Electrical Engineering EE3TR4

Day Class
Duration of Examination: 3 Hours
McMaster University Final Examination

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April, 2012

This examination paper includes 4 pages and 6 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions

(a) The McMaster Standard Calculator (Casio FX991) is the only calculator approved for this exam. **No other aids are permitted**.

(b) There are 6 questions. A full paper consists of all six.

(c) You must show your work for full marks.

(d) All major questions are of equal weight (10 marks).

(e) **Make sure you read the entire exam over in its entirety before you start!**

(f) The tables of Fourier transforms, trigonometric identities and the $Q(\cdot)$ function at the back of this exam may be useful.

(g) Good luck on this exam and have a great summer!

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1. Consider a commercial AM broadcasting system, where in a particular region we have 5 independent transmitters (stations), each simultaneously transmitting an AM-modulated signal, each with a total bandwidth of 10kHz. These 5 transmitted signals are stacked side-by-side in frequency from 1.00Mhz (1×10^6 Hz) to 1.05 Mhz.

Discuss a practical receiver structure that can select any specified station of interest from amongst the 5 that are simultaneously transmitted. Include as many numerical values as possible from the information given.

2. The autocorrelation function $R_X(\tau)$ of a random process is given as

$$R_X(\tau) = 10\delta(\tau) + cos(2\pi 10\tau). \tag{1}$$

a) Describe as fully as possible the characteristics of a sample $x(t)$ of the random process. (8 marks)

b) Sketch the power spectral density of the process (2 marks).

3. A signal $g(t) = \text{sinc}^2(Wt)$ is sampled at a frequency of $2W$ samples per second. Sketch the spectrum corresponding to the sampled signal, showing all relevant values.

4. We have a bandwidth of 20 Mhz (20×10^6 Hz) available to transmit a digital signal. We wish to transmit at a rate 10Mbits/sec, where we are using a raised-cosine pulse-shaping filter.

a) What value of the parameter α would you use and why? (5 marks)

b) Given that the power spectral density of the noise has a value 0.1μWatts per Hz., and the received signal has a level of 1 mV, what is the probability of error? Use the approximation

$$Q(a) \simeq \frac{1}{\sqrt{2\pi}\, a} \exp\left\{-\frac{a^2}{2}\right\}.$$

If you are used to working with the erfc$(\cdot)$ function instead of $Q(\cdot)$, then $Q(a) = \frac{1}{2}\text{erfc}(\frac{a}{\sqrt{2}})$. (5 marks)

5. The sinusoidal signal $m(t) = \cos(2\pi 1000t)$ is applied to an FM modulator with a modulation sensitivity of $k_f = 5000$ Hz/Volt.

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a) Sketch the spectrum of the FM-modulated signal showing all relevant values. Assume $f_c = 10$Mhz. You may find the table at the end of this exam sheet useful. (7 marks)

b) What is the instantaneous frequency of the wave at $t = 0.25$ msecs.? (3 marks)

6. A zero–mean random process $\mathcal{X}$ has an autocorrelation function $R_X(\tau) = 5 \times 10^{-4}\text{sinc}(200\tau)$. It is applied to an ideal low–pass filter with a gain of one in the passband and a cutoff frequency of 10 Hz. Give the power spectral density (4 marks), the autocorrelation function (4 marks), and the variance (2 marks) of the random process at the output of the filter.

Fourier Transform Pairs

Time Function	**Fourier Transform**
$\text{rect}\left(\frac{t}{T}\right)$	$\text{Tsinc}\big(\text{fT}\big)$
$\text{sinc}\big(2Wt\big)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(2\pi f_c t)$	$\delta(f - f_c)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\|t\|), \quad a > 0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_c t)$	$\frac{1}{2}\big[\delta(f - f_c) + \delta(f + f_c)\big]$

Trigonometric Identities

$$\cos(\theta) = \tfrac{1}{2}\big[\exp(j\theta) + \exp(-j\theta)\big]$$
$$\sin(\theta) = \tfrac{1}{2j}\big[\exp(j\theta) - \exp(-j\theta)\big]$$
$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\cos^2(\theta) - \sin^2(\theta) = \cos(2\theta)$$
$$\cos^2(\theta) = \tfrac{1}{2}\big[1 + \cos(2\theta)\big]$$
$$2\sin(\theta)\cos(\theta) = \sin(2\theta)$$
$$\sin(\alpha)\sin(\beta) = \tfrac{1}{2}\big[\cos(\alpha - \beta) - \cos(\alpha + \beta)$$
$$\cos(\alpha)\cos(\beta) = \tfrac{1}{2}\big[\cos(\alpha - \beta) + \cos(\alpha + \beta)$$
$$\sin(\alpha)\cos(\beta) = \tfrac{1}{2}\big[\sin(\alpha - \beta) + \sin(\alpha + \beta)$$

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The $Q(\cdot)$ function

The probability that a zero-mean Gaussian random variable x with a variance of 1 will exceed a value of a is given by the function $Q(a)$; *i.e.*, $P(x > a) = Q(a) = \frac{1}{\sqrt{2\pi}} \int_a^\infty \exp(-\frac{x^2}{2}) dx$. Below is a table of various values of Q.

a	Q(a)
1.5000	0.0668
1.6000	0.0548
1.7000	0.0446
1.8000	0.0359
1.9000	0.0287
2.0000	0.0228
2.1000	0.0179
2.2000	0.0139
2.3000	0.0107
2.4000	0.0082
2.5000	0.0062
2.6000	0.0047
2.7000	0.0035
2.8000	0.0026
2.9000	0.0019
3.0000	0.0013

The End.