Robust Coding Schemes for Distributed Sensor Networks with Unreliable Sensors

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Abstract — **We consider a distributed sensor network in which several observations are communicated to the fusion center using limited transmission rate. The observations must be separately coded. We introduce a class of robust distributed coding schemes which flexibly trade off between system robustness and compression efficiency.**

I. Introduction

Consider the distributed sensor network shown in Fig. 1. $\{X(t)\}_{t=1}^{\infty}$ is the target data sequence which the fusion center tries to recover. This data sequence may not be observed directly. Corrupted versions of $\{X(t)\}_{t=1}^{\infty}$ are separately coded by 2 sensors. The data rate at which Sensor i $(i = 1, 2)$ may transmit information about its observations is limited to R_i bits per second. The sensors are not permitted to communicate with each other; i.e., Sensor i has to send data based solely on its own noisy observations $\{Y_i(t)\}_{t=1}^{\infty}$.

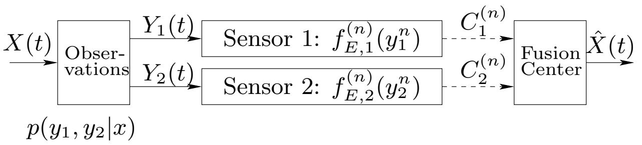

Figure 1: Model of distributed sensor network with unreliable sensors

Let $\{X(t), Y_1(t), Y_2(t)\}_{t=1}^{\infty}$ be temporally memoryless source with instantaneous joint probability distribution $P(x, y_1, y_2)$ on $\mathcal{X} \times \mathcal{Y}_1 \times \mathcal{Y}_2$, where $\mathcal{X}$ is the common alphabet of the random variables $X(t)$ for $t = 1, 2, \cdots$, $\mathcal{Y}_i$ $(i = 1, 2)$ is the common alphabet of the random variables $Y_i(t)$ for $t = 1, 2, \cdots$. If Sensor i is able to function, then it encodes a block $y_i^n = [y_i(1), \cdots, y_i(n)]$ of length n from its observed data using a source code $c_i^{(n)} = f_{E,i}^{(n)}(y_i^n)$ of rate $\frac{1}{n}\log|\mathcal{C}_i^{(n)}|$. If the fusion center only receives the data from Sensor i, then it tries to recover the target sequence $x^n = [x(1), \cdots, x(n)]$ by implementing a mapping $f_{D,i}: \ \mathcal{C}_i \to \mathcal{X}^n \quad (i = 1, 2)$. If the fusion center receives the data from both sensors, then it tries to recover the target sequence by implementing a mapping $f_{D,3}: \ \mathcal{C}_1^{(n)} \times \mathcal{C}_2^{(n)} \to \mathcal{X}^n$.

This model subsumes the multiple description problem [1]and the CEO problem [2], and was first studied in [3].

Definition 1 *The quintuple* $(R_1, R_2, D_1, D_2, D_3)$ *is called achievable, if* $\forall \varepsilon > 0$, $\exists n_0$ *such that* $\forall n > n_0$ *there exist encoding functions:*

$$f_{E,i}^{(n)} : \mathcal{Y}_i^{(n)} \to \mathcal{C}_i^{(n)} \quad \log|\mathcal{C}_i^{(n)}| \le n(R_i + \varepsilon) \quad i = 1, 2$$

and decoding functions:

$$f_{D,i}: \ \mathcal{C}_i^{(n)} \to \mathcal{X}^n \quad i = 1, 2$$

$$f_{D,3}: \ \mathcal{C}_1^{(n)} \times \mathcal{C}_2^{(n)} \to \mathcal{X}^n$$

such that for $\hat{X}_i^n = f_{D,i}(f_{E,i}(Y_i^n)) \quad (i = 1, 2)$, *and for* $\hat{X}_3^n = f_{D,3}(f_{E,1}(Y_i^n), f_{E,1}(Y_i^n))$,

$$\frac{1}{n} E \sum_{t=1}^{n} d(X(t), \hat{X}_i(t)) < D_i + \varepsilon \quad i = 1, 2, 3.$$

Here $d(\cdot, \cdot) : \mathcal{X} \times \mathcal{X} \to [0, d_{max}]$ *is a given distortion measure.*
Let $\mathcal{Q}$ *denote the set of all achievable quintuples.*

II. Main Results

Theorem 1 $(R_1, R_2, D_1, D_2, D_3)$ *is achievable, if there exist random variables* $(W_{1,1}, W_{1,2}, W_{2,1}, W_{2,2})$ *jointly distributed with the generic source variables* (X, Y_1, Y_2) *such that the following properties are satisfied:*

(i) $(W_{1,1}, W_{1,2}) \to Y_1 \to (X, Y_2, W_{2,1}, W_{2,2})$,
$(W_{2,1}, W_{2,2}) \to Y_2 \to (X, Y_1, W_{1,1}, W_{1,2})$;

(ii)

$$\begin{aligned} R_1 &\ge I(Y_1; W_{1,1}) + I(Y_1; W_{1,2}|W_{1,1}, W_{2,1}, W_{2,2}), \\ R_2 &\ge I(Y_2; W_{2,1}) + I(Y_2; W_{2,2}|W_{1,1}, W_{2,1}, W_{1,2}), \\ R_1 + R_2 &\ge I(Y_1; W_{1,1}) + I(Y_2; W_{2,1}) \\ &\quad + I(Y_1, Y_2; W_{1,2}, W_{2,2}|W_{1,1}, W_{2,1}); \end{aligned}$$

(iii) There exist functions:

$$f_i: \ \mathcal{W}_{i,1} \to \mathcal{X} \quad i = 1, 2,$$

$$f_3: \ \mathcal{W}_{1,1} \times \mathcal{W}_{1,2} \times \mathcal{W}_{2,1} \times \mathcal{W}_{2,2} \to \mathcal{X},$$

such that $Ed(X, \hat{X}_i) \le D_i \quad (i = 1, 2, 3)$, *where* $\hat{X}_1 = f_1(W_{1,1})$, $\hat{X}_2 = f_2(W_{2,1})$ *and* $\hat{X}_3 = f_3(W_{1,1}, W_{1,2}, W_{2,1}, W_{2,2})$.

If $\mathcal{C}$ *denotes the set of these achievable quintuples, then time sharing yields that* $conv(\mathcal{C})$ *is also an achievable region.*

References

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