}) \tag{46}$$

for all sufficiently large c. Combining (44), (45), and (46) proves $\overline{c} = \infty$.

Next consider the case $r'(\mu) > r'(\overline{x})$. There must exist $\epsilon > 0$ such that $r'(\mu + \epsilon) > r'(\overline{x})$. For $c > \underline{x}$ and $x \in [\underline{x}, \min\{\mu, c\}]$, it is easy to establish the following uniform lower bound:

$$\begin{aligned} &\underline{r'}_{[\underline{x},c]}(x) \\ &\geq \min\left\{\frac{\epsilon}{\mu+\epsilon-\underline{x}} r'(\mu) + \frac{\mu - \underline{x}}{\mu+\epsilon-\underline{x}} r'(\overline{x}), r'(\mu+\epsilon)\right\}. \end{aligned}$$

Clearly, we have $\overline{\xi} \in [\underline{x}, \min\{\mu, c\}]$ for $c > \underline{x}$. Now it can be readily verified that

$$\begin{aligned} &\lim_{c \uparrow \overline{x}} \rho(c) \underline{r'}_{[\underline{x},c]}(\overline{\xi}) \\ &\geq \min\left\{\frac{\epsilon}{\mu+\epsilon-\underline{x}} r'(\mu) + \frac{\mu - \underline{x}}{\mu+\epsilon-\underline{x}} r'(\overline{x}), r'(\mu+\epsilon)\right\} \\ &> r'(\overline{x}), \end{aligned}$$

which implies $\overline{x} < \infty$.

Appendix B
Proof of (14) and (15)

We shall first prove (14). Note that

$$\begin{aligned}
&\phi(g)-\phi(g-\epsilon)\\
&=r(g)-r(g-\epsilon)\\
&\quad+\rho(c-b+g)\mathbb{E}[r(b-g+X)|X<c-b+g]\\
&\quad-\rho(c-b+g-\epsilon)\mathbb{E}[r(b-g+\epsilon+X)\\
&\qquad\qquad|X<c-b+g-\epsilon]\\
&\quad-\mathbb{P}(c-b+g-\epsilon\le X<c-b+g)r(c)\\
&=r(g)-r(g-\epsilon)\\
&\quad+\rho(c-b+g)\mathbb{E}[r(b-g+X)|X<c-b+g]\\
&\quad-\rho(c-b+g)\mathbb{E}[r(b-g+\epsilon+X)|X<c-b+g]\\
&\quad+\rho(c-b+g)\mathbb{E}[r(b-g+\epsilon+X)|X<c-b+g]\\
&\quad-\rho(c-b+g-\epsilon)\mathbb{E}[r(b-g+\epsilon+X)\\
&\qquad\qquad|X<c-b+g-\epsilon]\\
&\quad-\mathbb{P}(c-b+g-\epsilon\le X<c-b+g)r(c)\\
&=r(g)-r(g-\epsilon)\\
&\quad+\rho(c-b+g)\mathbb{E}[r(b-g+X)-r(b-g+\epsilon+X)\\
&\qquad\qquad|X<c-b+g]\\
&\quad+\mathbb{P}(c-b+g-\epsilon\le X<c-b+g)\\
&\quad\times\mathbb{E}[r(b-g+\epsilon+X)-r(c)\\
&\qquad\qquad|c-b+g-\epsilon\le X<c-b+g].
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}(\phi(g)-\phi(g-\epsilon))\\
&=\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}(r(g)-r(g-\epsilon))\\
&\quad+\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}\rho(c-b+g)\mathbb{E}[r(b-g+X)\\
&\qquad\qquad-r(b-g+\epsilon+X)|X<c-b+g]\\
&\quad+\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}\mathbb{P}(c-b+g-\epsilon\le X<c-b+g)\\
&\quad\times\mathbb{E}[r(b-g+\epsilon+X)-r(c)\\
&\qquad\qquad|c-b+g-\epsilon\le X<c-b+g].
\end{aligned}\tag{47}$$

Clearly, we have

$$\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}(r(g)-r(g-\epsilon))=r'(g).\tag{48}$$

Moreover,

$$\begin{aligned}
&\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}\rho(c-b+g)\mathbb{E}[r(b-g+X)\\
&\qquad\qquad-r(b-g+\epsilon+X)|X<c-b+g]\\
&=-\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}\rho(c-b+g)\\
&\quad\times\mathbb{E}\left[\int_0^{\epsilon}r'(b-g+v+X)\mathrm{d}v\,\middle|\,X<c-b+g\right]\\
&=-\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}\int_0^{\epsilon}\rho(c-b+g)\\
&\quad\times\mathbb{E}[r'(b-g+v+X)|X<c-b+g]\mathrm{d}v \qquad (49)\\
&=-\rho(c-b+g)\mathbb{E}[r'(b-g+X)|X<c-b+g], \qquad (50)
\end{aligned}$$

where (49) follows by Fubini's theorem, and (50) is due to the fact that

$$\begin{aligned}
&\lim_{\epsilon\downarrow 0}\rho(c-b+g)\mathbb{E}[r'(b-g+\epsilon+X)|X<c-b+g]\\
&=\rho(c-b+g)\mathbb{E}[r'(b-g+X)|X<c-b+g]
\end{aligned}$$

as a consequence of the monotone convergence theorem. It can also be verified that

$$\begin{aligned}
0&\le\liminf_{\epsilon\downarrow 0}\frac{1}{\epsilon}\mathbb{P}(c-b+g-\epsilon\le X<c-b+g)\\
&\quad\times\mathbb{E}[r(b-g+\epsilon+X)-r(c)\\
&\qquad\qquad|c-b+g-\epsilon\le X<c-b+g]\\
&\le\limsup_{\epsilon\downarrow 0}\frac{1}{\epsilon}\mathbb{P}(c-b+g-\epsilon\le X<c-b+g)\\
&\quad\times\mathbb{E}[r(b-g+\epsilon+X)-r(c)\\
&\qquad\qquad|c-b+g-\epsilon\le X<c-b+g]\\
&\le\limsup_{\epsilon\downarrow 0}\frac{1}{\epsilon}\mathbb{P}(c-b+g-\epsilon\le X<c-b+g)\\
&\quad\times\mathbb{E}[r'(c)(b-g+\epsilon+X-c)\\
&\qquad\qquad|c-b+g-\epsilon\le X<c-b+g]\\
&\le\limsup_{\epsilon\downarrow 0}\mathbb{P}(c-b+g-\epsilon\le X<c-b+g)r'(c)\\
&=0,
\end{aligned}$$

which implies

$$\begin{aligned}
&\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}\mathbb{P}(c-b+g-\epsilon\le X<c-b+g)\\
&\quad\times\mathbb{E}[r(b-g+\epsilon+X)-r(c)\\
&\qquad\qquad|c-b+g-\epsilon\le X<c-b+g]\\
&=0.
\end{aligned}\tag{51}$$

Substituting (48), (50), and (51) into (47) proves (14).

Now we proceed to prove (15). Note that

$$\begin{aligned}
&\phi(g+\epsilon)-\phi(g)\\
&=r(g+\epsilon)-r(g)\\
&\quad+\rho(c-b+g+\epsilon)\mathbb{E}[r(b-g-\epsilon+X)\\
&\qquad\qquad|X<c-b+g+\epsilon]\\
&\quad-\rho(c-b+g)\mathbb{E}[r(b-g+X)|X<c-b+g]\\
&\quad-\mathbb{P}(c-b+g\le X<c-b+g+\epsilon)r(c)\\
&=r(g+\epsilon)-r(g)\\
&\quad+\rho(c-b+g+\epsilon)\mathbb{E}[r(b-g-\epsilon+X)\\
&\qquad\qquad|X<c-b+g+\epsilon]\\
&\quad-\rho(c-b+g)\mathbb{E}[r(b-g-\epsilon+X)|X<c-b+g]\\
&\quad+\rho(c-b+g)\mathbb{E}[r(b-g-\epsilon+X)|X<c-b+g]\\
&\quad-\rho(c-b+g)\mathbb{E}[r(b-g+X)|X<c-b+g]\\
&\quad-\mathbb{P}(c-b+g\le X<c-b+g+\epsilon)r(c)\\
&=r(g+\epsilon)-r(g)\\
&\quad+\rho(c-b+g)\mathbb{E}[r(b-g-\epsilon+X)-r(b-g+X)\\
&\qquad\qquad|X<c-b+g]\\
&\quad+\mathbb{P}(c-b+g\le X<c-b+g+\epsilon)
\end{aligned}$$

$$
\begin{aligned}
&\quad \times \mathbb{E}[r(b-g-\epsilon+X)-r(c) \\
&\qquad\qquad |c-b+g \le X < c-b+g+\epsilon] \\
&= r(g+\epsilon)-r(g) \\
&\quad + \rho(c-b+g)\mathbb{E}[r(b-g-\epsilon+X)-r(b-g+X) \\
&\qquad\qquad |X < c-b+g] \\
&\quad + \mathbb{P}(c-b+g < X < c-b+g+\epsilon) \\
&\quad \times \mathbb{E}[r(b-g-\epsilon+X)-r(c) \\
&\qquad\qquad |c-b+g < X < c-b+g+\epsilon] \\
&\quad + \mathbb{P}(X = c-b+g)(r(c-\epsilon)-r(c)).
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}(\phi(g+\epsilon)-\phi(g)) \\
&= \lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}(r(g+\epsilon)-r(g)) \\
&\quad + \lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}\rho(c-b+g)\mathbb{E}[r(b-g-\epsilon+X) \\
&\qquad - r(b-g+X)|X < c-b+g] \\
&\quad + \lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}\mathbb{P}(c-b+g < X < c-b+g+\epsilon) \\
&\quad \times \mathbb{E}[r(b-g-\epsilon+X)-r(c) \\
&\qquad |c-b+g < X < c-b+g+\epsilon] \\
&\quad + \lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}\mathbb{P}(X = c-b+g)(r(c-\epsilon)-r(c)). \qquad (52)
\end{aligned}
$$

Similarly to (48), (50), and (51), we have

$$
\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}(r(g+\epsilon)-r(g)) = r'(g), \qquad (53)
$$

$$
\begin{aligned}
&\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}\rho(c-b+g)\mathbb{E}[r(b-g-\epsilon+X) \\
&\qquad - r(b-g+X)|X < c-b+g] \\
&= -\rho(c-b+g)\mathbb{E}[r'(b-g+X)|X < c-b+g], \qquad (54)
\end{aligned}
$$

$$
\begin{aligned}
&\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}\mathbb{P}(c-b+g < X < c-b+g+\epsilon) \\
&\quad \times \mathbb{E}[r(b-g-\epsilon+X)-r(c) \\
&\qquad |c-b+g < X < c-b+g+\epsilon] \\
&= 0. \qquad (55)
\end{aligned}
$$

Moreover,

$$
\begin{aligned}
&\lim_{\epsilon\downarrow 0}\frac{1}{\epsilon}\mathbb{P}(X = c-b+g)(r(c-\epsilon)-r(c)) \\
&= -\mathbb{P}(X = c-b+g)r'(c). \qquad (56)
\end{aligned}
$$

Substituting (53), (54), (55), and (56) into (52) proves (15).

APPENDIX C
PROOF OF (29), (30), AND (31)

We have

$$
\begin{aligned}
&\mathbb{P}(\lceil \mu - \mu^{\frac{2}{3}} \rceil \le X \le \lfloor \mu + \mu^{\frac{2}{3}} \rfloor) \\
&\ge \mathbb{P}(|X-\mu| < \mu^{\frac{2}{3}}) \\
&= 1 - \mathbb{P}(|X-\mu| \ge \mu^{\frac{2}{3}}) \\
&\ge 1 - \frac{\mathbb{E}[(X-\mu)^2]}{\mu^{\frac{4}{3}}} \qquad (57) \\
&= 1 - \mu^{-\frac{1}{3}},
\end{aligned}
$$

where (57) is due to Chebyshev's inequality. Therefore,

$$
\lim_{\mu\uparrow\infty}\mathbb{P}(\lceil \mu - \mu^{\frac{2}{3}} \rceil \le X \le \lfloor \mu + \mu^{\frac{2}{3}} \rfloor) = 1.
$$

Now we are in a position to prove (29). It can be verified that

$$
\begin{aligned}
&\lim_{\mu\uparrow\infty}(1+a\mu)\sum_{k=\lceil \mu-\mu^{\frac{2}{3}}\rceil}^{\lfloor \mu+\mu^{\frac{2}{3}}\rfloor}\frac{e^{-\lambda}\lambda^k}{(1+k)(k!)} \\
&= \lim_{\mu\uparrow\infty} a\sum_{k=\lceil \mu-\mu^{\frac{2}{3}}\rceil}^{\lfloor \mu+\mu^{\frac{2}{3}}\rfloor}\frac{e^{-\lambda}\lambda^k}{k!} \qquad (58) \\
&= \lim_{\mu\uparrow\infty} a\mathbb{P}(\lceil \mu - \mu^{\frac{2}{3}} \rceil \le X \le \lfloor \mu + \mu^{\frac{2}{3}} \rfloor) \\
&= a,
\end{aligned}
$$

where (58) is due to the fact that

$$
\frac{1+a\mu}{1+\lfloor \mu+\mu^{\frac{2}{3}}\rfloor} \le \frac{1+a\mu}{1+k} \le \frac{1+a\mu}{1+\lceil \mu-\mu^{\frac{2}{3}}\rceil}, \quad k = \lceil \mu-\mu^{\frac{2}{3}}\rceil, \cdots, \lfloor \mu+\mu^{\frac{2}{3}}\rfloor,
$$

and

$$
\lim_{\mu\uparrow\infty}\frac{1+a\mu}{1+\lfloor \mu+\mu^{\frac{2}{3}}\rfloor} = \lim_{\mu\uparrow\infty}\frac{1+a\mu}{1+\lceil \mu-\mu^{\frac{2}{3}}\rceil} = a.
$$

This proves (29).

Next we proceed to prove (30). It can be verified that

$$
\begin{aligned}
&\lim_{\mu\uparrow\infty}(1+a\mu)\sum_{k=\lfloor \mu+\mu^{\frac{2}{3}}\rfloor+1}^{\infty}\frac{e^{-\lambda}\lambda^k}{(1+k)(k!)} \\
&\le \lim_{\mu\uparrow\infty} a\sum_{k=\lfloor \mu+\mu^{\frac{2}{3}}\rfloor+1}^{\infty}\frac{e^{-\lambda}\lambda^k}{k!} \qquad (59) \\
&= \lim_{\mu\uparrow\infty} a\mathbb{P}(X \ge \lfloor \mu + \mu^{\frac{2}{3}} \rfloor + 1) \\
&\le \lim_{\mu\uparrow\infty} a(1-\mathbb{P}(\lceil \mu - \mu^{\frac{2}{3}} \rceil \le X \le \lfloor \mu + \mu^{\frac{2}{3}} \rfloor)) \\
&= 0,
\end{aligned}
$$

where is (59) due to the fact that

$$
\frac{1+a\mu}{1+k} \le \frac{1+a\mu}{\lfloor \mu+\mu^{\frac{2}{3}}\rfloor+2}, \quad k \ge \lfloor \mu+\mu^{\frac{2}{3}}\rfloor+1,
$$

and

$$
\lim_{\mu\uparrow\infty}\frac{1+a\mu}{\lfloor \mu+\mu^{\frac{2}{3}}\rfloor+2} = a.
$$

This proves (30).

Finally, we shall prove (31). Note that

$$\begin{aligned}
&\lim_{\mu\uparrow\infty}(1+a\mu)\sum_{k=0}^{\lceil\mu-\mu^{\frac{2}{3}}\rceil-1}\frac{e^{-\lambda}\lambda^k}{(1+k)(k!)}\\
&=\lim_{\mu\uparrow\infty}(1+a\mu)\sum_{k=0}^{\lceil\mu-\mu^{\frac{2}{3}}\rceil-1}\frac{e^{-\mu}\mu^k}{(1+k)(k!)}\\
&\le\lim_{\mu\uparrow\infty}(1+a\mu)\sum_{k=0}^{\lfloor\mu-\mu^{\frac{2}{3}}\rfloor-1}\frac{e^{-\mu}\mu^k}{k!}\\
&\le\lim_{\mu\uparrow\infty}(1+a\mu)\lfloor\mu-\mu^{\frac{2}{3}}\rfloor\frac{e^{-\mu}\mu^{\lfloor\mu-\mu^{\frac{2}{3}}\rfloor}}{\lfloor\mu-\mu^{\frac{2}{3}}\rfloor!},
\end{aligned}\tag{60}$$

where (60) is due to the fact that

$$\frac{\mu^k}{k!}\le\frac{\mu^{\lfloor\mu-\mu^{\frac{2}{3}}\rfloor}}{\lfloor\mu-\mu^{\frac{2}{3}}\rfloor!},\quad k=0,1,\cdots,\lfloor\mu-\mu^{\frac{2}{3}}\rfloor-1.$$

Let $\delta\triangleq\frac{\mu-\lfloor\mu-\mu^{\frac{2}{3}}\rfloor}{\mu}$. We have

$$\begin{aligned}
&\lim_{\mu\uparrow\infty}(1+a\mu)\lfloor\mu-\mu^{\frac{2}{3}}\rfloor\frac{e^{-\mu}\mu^{\lfloor\mu-\mu^{\frac{2}{3}}\rfloor}}{\lfloor\mu-\mu^{\frac{2}{3}}\rfloor!}\\
&=\lim_{\mu\uparrow\infty}(1+a\mu)\mu(1-\delta)\frac{e^{-\mu}\mu^{\mu(1-\delta)}}{(\mu(1-\delta))!}\\
&=\lim_{\mu\uparrow\infty}(1+a\mu)\mu(1-\delta)\frac{e^{-\mu\delta}(1-\delta)^{-\mu(1-\delta)-\frac{1}{2}}}{\sqrt{2\pi\mu}},
\end{aligned}\tag{61}$$

where (61) follows by Stirling's approximation $(\mu(1-\delta))!\sim_\infty\sqrt{2\pi\mu(1-\delta)}e^{-\mu(1-\delta)}(\mu(1-\delta))^{\mu(1-\delta)}$. Since

$$\begin{aligned}
&\log((1-\delta)^{\mu(1-\delta)+\frac{1}{2}})\\
&=(\mu(1-\delta)+\frac{1}{2})\log(1-\delta)\\
&=(\mu(1-\delta)+\frac{1}{2})(-\delta-\frac{\delta^2}{2}+o_{\delta\downarrow 0}(\delta^2))\\
&=-\mu\delta+\frac{\mu\delta^2}{2}+o_{\mu\uparrow\infty}(\mu^{\frac{1}{3}}),
\end{aligned}$$

it follows that

$$(1-\delta)^{-\mu(1-\delta)-\frac{1}{2}}=e^{\mu\delta-\frac{\mu\delta^2}{2}+o_{\mu\uparrow\infty}(\mu^{\frac{1}{3}})}.\tag{62}$$

Substituting (62) into (61) and taking the limit gives

$$\begin{aligned}
&\lim_{\mu\uparrow\infty}(1+a\mu)\lfloor\mu-\mu^{\frac{2}{3}}\rfloor\frac{e^{-\mu}\mu^{\lfloor\mu-\mu^{\frac{2}{3}}\rfloor}}{\lfloor\mu-\mu^{\frac{2}{3}}\rfloor!}\\
&=\lim_{\mu\uparrow\infty}(1+a\mu)\mu(1-\delta)\frac{e^{-\frac{\mu\delta^2}{2}+o_{\mu\uparrow\infty}(\mu^{\frac{1}{3}})}}{\sqrt{2\pi\mu}}\\
&=0,
\end{aligned}$$

which, together with (60), proves (31).

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