Electrical Engineering EE3TR4

Day Class
Duration of Examination: 3 Hours
McMaster University Final Examination

Instructor: Dr. J. Reilly
April, 2009

This examination paper includes 6 pages and 6 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions

(a) The McMaster Standard Calculator (Casio FX991) is the only calculator approved for this exam. **No other aids are permitted**.

(b) There are 6 questions. A full paper consists of all six.

(c) You must show your work for full marks.

(d) All major questions are of equal weight.

(e) **Make sure you read the entire exam over in its entirety before you start!**

(f) The tables of Fourier transforms, trigonometric identities and the erf(·) function at the back of this exam may be useful.

(g) Good luck on this exam and have a great summer!

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1. Consider the system shown in Fig. 1. The cutoff frequency f_c of the ideal low–pass filter $H(f)$ satisfies $f_c > f_o$, where f_o is the frequency of the input sinusoid. The passband gain of $H(f)$ is one. The noise $n(t)$ is white and has a power spectral density (PSD) $S_n = N_o/2$. Give an expression for the autocorrelation $R_x(\tau)$ and the power spectral density $S_x(f)$ of the output $x(t)$. *Hint:* The noise $n(t)$ and any sinusoid are uncorrelated.

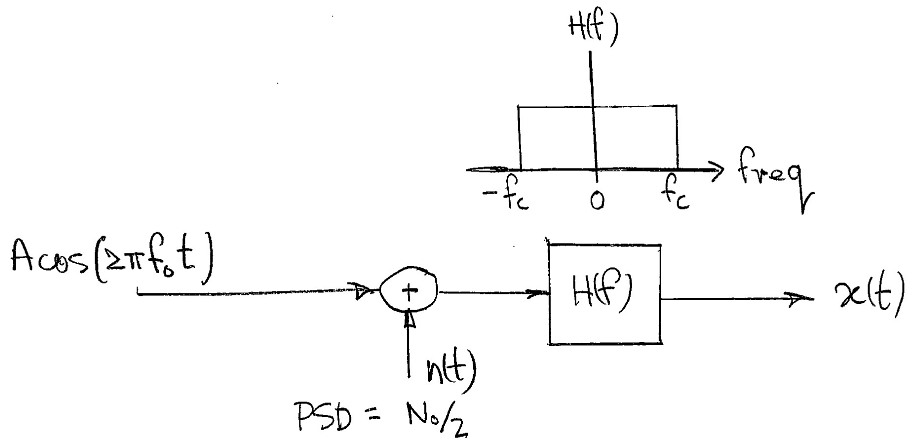

Figure 1: Figure for Question 1.

2. A block diagram of a digital transmission system is shown in Fig. 2. The signal $b(t)$ is a binary sequence of 1's and 0's with a bit rate of 64 Kbits/sec. The system sends one bit per symbol duration (T secs). The pulse shape corresponding to a single bit is rectangular.

a) Sketch the power spectral density of the signal $a(t)$ showing all relevant values. (4 marks)

b) Sketch the magnitude frequency response corresponding to the transmitter filter $G(f)$ if the signal $g(t)$ is to occupy the minimum possible bandwidth with zero intersymbol interference (ISI). Show all relevant values. (3 marks)

c) Explain how to transmit this bit stream in a bandwidth of 16 KHz. (3 marks)

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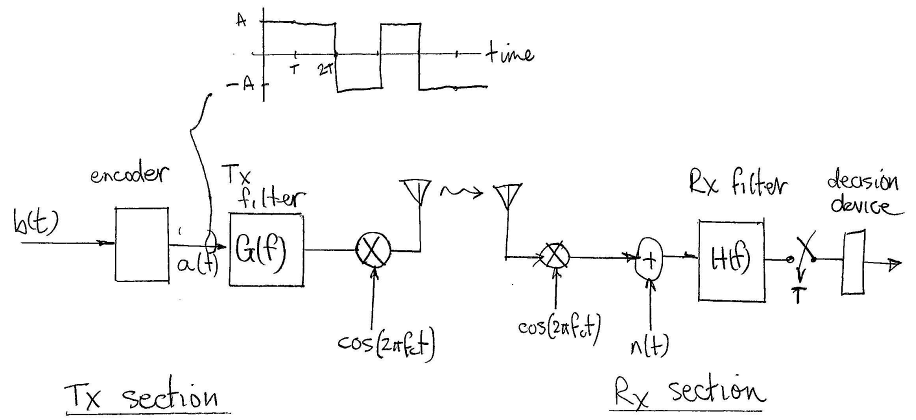

Figure 2: A block diagram of a digital transmission system.

3. a) Find the inverse Fourier transform $g(t)$ corresponding to the spectrum $G(f)$ shown below in Fig. 3. (8 marks)

b) What is the minimum sampling rate of the corresponding signal $g(t)$ so that $g(t)$ can be reconstructed from its samples without distortion? (2 marks)

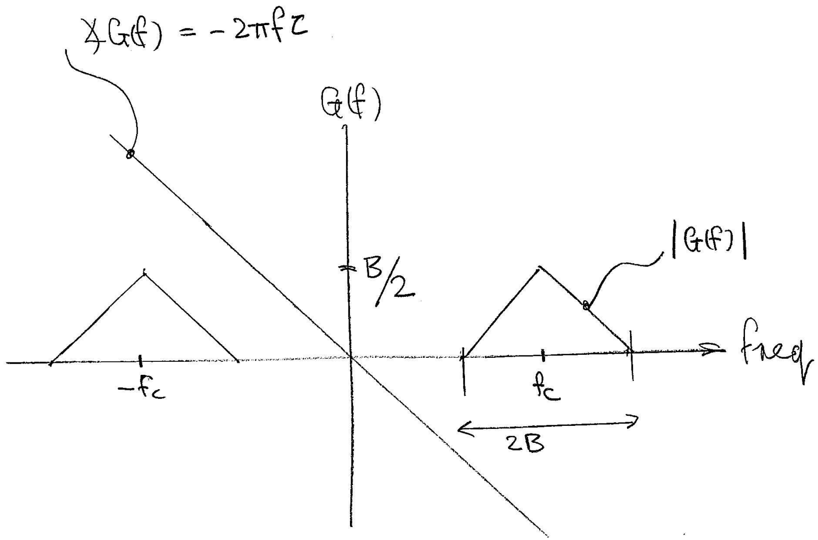

Figure 3: The spectrum $G(f)$ for Question 3.

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4. We wish to transmit two baseband messages (i.e., messages whose spectra are centred at 0 Hz) each with a spectrum extending up to W Hz.

a) Draw a block diagram of a modulation system that can transmit both these messages using a total bandwidth of $2W$ Hz. (8 marks)

b) explain a possible difficulty that is encountered with this approach. (2 marks)

5. Consider the digital transmission system of Question 2. For the purposes of this question, the filter response $G(f) = 1$ for all values of f. The transmission system transmits one bit per symbol. The pulse shape at the receiver corresponding to a single bit is rectangular, with duration T seconds.

a) what is the impulse response of the receiver filter $H(f)$ in this case that minimizes the probability of error (i.e., the *bit error rate*)? (4 marks)

b) what is the probability of a bit error corresponding to this choice of filter? The value of T is the same as that in Question 2. The noise $n(t)$ is white with a power spectral density level of $N_o = 1\mu$W/Hz, and the amplitude A of the pulse at the input to the decision device is 0.7589V *Hint:* Refer to the erf($\cdot$) tables at the end of this exam. (6 marks)

6. A DSB/SC modulation system has a sinusoidal message signal $m(t) = A_m \cos(2\pi f_m t)$. The modulated signal $s(t)$ is then multiplied by the sinusoid $\cos(2\pi \frac{f_c}{2} t)$ as shown in the figure below. Sketch the spectrum of the output $x(t)$ showing all relevant values.

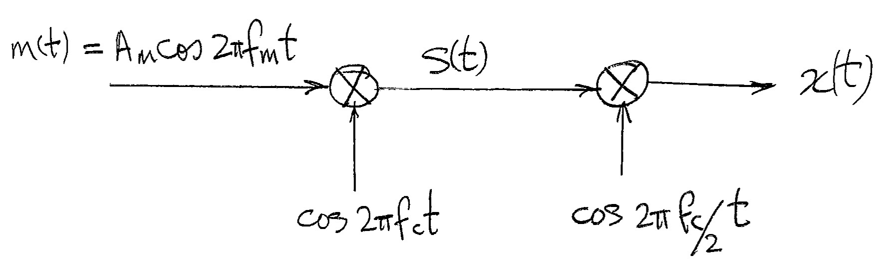

Figure 4: The modulation scheme for Question 6.

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Fourier Transform Pairs

Time Function	Fourier Transform
$\mathrm{rect}\left(\frac{t}{T}\right)$	$\mathrm{Tsinc}\big(\mathrm{fT}\big)$
$\mathrm{sinc}\big(2Wt\big)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(2\pi f_c t)$	$\delta(f - f_c)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\|t\|), \quad a > 0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_c t)$	$\frac{1}{2}\big[\delta(f - f_c) + \delta(f + f_c)\big]$

Trigonometric Identities

$$\cos(\theta) = \tfrac{1}{2}\big[\exp(j\theta) + \exp(-j\theta)\big]$$
$$\sin(\theta) = \tfrac{1}{2j}\big[\exp(j\theta) - \exp(-j\theta)\big]$$
$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\cos^2(\theta) - \sin^2(\theta) = \cos(2\theta)$$
$$\cos^2(\theta) = \tfrac{1}{2}\big[1 + \cos(2\theta)\big]$$
$$2\sin(\theta)\cos(\theta) = \sin(2\theta)$$
$$\sin(\alpha)\sin(\beta) = \tfrac{1}{2}\big[\cos(\alpha-\beta) - \cos(\alpha+\beta)$$
$$\cos(\alpha)\cos(\beta) = \tfrac{1}{2}\big[\cos(\alpha-\beta) + \cos(\alpha+\beta)$$
$$\sin(\alpha)\cos(\beta) = \tfrac{1}{2}\big[\sin(\alpha-\beta) + \sin(\alpha+\beta)$$

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The erf(·) function

- **Note: The erfc(·) function is related to the erf(·) function by erfc(u) $= 1 -$ erf(u).**
- **For those more familiar with the $Q(\cdot)$ function as used in the 2nd edition of the text, we have $Q(u) = \frac{1}{2}\text{erfc}(\frac{u}{\sqrt{2}})$.**

TABLE 6 Error Function

u	erf(u)	u	erf(u)
0.00	0.00000	1.10	0.88021
0.05	0.05637	1.15	0.89612
0.10	0.11246	1.20	0.91031
0.15	0.16800	1.25	0.92290
0.20	0.22270	1.30	0.93401
0.25	0.27633	1.35	0.94376
0.30	0.32863	1.40	0.95229
0.35	0.37938	1.45	0.95970
0.40	0.42839	1.50	0.96611
0.45	0.47548	1.55	0.97162
0.50	0.52050	1.60	0.97635
0.55	0.56332	1.65	0.98038
0.60	0.60386	1.70	0.98379
0.65	0.64203	1.75	0.98667
0.70	0.67780	1.80	0.98909
0.75	0.71116	1.85	0.99111
0.80	0.74210	1.90	0.99279
0.85	0.77067	1.95	0.99418
0.90	0.79691	2.00	0.99532
0.95	0.82089	2.50	0.99959
1.00	0.84270	3.00	0.99998
1.05	0.86244	3.30	0.999998

The *error function* erf(u) is defined by

$$\text{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-z^2)\, dz$$

Figure 5:

The End.